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HEALTH SUBSIDIES, PREVENTION AND WELFARE

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Health subsidies, prevention and welfare *

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Abstract

Health subsidies involve public budgetary costs. However, they generate a positive externality by encouraging participation in health-improving initiatives, which help reduce future health care costs. We build an overlapping generations model with a government subsidizing investment in health by the young generation and paying the health care costs of the old generation. We find that the welfare-maximizing subsidy rate depends positively on health externality and the size of health care costs, and negatively on the discount factor. The subsidy rate should therefore be high when prevention more effective at cost saving and when the population is myopic about the future. Moreover, the welfare-maximizing subsidy rate is lower than the health-maximizing rate but higher than the capital-maximizing rate.

Keywords: Overlapping generations model, health subsidy, welfare.

JEL-Code: H23, I18, O41.

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Résumé non-technique

Dans la plupart des pays développés, les dépenses liées aux soins de santé sont importantes. Ainsi, dans l'Union Européenne, les dépenses totales en soins de santé s'élevaient à 10,1 % du PIB en 2015, dont 80% (soit environ 8 % du PIB) étaient à charge du secteur public. Par ailleurs, les dépenses en soins de santé sont relativement plus importantes pour les personnes âgées, du fait que leurs traitements sont plus onéreux et généralement plus longs. C'est pourquoi il peut être dans l'intérêt public d'encourager les actions préventives (dépistages, vaccinations, etc.) afin de diminuer les coûts curatifs qui devraient être mis en oeuvre plus tard. Si les prix élevés de certains soins préventifs peuvent décourager leur utilisation, les subventions publiques (par exemple à travers le remboursement partiel de ces soins) pourraient cependant être un moyen d'encourager la participation. En d'autres termes, 1 euro de subside payé aujourd'hui par l'Etat visant à encourager les soins préventifs peut améliorer la santé d'un individu et permettre ainsi à ce même Etat d'économiser demain en soins curatifs. Le but de ce papier est de déterminer le taux optimal de subvention afin de maximiser le bien-être d'un agent représentatif.

Pour ce faire, nous construisons un modèle d'équilibre général à générations imbriquées. Dans notre modèle, un individu vit pendant deux périodes. Lorsque l'individu est jeune, il utilise ses revenus pour consommer, épargner ou bien investir dans sa santé (soins préventifs). Ces soins préventifs peuvent être en partie subventionnés par le gouvernement. Lorsque l'individu est âgé, il utilise son épargne pour consommer et son investissement en soins préventifs lui permet d'améliorer son état de santé et donc son bien-être. Un individu âgé en bonne santé coutera également moins cher à l'Etat en soins curatifs. Le budget de l'Etat est en équilibre à chaque période, et il finance les soins préventifs (partiellement selon le taux de subvention) et les soins curatifs à travers une taxe sur les revenus du travail, qui engendrera par ailleurs des distorsions dans l'économie.

Une hausse du taux de subvention affecte l'économie via différents canaux. Premièrement, pour un niveau de dépenses en soins préventifs donné, une hausse du taux de subvention est couteuse pour les finances publiques et le gouvernement devra donc augmenter la taxe sur les revenus du travail. Deuxièmement, à revenus donnés, une hausse du taux de subvention va stimuler l'utilisation de soins préventifs au détriment de la consommation et de l'épargne.

Troisièmement, l'utilisation de soins préventifs va améliorer l'état de santé et diminuer les soins curatifs, donc réduire les dépenses publiques plus tard. Ce canal, que l'on nomme 'health externality channel' dans notre papier, est évidemment crucial pour nos résultats. Une externalité forte signifie que la prévention est efficace et/ou que la prévention est dirigée vers des pathologies spécifiques qui permettent une épargne importante plus tard.

Certaines hypothèses techniques nous permettent de calculer la solution du modèle et d'obtenir tous nos résultats de manière analytique. Deux résultats principaux se dégagent de notre analyse. Premièrement, nous montrons que le taux de subvention qui maximise le bien-être d'un agent représentatif dépend positivement de l'externalité décrite ci-dessus et des couts des soins curatifs. Par contre, il dépend négativement du poids que l'agent accorde au futur, c'est-à-dire à sa situation (richesse financière et santé) quand il sera âgé. Deuxièmement, nous montrons que le taux de subvention qui maximise le bien-être est toujours plus grand que le taux qui maximise l'accumulation de capital (c'est-à-dire la production) et toujours plus petit que le taux qui maximise le niveau de santé des individus. Maximiser la production signifie que l'on accepterait une réduction du niveau de santé avec en contrepartie une diminution des taxes et une augmentation de l'épargne et donc de l'investissement. Maximiser le niveau de santé signifie que l'on accepterait une taxation élevée et une réduction de la consommation.

1 Introduction

Subsidizing private health expenditures, for instance through medical reimbursements, encourages participation in preventive initiatives that improve health status. Such health subsidies involve public budgetary costs, but help reduce future health care costs. This trade-off affects the economy in many ways. The aim of this paper is to determine the welfare-maximizing subsidy rate within a dynamic general equilibrium model.

This question is important because advanced countries spend considerable resources on health, of which a large part is publicly-financed. In the EU28, *total* expenditure on health care reached 10.1% of GDP in 2015 and *public* expenditure alone 8.0% of GDP (see European Commission, 2018). Several factors explain health expenditures from the supply side, including the costs of health technologies, the price of pharmaceuticals or the specific institutional setting of health care provision. On the demand side, the main driver is the need for health care to live a longer healthy life. The need for care, and therefore health spending, is proportionally larger at older ages. For instance, chronic diseases, such as cardiovascular diseases, cancers or diabetes, are more common among older adults (WHO, 2018). They tend to be of long duration and involve high treatment costs. It is therefore in the interest of governments to encourage participation in preventive initiatives that may contain future health care costs. To stimulate this participation, subsidies play a crucial role as high out-of-pocket payments may discourage the use of health services and health investments, thereby generating poor health outcomes later (Cutler et al., 2013).

Prevention acts on both morbidity and mortality. It may directly decrease costs through lower morbidity but also indirectly increase them if prevention reduces mortality and people continue to consume health care during the additional years of life (usually spent in poor health). In our paper, we assume longevity is exogenous and focus on morbidity since several empirical studies show the important role of prevention for individual health. For instance, Grootjans-van Kampen et al. (2014) consider five different disease categories and show that in four out of the five categories, prevention leads only to health care savings without longevity gains. Only prevention of highly fatal diseases (neoplasms in Grootjans-van Kampen et al., 2014) may raise life expectancy and also health-related costs. Also, results on cost-saving initiatives in the UK indicate that a return of 1.35 pounds can be expected on every pound invested in flu vaccine or that screening is cost-saving in the short run for cervical cancer (below 5 years) and in the longer run for breast and colon cancer (see for instance WHO, 2014, for a review of these different studies). While these results do not exclude longevity gains, they indicate that lower morbidity has a considerable cost reducing effect.

To study the theoretical effects of a more generous health subsidy, we use an overlapping generations (OLG) framework with households living for 2 periods. When young, households undertake private health-improving medical expenses, in short *health investments*. These expenditures may be partially subsidized by the government. Health improvement has two implications for the elderly. It provides utility and it reduces the need for publicly-financed *health care*.[1] Without utility, there are no health investments by young households. Without a reduction in health care needs of the elderly, subsidies cannot be effective at reducing costs for the government. The government taxes labor income to finance subsidies on health investments by the young and health care needs of the elderly. A generous subsidy policy therefore affects the economy through different channels. First, for given private health expenditures, a higher subsidy raises taxation and depresses disposable income. This is the *direct cost channel*. Second, for a given disposable income, a higher subsidy spurs health investments at the expense of savings and consumption. This is the *health investment channel*. Third, increased prevention today improves health status tomorrow, which reduces publicly-financed health care costs of the elderly. This is the *health externality channel*. A strong health externality indicates that prevention is on average effective at reducing future costs or alternatively that prevention is only targeted to specific cost-saving initiatives.

We look at the steady-state effects of a health subsidy on (i) per worker capital, (ii) health status and (iii) welfare. We show that the subsidy rate maximizing welfare depends positively on the health externality and the size of health care costs, and negatively on the discount factor. The welfare-maximizing subsidy must therefore be large when prevention is very effective at reducing costs. It must also be large when private agents do not care enough about their future.

[1] In our setup, health care costs must then be understood as medical expenditures for elderly that are entirely publicly-financed. Health care differs from long-term care, which consists of nursing care for people unable to carry out daily activities. Long-term care can be also provided by unpaid relatives or financed through private insurance (Canta et al., 2016).

We also find that the subsidy rate maximizing welfare is larger than the rate maximizing per worker capital, but lower than the rate maximizing health status. Indeed, maximizing welfare means we need good health, since health enters in the utility, but also a large capital stock to generate enough production and consumption. Maximizing only health means that we accept high taxes and low consumption to pay for a very generous policy. Maximizing only capital means that we accept a less generous policy and poorer health in order to lower taxes and stimulate savings. Finally, we are able to characterize all these results analytically using a textbook OLG model with only minor departures.

The macroeconomic literature typically examines the economic implications of an exogenous increase in (public) health investment. This translates into either reduced mortality, influencing e.g. savings, or lower morbidity, affecting e.g. productivity (see next section for a review of the literature). The main contribution of our paper is the analysis of a novel mechanism through which a higher subsidy may raise health investments and reduce health care costs. Moreover, our analysis examines the effects of higher health subsidies with endogenous health investments to account for the effects of health on income as well as of income on health. Furthermore, compared to related studies, our paper provides an analytical treatment of the general equilibrium effects of a health subsidy with results in terms of per worker capital, welfare and health investment.

The rest of the paper is organized as follows. We review the literature in section 2 and introduce the model in section 3. In section 4, we explain the equilibrium properties and look at the effects of a higher subsidy rate. Section 5 concludes.

2 The Literature

We do not attempt to present an exhaustive review of the health economics literature, which is large and growing. We limit ourselves to the branch of the literature in which our paper belongs, before commenting briefly on some other studies sharing similar features but differing in scope from ours.

Our paper is linked mainly to studies investigating the implications of health (policy) on the macro-economy and, in particular, on capital accumulation and welfare. A major distinction between these studies is the health dimension (mortality or morbidity) they consider (see Bloom et al., 2018, for a recent review). Focusing on mortality, Chakraborty (2004) develops a two-period OLG model where public health investments raise life expectancy, which can enhance capital per capita through two channels. By increasing longevity, health investments spur savings and capital accumulation, but also raise the returns from education, fostering human capital accumulation and thus productivity.[2] Centered on morbidity, the two-period OLG model with exogenous longevity of Fanti and Gori (2011) assumes that improved health affects the labor productivity of old age workers. Public health investments reduce savings, because increased taxation depresses disposable income in the first period of life, but also because individuals need to save less, given that higher productivity raises second period wages. Some studies consider both mortality and morbidity changes. In Kuhn and Prettner (2016), better health raises longevity and lowers morbidity, the latter effect translating into higher worker productivity and labor market participation. An exogenous increase in per capita health care consumption leads to an expansion of the health care sector, which can be growth-enhancing despite diverting resources from other sectors. Our study focuses on the link between health investment and morbidity, which has been empirically shown to be important (Grootjans-van Kampen et al., 2014). Moreover, since the reduction in morbidity mainly happens after the age of 65, we do not represent lower morbidity through higher productivity but through better health status, which reduces health care costs at old age.

To our knowledge, the only other study considering that health investments may lead to lower morbidity and health-related cost reductions is Melindi-Ghidi and Sas (2015), which is centered on pensions. Our analysis differs in two aspects. While their study only considers exogenous public health investments, in our paper an exogenous – public – subsidy also affects private health investment and, in some specific cases, health may even decrease despite a higher subsidy. Moreover, we provide an analytical solution to study the general equilibrium effects of a health subsidy on capital accumulation, health investments and welfare. Our results are therefore general and do not rely on a specific calibration.

Finally, we briefly discuss some papers with a different scope from ours but similar approaches.

[2]Except that health investments are tax-financed, health analyzes with endogenous mortality are close to studies that do not directly refer to health and that have a more general look at the implications of longevity on economic development (see e.g. Cervellati and Sunde, 2011; Zhang et al., 2003; Boucekkine et al., 2002).

Our focus is on the theoretical implications of health investment on the macro-economy. Bhattacharjee et al. (2017) instead provide an empirical analysis to examine how the mix of private and public shares in health expenditures affects income inequality. Other studies focus more specifically on the role of private insurance in the provision of health care. Jack and Sheiner (1997) investigate the effects of reimbursing out-of-pocket health expenditures when individuals already benefit from a subsidy on insurance premium payments that implies overconsumption of health services. In their static model, reimbursing out-of-pocket spending can lower health expenditures and raise welfare, because it limits the welfare-reducing effect of the existing premium subsidy. Jaspersen and Richter (2015) analyze the effects of premium subsidies on moral hazard and find that they reduce prevention efforts by individuals. Canta et al. (2016) show that increased long-term care (LTC) needs can foster capital accumulation depending on the source of LTC funding, i.e. whether health services are provided by the family, the State or the market (private insurance). The latter three studies consider how the government can influence health investments, but they ignore the possible impact on mortality or morbidity and their focus is the design of health care provision. Finally, some papers focus on the effect of aging on the economy in the presence of a health care system. For instance, Aisa and Pueyo (2013) finds that, when savings are insufficiently stimulated, aging is growth-deteriorating because LTC drags resources from productive sectors.

3 The Model

This section develops an overlapping generations model. Individuals are identical and have perfect foresight. They live for two periods, working when young (first period of life) and retiring when old (second period of life). The size of the new generation is constant and normalized to 1. Without loss of generality, the survival rate of the new generation is 1 so that the number of old aged individuals in each period t also corresponds to 1.[3] Young individuals obtain a wage w_t in exchange for their inelastic labor supply. When they retire, they live from their accumulated savings. Health components are introduced in the following manner. Young individuals engage in private health investment m_t. A fraction ϕ of these expenditures are subsidized by the government. Health investment improves health status h_{t+1} when old,

[3]Assuming a population growth rate of $n > 0$ and a survival rate $0 < \theta < 1$ would not modify our analysis. See Appendix A for a formal development.

which generates utility. Health investment also reduces, through better health status, the need for treatments z_{t+1} when old. This health care is entirely publicly financed. The government uses labor income taxation to finance subsidies for the youngs' health investment and the cost of health care for the elderly.

Individuals

Each new-born individual has the following lifetime utility function

$$U_t = \ln c_{1,t} + \beta(\ln c_{2,t+1} + \ln h_{t+1}) \tag{1}$$

where c_1 denotes consumption when young, c_2 consumption when old and h the health status. The individual's utility in the second period of life is weighted by the subjective discount factor $\beta \in (0, 1)$.[4] The young agent faces the following budget constraint

$$c_{1,t} + s_t + (1 - \phi)m_t = w_t(1 - \tau_t) \tag{2}$$

where s stands for savings, m for health investment, w the wage and the labor income tax τ. $\phi \in [0, 1]$ is the share of health investment subsidized by the government.[5] At old age, the individual faces health care costs. These health costs z are entirely publicly-financed. Therefore, z does not appear in the budget constraint of the retired individual born in t, which is only given by

$$c_{2,t+1} = R_{t+1}s_t \tag{3}$$

where R is the gross return on savings. Substituting for s_t in the above two equations yields the individual's intertemporal budget constraint

$$c_{1,t} + \frac{c_{2,t+1}}{R_{t+1}} + (1 - \phi)m_t = w_t(1 - \tau_t).$$

We posit that the individual's health status h at old age is a linear function of health investment m undertaken during youth

$$h_{t+1} = m_t. \tag{4}$$

[4] Introducing a health-specific utility parameter $v \geq 0$ would not change our results. See Appendix A. Moreover, the choice of logarithmic utility is not problematic since the survival rate is exogenous here. In a model with an endogenous survival rate increasing with medical expenses, the utility from health status should be restricted to positive values.

[5] We only consider here the possibility of a subsidy, ruling out $\phi < 0$, which would correspond to a tax on health investment.

The individual's problem can be described as follows. Having substituted (2), (3) and (4) in (1), the individual chooses s_t and m_t so as to maximize her/his lifetime utility, which yields the first order conditions

$$\begin{aligned} c_{2,t+1} &= \beta R_{t+1} c_{1,t} \quad (5) \\ \beta \frac{1}{m_t} &= \frac{1-\phi}{c_{1,t}}. \quad (6) \end{aligned}$$

where (5) is the Euler equation for consumption and (6) states that health investment is such that the marginal utility gain in terms of tomorrow's health status equals its marginal cost in terms of today's consumption.

Firms

The representative firm produces final goods according to a Cobb-Douglas production technology using capital and labor as inputs. Since labor is normalized to 1, output is given by

$$Y_t = k_t^{\alpha} \quad (7)$$

where k is at the same time total capital and capital per worker. $\alpha \in (0,1)$ is the capital share. Capital fully depreciates at the end of each period.[6] There is perfect competition on the goods market and the price of output is normalized to one. The firm rents capital and labor from the households and pays them their respective marginal product

$$R_t = \alpha k_t^{\alpha-1} \quad (8)$$

$$w_t = (1-\alpha) k_t^{\alpha}. \quad (9)$$

Government

The government subsidizes a fraction ϕ of health investment m by the young generation. It also entirely finances old age health care costs z. We assume these old age health care costs take the form $z_t = \mu\,(w_t - \chi h_t^{\xi})$. This functional form implies that health care depends positively

[6]This assumption is innocuous. We could have partial depreciation and define a total production function with the same properties. Similarly, introducing a scale factor $A > 0$ in front of the production function would leave the equilibrium unchanged. See Appendix A.

on gross wage w and negatively on health status h. The link between wages and health care consumption reflects that there is a labor component in health-related costs, although we do not have an explicit production function for health care. The fact that health care consumption decreases as health status improves is obvious (Melindi-Ghidi and Sas, 2015). $\chi \geq 0$ represents this health externality and $\xi \in (0,1)$ affects the elasticity of health care with respect to health status, which is restricted to be smaller than one to avoid multiple non trivial equilibria.[7] Finally, $\mu \in (0,1)$ represents the global level of health-related costs. $\mu < 1$ is a sufficient condition preventing health care costs exceeding the gross wage of current workers. Using (4), the health care expression becomes

$$z_t = \mu \left(w_t - \chi \, m_{t-1}^{\xi} \right). \tag{10}$$

The government balances its budget in every period and finances the health investment subsidy and health care costs through payroll taxes τ_t, which implies[8]

$$\phi \, m_t + z_t \quad = \quad \tau_t \, w_t. \tag{11}$$

The government faces an obvious trade-off. On the one hand, more health investment m when young is costly because of the – partial – subsidy. On the other hand, more m also reduces health care costs z when old (equation (10)) and therefore government expenditures. Wage taxation to finance health subsidies can therefore be worthwhile because subsidies stimulate health investment when young, which reduces health care costs when old. Finding this 'optimal' subsidy rate is precisely the goal of our paper.

Capital accumulation

With a fully depreciated capital stock at the end of each period, young individuals' savings determine next period's capital stock

$$k_{t+1} \quad = \quad s_t. \tag{12}$$

[7] See Proposition 1.

[8] In most OECD countries, social security is typically financed through dedicated taxes on labor income (social security contribution).

4 Theoretical analysis

In this section, we define the transitional dynamics, we analytically characterize the stable steady state and we derive the effects of a higher subsidy rate on the steady state.

4.1 Transitional dynamics

To define the dynamic equilibrium, we first compute savings by inserting (2) and (3) in (5)

$$R_{t+1}s_t = \beta\left[w_t(1-\tau_t) - s_t - (1-\phi)m_t\right]$$

We then use equations (8), (9), (11), (10) and (16) to replace $R_{t+1}, w_t, \tau_t, z_t$ and m_t, respectively. After rearranging, we have

$$s_t \quad = \quad \frac{\beta}{1+\beta}\left\{(1-\alpha)(1-\mu)\,k_t^{\alpha} + \chi\mu\left(\frac{k_t}{1-\phi}\right)^{\xi} - \frac{k_{t+1}}{1-\phi}\right\} \tag{13}$$

Inserting (13) in (12) and rearranging the terms yields

$$k_{t+1} \quad = \quad \frac{(1-\alpha)\,(1-\mu)\,k_t^{\alpha} + \chi\mu\left(\frac{k_t}{1-\phi}\right)^{\xi}}{\frac{1+\beta}{\beta} + \frac{1}{1-\phi}} \tag{14}$$

As stated in Proposition 1, it is possible to prove the existence and stability of a unique non-trivial steady state $\bar{k} > 0$.

Proposition 1 *The economy has a unique stable steady state equilibrium with dynamics characterized by equation* (14)*.*

Proof. It is useful to rewrite equation (14)

$$k_{t+1} = \frac{1}{J_1}\left(J_2\,k_t^{\alpha} + J_3\,k_t^{\xi}\right) \equiv J(k_t)$$

with $J_1 \equiv (1+\beta)/\beta + 1/(1-\phi) > 1$, $J_2 \equiv (1-\alpha)\,(1-\mu) > 0$ and $J_3 \equiv \chi\mu/(1-\phi)^{\xi} \geq 0$. In the steady-state, we have $\bar{k} = J(\bar{k})$. Moreover, $J'_{k_t}(k_t) = \frac{1}{J_1}\left(\alpha\,J_2\,k_t^{\alpha-1} + \xi\,J_3\,k_t^{\xi-1}\right) > 0$ and $J''_{k_t}(k_t) = \frac{1}{J_1}\left[(\alpha-1)\alpha\,J_2\,k_t^{\alpha-2} + (\xi-1)\xi\,J_3\,k_t^{\xi-2}\right] < 0$ since $\xi \in (0,1)$ and $\alpha \in (0,1)$. Since $J(0) = 0$ and $\lim_{k_t \to 0^+} J'_{k_t}(k_t) = +\infty$, the steady state $\bar{k} = 0$ is unstable, and because $\lim_{k_t \to +\infty} J'_{k_t}(k_t) = 0$, there exists a unique positive steady state $\bar{k} > 0$. ∎

Figure 1: Transitional dynamics of k_{t+1} and m_t

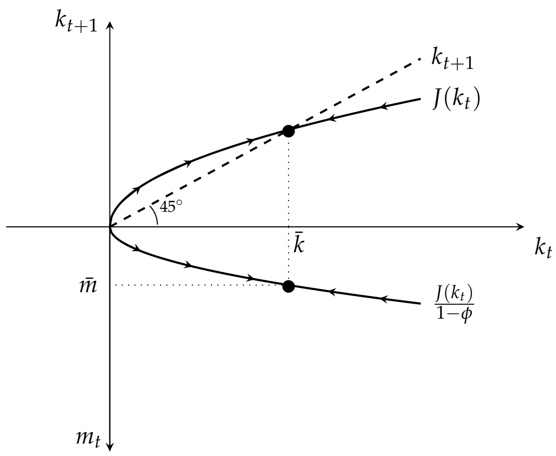


Figure 1 depicts the transitional dynamics of per worker capital k_{t+1} and health investment $m_t = k_{t+1}/(1-\phi)^{\xi} = J(k_t)/(1-\phi)$. It also shows there exists an unstable steady state at $\bar{k} = \bar{m} = 0$, as well as a stable one with strictly positive $\bar{k} > 0$ and $\bar{m} > 0$.[9]

4.2 Steady state equilibrium

Under Assumption 1, it is possible to analytically characterize this unique stable steady state.

Assumption 1 $\xi = \alpha$.

In this case equation (14) boils down to

$$k_{t+1} = \sigma k_t^{\alpha}$$

where

$$\sigma \equiv \frac{(1-\alpha)(1-\mu) + \frac{\chi\mu}{(1-\phi)^{\alpha}}}{\frac{1+\beta}{\beta} + \frac{1}{1-\phi}} > 0. \tag{15}$$

Proposition 2 *Under Assumption 1, the unique stable steady state is defined by* $\bar{k} = \sigma^{\frac{1}{1-\alpha}} > 0$.

Proof. When $\xi = \alpha$, it is straightforward to show that $\bar{k} = \sigma^{\frac{1}{1-\alpha}} > 0$ is the interior steady state of the dynamic system described in (14). ■

[9] $\xi > 1$ would no longer guarantee the concavity of $J(k_t)$. Since $\lim_{k_t \to 0^+} J'_{k_t}(k_t) = +\infty$, this implies that an unstable steady state with strictly positive $\bar{k} > 0$ and $\bar{m} > 0$ could also exist in this economy.

4.3 Admissible subsidy rates

As mentioned in Section 3, the subsidy rate respects $0 \leq \phi \leq 1$. Moreover, when the externality χ is high, a generous subsidy might imply negative health care expenditure z_t, which is something we want to rule out. Put differently, at a given ϕ, the health externality χ must have an upper bound $\overline{\chi}_z(\phi)$. More precisely, under Assumption 1, when $0 \leq \chi \leq (1-\alpha)(1-\phi)^\alpha \equiv \overline{\chi}_z(\phi)$, health care expenditures z_t are always positive.[10] To show this, we combine (4) with (5) and (6) to obtain $m_t = \frac{1}{(1-\phi)} \frac{c_{2,t+1}}{R_{t+1}}$. This equation together with (3), (8) and (12) yields

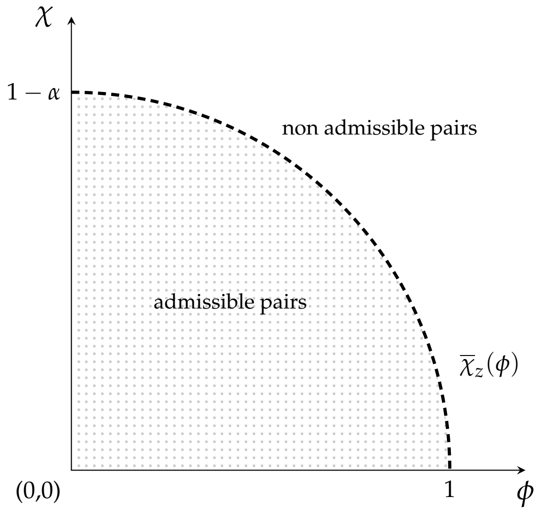
$$m_t \quad = \quad \frac{k_{t+1}}{1-\phi}. \tag{16}$$

Plugging (16) and (9) into (10) immediately shows the relationship between $\chi \leq \overline{\chi}_z(\phi)$ and $z_t \geq 0$. We combine these restrictions on ϕ and χ to define admissible pairs (ϕ, χ) as

Definition 1 *(ϕ, χ) is an admissible pair if (i) $0 \leq \phi \leq 1$ and (ii) $0 \leq \chi \leq \overline{\chi}_z(\phi)$ with $\overline{\chi}_z(\phi) = (1-\alpha)(1-\phi)^\alpha$.*

Since $0 \leq \phi \leq 1$, $\overline{\chi}_z(\phi)$ is decreasing and concave in ϕ, $\overline{\chi}_z(0) = 1-\alpha$ and $\overline{\chi}_z(1) = 0$. Figure 2 plots the admissible area for the pair (ϕ, χ).

Figure 2: Admissible pairs (ϕ, χ)



[10]The non negativity of z_t is therefore ensured not only at the stable steady state but also during the transitional dynamics.

4.4 Effects of a higher subsidy rate

The remaining analysis focuses on the steady state under Assumption 1. To simplify notation, we omit the bar on steady state variables. The main objective is to study the implications of the health subsidy rate ϕ on per worker capital k, health investment $m = k/(1-\phi)$ and welfare, represented by the lifetime utility U of a new-born generation.[11] The lifetime utility of a representative young generation is $U = \ln c_1 + \beta \ln c_2 + \beta \ln h$. Since the three arguments of the utility function can be expressed in terms of per worker capital,

$$c_1 = \frac{k}{\beta}, \quad c_2 = \alpha k^{\alpha}, \quad h = m = \frac{k}{1-\phi} \tag{17}$$

it is possible to rewrite the lifetime utility as

$$U = \tilde{U} - \beta \ln(1-\phi) + [1 + \beta(1+\alpha)] \ln k \tag{18}$$

where $\tilde{U}$ is a parameter combination $\tilde{U} = -\ln(\beta) + \beta \ln(\alpha)$.

Per worker capital

A more generous subsidy rate affects the economy through several channels. First, for any given m, a larger ϕ implies higher public costs. As a consequence, the tax rate increases, which reduces disposable income and thereby savings and capital accumulation. This is the *direct cost channel*. Second, for a given level of disposable income, a higher subsidy rate ϕ may induce households to save and consume less in order to invest more in health, which also leads to an increase in the tax rate. Savings and per worker capital are depressed. This substitution effect is the *health investment channel*.[12] Finally, higher health investments m improve the population's health status and reduce health care costs. The tax rate is lower and savings and per worker capital are enhanced. This latter *health externality channel* dominates if the health externality parameter χ is high enough. In other words, higher subsidies raise per worker capital when χ is sufficiently high, provided it remains in the admissible area. Proposition 3 summarizes how more generous subsidies affect per worker capital.

[11] We focus on the welfare of the young generation. We could instead focus on a 'social welfare' measure, i.e. a weighted sum of the welfare of the young and the old generations. This would not change the conclusions of our analysis.

[12] A higher subsidy affects the economy also through an income effect by which cheaper health investments raise the household's purchasing power and the household can spend more on consumption and savings.

Proposition 3 (Per worker capital) *In the admissible value range of (ϕ, χ) characterized in Definition 1, a more generous health subsidy raises per worker capital provided the health subsidy is not too large and the health-externality is strong enough, i.e.*

$$\frac{dk}{d\phi} \geq 0 \;\Leftrightarrow\; (i)\ \phi \leq \phi_k^{AS} \;\; and \;\; (ii)\ \chi \geq \underline{\chi}_k(\phi)$$

where $\phi_k^{AS} = 1 - \frac{1-\alpha}{\alpha}\frac{\beta}{1+\beta}$ *and* $\underline{\chi}_k(\phi) = \frac{(1-\alpha)(1-\mu)}{\mu(1-\phi)^{1-\alpha}}\left(\frac{\alpha(1+\beta)}{\beta} - \frac{1-\alpha}{1-\phi}\right)^{-1}.$

Proof. See Appendix B. ∎

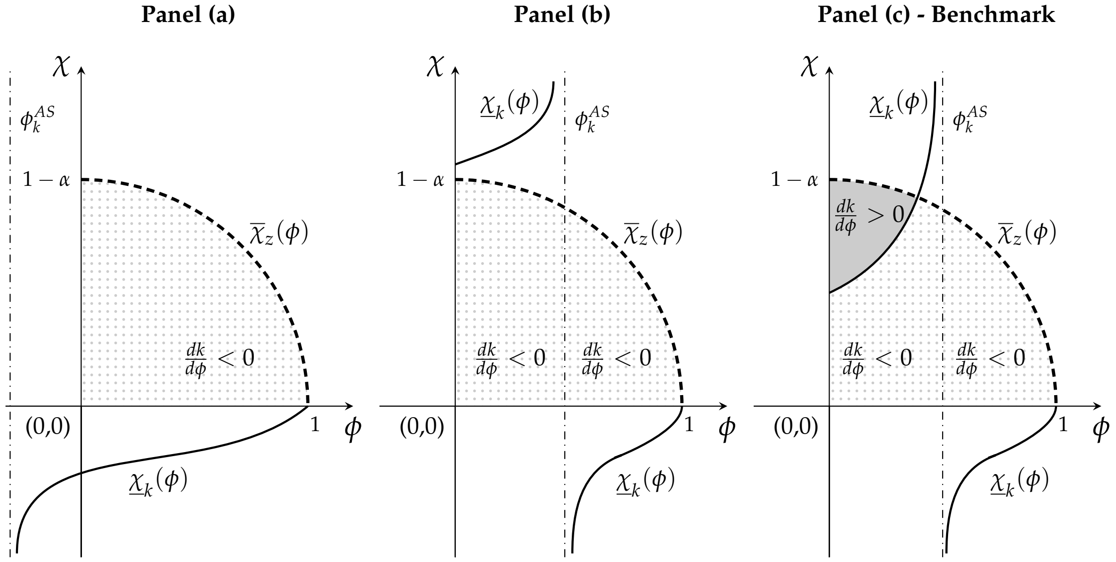
Figure 3 illustrates Proposition 3. Before describing the figure, let us briefly discuss the main properties of the $\underline{\chi}_k(\phi)$ curve. $\underline{\chi}_k(\phi)$ admits an asymptote $\phi_k^{AS} \leq 1$. It is increasing and strictly positive on the left branch and increasing and strictly negative on the right branch located at $]\phi_k^{AS}, 1[$. This implies that, in the absence of externality ($\chi = 0$), capital cannot be increased with a subsidy, since $dk/d\phi \geq 0$ requires χ to be on the left branch and $\chi \geq \underline{\chi}_k > 0$. In this case, a higher subsidy leads to increased taxation and discourages savings and capital accumulation through the direct cost and health investment channels, without any health externality channel to compensate.

Figure 3 depicts three possible cases. Panel (a) illustrates the case where the asymptote is negative ($\phi_k^{AS} < 0$) which happens when β is large, i.e. when $\beta/(1+2\beta) > \alpha$.[13] Condition (i) in Proposition 3 states that pairs (ϕ, χ) associated with $dk/d\phi \geq 0$ must be located on the left of the asymptote. In the present case, where $\phi_k^{AS} < 0$, capital can only be increased if health investments are taxed, i.e. with a negative subsidy rate ϕ, which is not admissible. One interpretation is that a large time discount factor induces individuals to make considerable health investments, putting pressure on public finance and thus maximizing capital requires a negative subsidy rate.

The next two panels of Figure 3 consider a positive asymptote ($\phi_k^{AS} > 0$) but differ in terms of the vertical position of the left branch of $\underline{\chi}_k(\phi)$. Condition (ii) in Proposition 3 states that a positive effect of subsidy on capital requires that χ be above a lower bound $\underline{\chi}_k(\phi)$ to generate

[13] More precisely, given $0 \leq \beta \leq 1$, $\phi_k^{AS} < 0$ requires $\alpha < 1/3$ and β large. For example, with $\alpha = 0.33$, the condition implies a discount factor of $\beta \geq 0.97$ (equivalent to a discount rate of less than 0.1% on an annual basis, if one period lasts 40 years). This discount factor is large, especially since the condition would actually apply to the product of discount factor and the survival rate, which is set to one here.

Figure 3: The implications of subsidy (ϕ) on per worker capital (k): different cases



Dotted and grey areas contain admissible pairs (ϕ, χ). Panel (a) shows the case where the asymptote ϕ_k^{AS} is negative. Panel (b) assumes that $\phi_k^{AS} > 0$ and the left and positive branch of $\underline{\chi}_k(\phi)$ is above $\overline{\chi}_z(\phi)$. In Panel (c), $\phi_k^{AS} > 0$ and $0 < \underline{\chi}_k(0) < \overline{\chi}_z(0)$.

a sufficient externality. Panel (b) of Figure 3 considers a case where this lower bound is too elevated, i.e. where $\underline{\chi}_k$ is above $\overline{\chi}_z$. Indeed χ is bounded from above by $\overline{\chi}_z(\phi)$ and because $\overline{\chi}_z$ is decreasing and $\underline{\chi}_k$ increasing in ϕ, a direct implication of condition (ii) is that there is no admissible pair (ϕ, χ) raising capital when $\underline{\chi}_k(0) \geq \overline{\chi}_z(0) = 1 - \alpha$. Thus there is no intersection between the area above $\underline{\chi}_k$ and the area below $\overline{\chi}_z$ (and left of ϕ_k^{AS}). This case may happen when μ is too small, implying modest public expenditures on health care and thus an externality that must be very large to generate enough cost and tax reductions such that $dk/d\phi \geq 0$. In the present case, the externality compatible with $dk/d\phi \geq 0$ is beyond the admissible values of χ.

Panel (c) of Figure 3 assumes that $\underline{\chi}_k(0)$ is below $\overline{\chi}_z(0)$ and thus the set of admissible pairs (ϕ, χ) such that $dk/d\phi \geq 0$ is not empty (i.e. β and μ have standard values). This set is represented by the shaded gray area. We also report this – benchmark – case in Figure 6. Per worker capital is maximized on any admissible point of the $\underline{\chi}_k(\phi)$ curve. In Panel (c) of Figure 3 (benchmark case), there should be no subsidy as long as χ is below $\underline{\chi}_k(0)$. Let us then assume that χ increases further from $\underline{\chi}_k(0)$. In this case, the policy maker must start increasing ϕ_k to stay on the $\underline{\chi}_k(\phi)$ curve. However, when χ becomes too high, we move from curve $\underline{\chi}_k(\phi)$ to curve $\overline{\chi}_z(\phi)$, and ϕ_k needs to decrease in order to avoid negative health care expenditures.

Health investment

Before moving to the welfare analysis, it might be interesting to look at the effects of the subsidy on health investment m. Since $m = k/(1-\phi)$, a more generous subsidy policy has an ambiguous indirect effect through k (see Proposition 3 above) but also a positive direct effect through ϕ. As an obvious consequence, it is easier to increase m than k when raising the subsidy. We show this in Proposition 4 below.

Proposition 4 (Health investment) *In the admissible area characterized in Definition 1, a more generous health subsidy improves the population's health status if the health-externality is sufficiently strong, i.e.*

$$\frac{d\,m}{d\,\phi} \geq 0 \;\Leftrightarrow\; \chi \geq \underline{\chi}_m(\phi)$$

where $\underline{\chi}_m(\phi) = \frac{(1-\alpha)(1-\mu)}{\mu(1-\phi)^{1-\alpha}} \left(\frac{\alpha\beta}{1+\beta} - (1-\alpha)(1-\phi) \right)$.

Proof. See Appendix C. ■

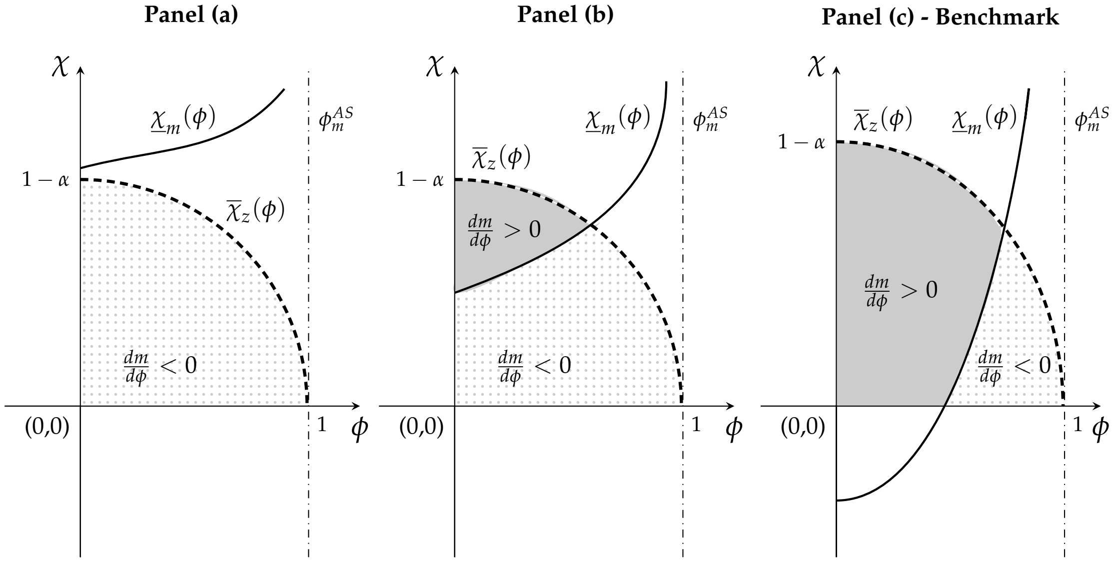
This lower bound on χ, the curve $\underline{\chi}_m(\phi)$, is increasing in ϕ over the value range $0 \leq \phi \leq 1$ and has an asymptote at $\phi = 1$. Thus the asymptote cannot be negative, in contrast to the previous case, and only the left branch of $\underline{\chi}_m$ lies in the interval $0 \leq \phi \leq 1$. Moreover, this left branch can be negative with $\underline{\chi}_m(0) < 0$ when $\beta/(1+\beta) \geq 1/\alpha - 1$. This implies that m may increase with the subsidy even when $\chi = 0$, as occurs in our benchmark case discussed shortly below. We nevertheless consider different cases depending on $\underline{\chi}_m(0)$ and illustrate them in Figure 4.

Panel (a) in Figure 4 considers that $\underline{\chi}_m(0) > \overline{\chi}_z(0) > 0$. Thus there is no intersection between the area above $\underline{\chi}_m$ and the area below $\overline{\chi}_z$. This case may occur when μ is sufficiently small.[14] The externality must be strong when μ is small, since then public expenditures on health care are low and thus χ must generate enough tax reductions for $dm/d\phi \geq 0$. In the present case, $dm/d\phi \geq 0$ cannot be achieved with any admissible value of χ.

Panel (b) assumes that $0 < \underline{\chi}_m(0) < \overline{\chi}_z(0)$ and thus there is a non-empty set of admissible pairs (ϕ, χ) such that $dm/d\phi \geq 0$. This set is represented by the shaded gray area between the curves

[14] This case also requires $\alpha \geq 2/3$, a necessary condition for $\underline{\chi}_m(0) > 0$.

Figure 4: The implications of subsidy (ϕ) on health investment (m): different cases



Dotted and grey areas contain admissible pairs (ϕ, χ). Panel (a) shows the case where $\underline{\chi}_m(0) > \overline{\chi}_z(0) > 0$. In Panel (b), $\overline{\chi}_z(0) > \underline{\chi}_m(0) > 0$. Panel (c) displays the case where $\underline{\chi}_m(0) < 0$.

$\underline{\chi}_m(0)$ and $\overline{\chi}_z(0)$. To move from Panel (a) to Panel (b), we need to increase μ. The larger μ, the lower $\underline{\chi}_m(0)$ and the more likely $\frac{\mathrm{d}m}{\mathrm{d}\phi} \geq 0$, i.e. the larger the shaded gray area. Health investment is maximized on any admissible point of the $\underline{\chi}_m(\phi)$ curve. There should be no subsidy as long as χ is below $\underline{\chi}_m(0)$. If χ increases further than $\underline{\chi}_m(0)$, the policy maker must raise ϕ_m to stay on the $\underline{\chi}_m(\phi)$ curve. However, when χ becomes too high, we move from curve $\underline{\chi}_m(\phi)$ to curve $\overline{\chi}_z(\phi)$, and start decreasing ϕ_m in order to avoid negative health care expenditures.

Panel (c) assumes that $\underline{\chi}_m(0) < 0$, which is our benchmark case (and also reported in Figure 6). This case happens for lower values of α, compared to the 2 other panels. One interpretation is that a lower α is associated with a larger labor share in production and thus a higher wage rate, which is the source of income of young individuals investing in health. The tax rate increase required to finance the subsidy is therefore smaller and the health investment channel is stronger than the direct cost channel. Here, even without a health externality, it is possible to find a subsidy policy ϕ_m maximizing health investment. In other words, the non-empty set of admissible pairs (ϕ, χ) such that $dm/d\phi \geq 0$ comprises $\chi = 0$, see shaded gray area. It is worth noting that a lower α not only moves the $\underline{\chi}_m(\phi)$ curve, but also shifts the $\overline{\chi}_z(\phi)$ curve upwards, enlarging the shaded gray area[15]

Welfare

The welfare analysis is directly related to our propositions 3 and 4 above. First, the health externality parameter χ must be above a minimum threshold in order for higher subsidies to have a positive effect on welfare. Since welfare is itself composed of consumption/capital and health investment, we can expect that the minimum threshold will be between the one for k and the one for m. Second, χ must be below a certain threshold to avoid negative health care expenditures. More precisely, the welfare effects of a more generous subsidy are observed through the lifetime utility of new-born individual in equation (18), reproduced here for convenience

$$U = \tilde{U} - \beta \ln(1-\phi) + [1+\beta(1+\alpha)] \ln k$$

where $\tilde{U}$ is a collection of parameters. A higher ϕ has a positive direct effect on U (second term) and an ambiguous indirect effect on U through k (third term). The final effect on welfare can be summarized by the following proposition

Proposition 5 (Welfare) *In the admissible area characterized in Definition 1, a more generous health subsidy enhances welfare provided the health subsidy is not too large and the health-externality is strong enough, i.e.*

$$\frac{dU}{d\phi} \geq 0 \;\Leftrightarrow\; (i)\ \phi \leq \phi_U^{AS} \;\; and \;\; (ii)\ \chi \geq \underline{\chi}_U(\phi)$$

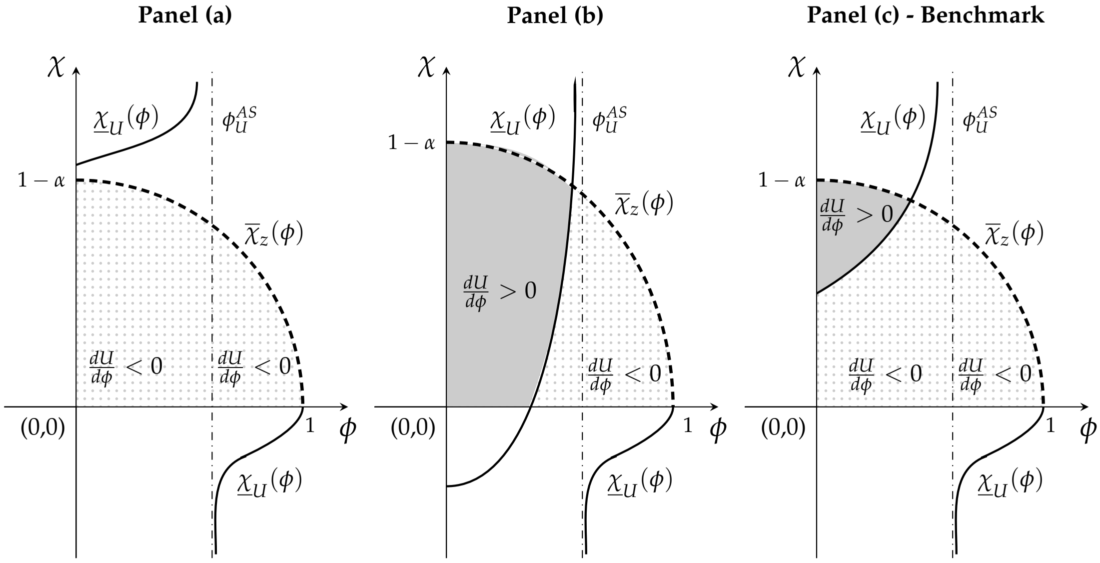
where $\Lambda = 1+\alpha+\frac{1}{\beta}$, $\phi_U^{AS} = 1 - \frac{(1-\alpha)(\Lambda-1)}{1+\alpha(\Lambda-1)} \frac{\beta}{1+\beta} > 0$
and $\underline{\chi}_U(\phi) = \frac{(1-\alpha)(1-\mu)}{\mu(1-\phi)^{1-\alpha}} (1-\phi) \frac{\Lambda(1-(1-\phi)(1-\alpha))-(1-\alpha)(1-\alpha(1-\phi))}{((\Lambda-\alpha)(1-\phi)+1)(1+\alpha(\Lambda-1))-\Lambda}$.

Proof. See Appendix D. ■

Figure 5 illustrates Proposition 5. The $\underline{\chi}_U(\phi)$ curve shares similarities with the $\underline{\chi}_k(\phi)$ curve. It admits an asymptote $\phi_U^{AS} \leq 1$, is increasing over $0 \leq \phi \leq 1$ and the right branch is negative on $]\phi_U^{AS}, 1[$. Two differences are that the asymptote is positive and the left branch can be negative. Moreover, we have $\phi_U^{AS} \geq \phi_k^{AS}$. Conditions (i) and (ii) in Proposition 5, namely that we

[15]We here develop intuitions with a lower α. However, the case shown in Panel (c) might also occur when α is higher but β small enough. For example, with $\alpha = 3/4$, $\underline{\chi}_m(0) < 0$ if $\beta < 1/2$.

Figure 5: The implications of subsidy (ϕ) on welfare (U): different cases



Dotted and grey areas contain admissible pairs (ϕ, χ). Panel (a) shows the case where $\underline{\chi}_U(0) > \overline{\chi}_z(0) > 0$. Panel (b) displays the case where $\underline{\chi}_U(0) < 0$. In Panel (c), $\overline{\chi}_z(0) > \underline{\chi}_U(0) > 0$.

must be on the left of the asymptote and above the left branch, are also similar to the ones in Proposition 3.

In Panel (a) of Figure 5, $\underline{\chi}_U(0) \geq \overline{\chi}_z(0) \geq 0$ and there is no admissible pair (ϕ, χ) that can increase utility with a subsidy. This may happen when μ is small, in which case public expenditures on health care are small and the externality needs to be excessively high to generate enough cost reductions. Moreover, in Panel (b) of Figure 5, we have $\underline{\chi}_U(0) \leq 0$ and utility increases with a subsidy even without an externality. This case occurs when α is sufficiently small, and thus when labor income, i.e. the source of income of the young generation who invests in health, is large.[16] Panel (c) in Figure 5 reports the benchmark case, where we assume that the set of admissible pairs (ϕ, χ) such that $dU/d\phi \geq$ is not empty and $\overline{\chi}_z(0) \geq \underline{\chi}_U(0) \geq 0$ (occurring for standard values of α and μ). Welfare is maximized on admissible points of curve $\underline{\chi}_U(\phi)$. We also report this case in Figure 6.

[16]More precisely, $\underline{\chi}_U(0)$ is always positive if $\alpha > 1/4$ and it is negative in $\phi = 0$ if $\alpha < 1/4$ and β sufficiently large.

Figure 6: The implications of subsidy (ϕ) on capital per worker (k), health investment (m) and welfare (U) in the benchmark case

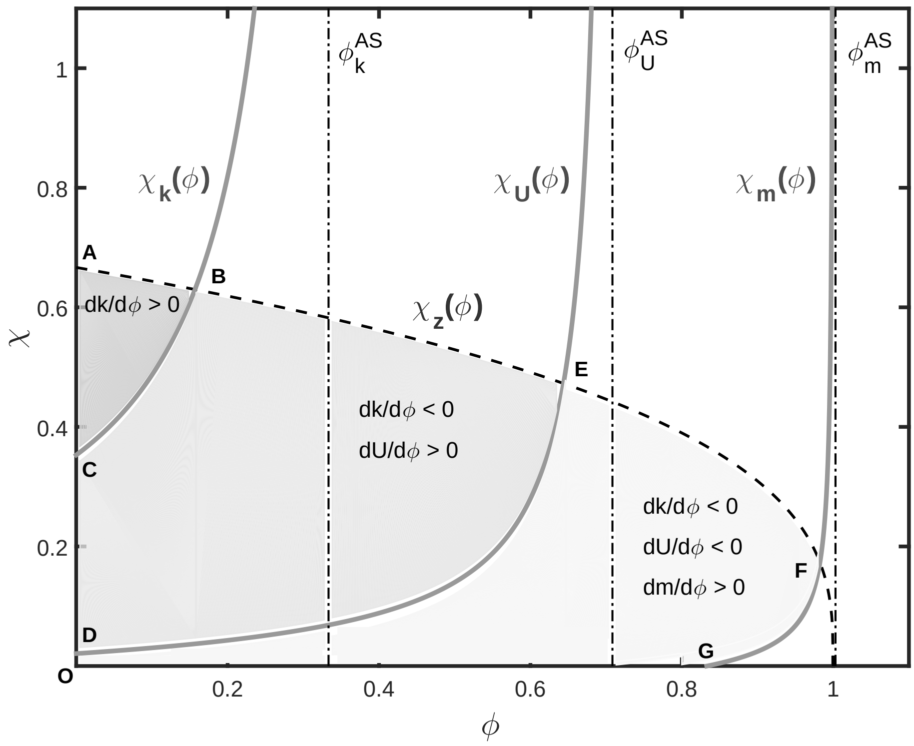


Figure 6 depicts the effects of a subsidy for the case where $\alpha = 1/3$, $\beta = 0.5$, and $\mu = 0.85$. The figure combines the three Panel (c)'s of Figures 3, 4 and 5. Point O is located at the origin i.e. (0,0).

Comparisons

Figure 6 summarizes the implications of a higher subsidy (ϕ) on k, U and m in our benchmark case, i.e. it combines the three Panel (c)'s of Figures 3, 4 and 5. $\overline{\chi}_z$ is the upper bound above which the pairs (ϕ, χ) are not admissible. Note that on $[0, \phi_k^{AS}[$, it is immediate that $dk/d\phi \geq 0 \Rightarrow dU/d\phi \geq 0$ which implies that $\underline{\chi}_U(\phi) \leq \underline{\chi}_k(\phi)$ and it is also true that $dk/d\phi \geq 0 \Rightarrow dm/d\phi \geq 0$ which implies that $\underline{\chi}_m(\phi) \leq \underline{\chi}_k(\phi)$. Moreover, on $[0, \phi_U^{AS}[$, $dU/d\phi \geq 0$ and $dk/d\phi \leq 0 \Rightarrow dm/d\phi \geq 0$ which implies that $\underline{\chi}_m(\phi) \leq \underline{\chi}_U(\phi) \leq \underline{\chi}_k(\phi)$. In Figure 6, a higher subsidy raises capital per worker for pairs (ϕ, χ) located below $\overline{\chi}_z$ and above $\underline{\chi}_k$, i.e. the dark grey area (area ABC). The medium gray area (BCDE) contains all admissible (ϕ, χ) pairs which allow $dU/d\phi \geq 0$ and $dm/d\phi \geq 0$ (while $dk/d\phi < 0$). The light gray area (ODEFG) comprises the pairs where only health investment increases when ϕ rises, $dm/d\phi \geq 0$, while the combined gray area (OAFG) represent all admissible (ϕ, χ) pairs which allow $dm/d\phi \geq 0$.

Suppose an economy with $\chi = 0.4$. We start from $\phi = 0$ and we increase it progressively, i.e. in Figure 6, we move horizontally and to the right starting from $(\phi, \chi) = (0, 0.4)$. Per worker capital increases (as well as U and m) and is maximized when ϕ reaches the curve $\underline{\chi}_k$. Further raising ϕ decreases k, though U and m would still increase. Welfare is maximized when ϕ reaches $\underline{\chi}_U$ and further increasing ϕ raises only m. Note that at some point (when ϕ reach about 0.8), we cannot increase ϕ further because we would be above the $\overline{\chi}_z$ curve.

Discussion

We can conclude from the above that the admissible – according to Definition 1 – subsidy rate ϕ_k maximizing capital is quite low. In fact, it is typically zero and only becomes strictly positive when the health externality channel, that is χ, is sufficiently large. Indeed, an additional subsidy means additional taxation, unless there is a heavy externality, which is detrimental to capital accumulation (direct cost channel). On the other hand, the admissible subsidy rate ϕ_m maximizing health is quite high and can even be strictly positive without any externality at all. In this case, there is an obvious increase in taxation and capital losses, but health investment nevertheless increases (health investment channel). In between is the admissible subsidy rate

ϕ_U maximizing welfare, i.e. $\phi_k \leq \phi_U \leq \phi_m$, since welfare includes both capital and health.

We now look at the evolution of ϕ_U depending on different parameters. We focus on the benchmark case but we would obtain similar intuitions in other cases or considering other values of ϕ_k or ϕ_m. Panel (a) of Figure 7 depicts ϕ_U as a function of χ, which is straightforward from Figure 5. When χ is too small, $\phi_U = 0$ because the direct cost channel dominates. Then ϕ_U increases along with χ to benefit from the health externality channel. When χ becomes too high, ϕ_U must decrease to stay at the frontier of the admissible area. Panel (b) of Figure 7 shows ϕ_U as a function of μ, the size of health care costs. We observe from Proposition 5 that $\partial \underline{\chi}_U(\phi)/\partial\mu < 0$ when $\underline{\chi}_U(\phi)$ is in the admissible area as characterized in Definition 1.[17] As an immediate result, ϕ_U is increasing in μ, that is the optimal subsidy rate is higher in an economy suffering from important health care costs. Note that when μ is sufficiently small, the optimal subsidy rate should even be negative, which is not admissible, and $\phi_U = 0$. Finally, Panel (c) shows ϕ_U as a function of β. Since β affects both the asymptote ϕ_U^{AS} and the curve $\underline{\chi}_U(\phi)$ in multiple ways, it is more difficult to derive clear-cut analytical results. Let us simply mention that when α is not too small[18], the asymptote moves to the left when β increases, which suggests a lower optimal subsidy. We see in Panel (c) that this is what happens in our benchmark case. When β is small, agents do not care about the future and therefore do not invest in health. As a result, their poor health condition implies huge health care costs and it is welfare maximizing to have a very generous subsidy. When β increases, the decentralized equilibrium is less harmful for public finance and there is less need for public policy intervention.

5 Conclusion

Health subsidies involve public budgetary costs, but they can also encourage participation in preventive initiatives that may improve health and reduce future health care costs. This paper develops an overlapping generations model where the government levies a labor income tax to finance subsidies on health investments by the young and the costs of health care for the old. Our study contributes to the literature by theoretically analyzing the general equilibrium effects of a mechanism by which a higher health subsidy may encourage health investments by

[17] Recall that in Panel (c) of Figure 5, this translate into a downward shift of $\underline{\chi}_U$ within the admissible area, while ϕ_U^{AS} does not move.

[18] More precisely, the condition is $\beta^2\alpha + \alpha(1+\alpha\beta)^2 - \beta^2 > 0$.

Figure 7: Welfare maximizing admissible subsidy (ϕ_U) as a function of selected parameters

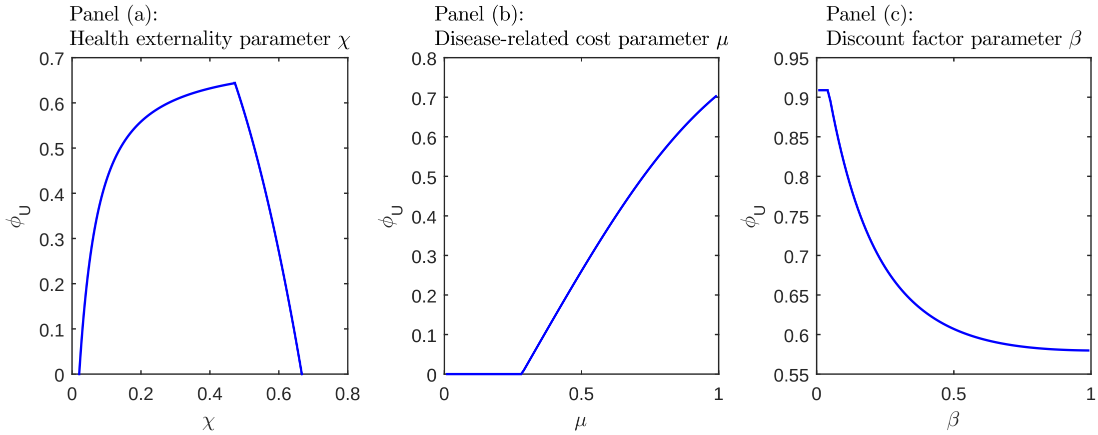


Figure depicts the subsidy ϕ_U maximizing welfare, under the condition that ϕ_U is admissible as characterized in Definition 1. For the non-varying parameters, we use the benchmark calibration $\alpha = 1/3$, $\beta = 0.5$, $\mu = 0.85$ and $\chi = 0.35$.

young households, which translate into better health when old, containing health care costs. We find that the welfare-maximizing subsidy rate depends positively on the health externality and on the size of the health care costs, and negatively on the discount factor. The subsidy rate should therefore be high when prevention is effective at reducing costs and when individuals do not care about the future. Moreover, the welfare-maximizing subsidy rate is lower than the health-maximizing rate and larger than the capital-maximizing rate. These analytical results are obtained with only minor departures from a standard OLG model.

Two extensions of our analysis might be interesting. First, in our study, the costs related to the health condition of the old generation are public. It would be important to consider the effects of a health subsidy on health investment decisions when costs may also be private (see e.g. Canta et al., 2016, with long-term care). Second, we focused on morbidity reduction resulting from health improvements. Future research could also include longevity gains in the analysis of the implications of a health subsidy. Increased life expectancy may raise health-related costs, because individuals consume health care during the additional years of life, and mitigate the cost reductions resulting from prevention (Grootjans-van Kampen et al., 2014).

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A A more general model

This appendix extends the model in the paper by adding population growth n, survival rate θ, leisure utility weight v and productivity level A. Below, we do not present in full details the more general model but only changes with respect to the model in the paper.

We assume that population grows at rate $n \geq 0$. The size N_t of a new generation therefore follows $N_t = (1+n)N_{t-1}$. $\theta \in (0,1]$ is the survival rate such the the population is composed of N_t young and θN_{t-1} old. $v \geq 0$ is a health specific utility parameter. As a result, the lifetime utility function (1) becomes

$$U_t = \ln c_{1,t} + \beta\theta(\ln c_{2,t+1} + v \ln h_{t+1})$$

and the budget constraint (3) of the old generation

$$c_{2,t+1} = \frac{1}{\theta} R_{t+1} s_t.$$

The representative firm uses labor N_t to produce output. The production function (7) is now

$$\frac{Y_t}{N_t} = A\, k_t^{\alpha}$$

where $A > 0$ is the productivity level. Population growth and survival probability also affect the government budget constraint (11), which becomes

$$(1+n)\phi\, m_t + \theta z_t = (1+n)\tau_t\, w_t.$$

Equation (10) represents the health care costs. We slightly modify it to index the second part (health externality) on the the productivity level[19]

$$z_t = \mu\, \left(w_t - A\, \chi\, m_{t-1}^{\xi} \right).$$

Finally, n modifies the capital-savings equation (12) according to

$$k_{t+1} = \frac{s_t}{1+n}.$$

Assumption 2 below ensures that health care expenditures are always positive

[19]The first part (wages) is implicitly indexed since wages are the marginal product of labor and are therefore proportional to A. In practice and as shown below, indexing the second part makes the effects of χ independent from the productivity level of the economy. In other words, A is a pure scale factor as in the standard OLG model.

Assumption 2 $0 \leq \chi \leq (1-\alpha)\left(\frac{1-\phi}{v(1+n)}\right)^{\xi} k_t^{\alpha-\xi}$.

After computation, we can obtain the dynamic equilibrium for capital

$$k_{t+1} = A \, \frac{(1-\alpha)\left(1-\frac{\theta\mu}{1+n}\right) k_t^{\alpha} + \frac{\chi\theta\mu}{1+n}\left(\frac{v(1+n)\,k_t}{1-\phi}\right)^{\xi}}{(1+n)\frac{1+\beta\theta}{\beta\theta} + \frac{v(1+n)}{1-\phi}} \tag{19}$$

First, we observe that A is only a scaling parameter which has no effect on the analysis and we may set $A = 1$ without loss of generality as in a standard OLG model. Second, we are able to show that the more general model can be transformed into the simpler model we present in the paper. Let us define $\tilde{\mu} \equiv \theta\mu/(1+n)$, $\tilde{\beta} \equiv \beta\theta/(1+n(1+\beta\theta))$ and $1-\tilde{\phi} \equiv (1-\phi)/(v(1+n))$. We have $\tilde{\mu}, \tilde{\beta} \in (0,1)$. To obtain $1-\tilde{\phi} \in [0,1]$, we need an extra condition $v \geq (1-\phi)/(1+n)$. We then simplify Assumption 2 and equation (19) into

$$0 \leq \chi \leq (1-\alpha)(1-\tilde{\phi})^{\xi}\, k_t^{\alpha-\xi}$$

and

$$k_{t+1} = \frac{(1-\alpha)\,(1-\tilde{\mu})\,k_t^{\alpha} + \chi\tilde{\mu}\left(\frac{k_t}{1-\tilde{\phi}}\right)^{\xi}}{\frac{1+\tilde{\beta}}{\tilde{\beta}} + \frac{1}{1-\tilde{\phi}}}$$

These expressions are equivalent to Definition 1 and equation (14). As a consequence, all the results presented in the paper – from Proposition 1 to Proposition 5 – also hold with the more general model, as long as $v \geq (1-\phi)/(1+n)$.

B Proof of Proposition 3

We use the steady state expression for k from Proposition 2 and the definitions for ϕ_k^{AS} and $\underline{\chi}_k(\phi)$ from Proposition 3. Moreover, $\beta, \alpha, \mu \in (0,1)$ and we have restrictions on ϕ and χ characterized in Definition 1. It is then straightforward to write

$$
\begin{aligned}
\frac{dk}{d\phi} \geq 0 \quad &\Leftrightarrow \quad \frac{\sigma^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \frac{\partial \sigma}{\partial \phi} \geq 0 \\
&\Leftrightarrow \quad \frac{\partial \sigma}{\partial \phi} \geq 0 \\
&\Leftrightarrow \quad \chi \mu \alpha (1-\phi)^{1-\alpha} \left(1 + \frac{1}{\beta} + \frac{1}{1-\phi}\right) \geq (1-\alpha)(1-\mu) + \frac{\chi \mu}{(1-\phi)^{\alpha}} \\
&\Leftrightarrow \quad \chi \left(\frac{\alpha(1+\beta)}{\beta} - \frac{1-\alpha}{1-\phi}\right) \geq \frac{(1-\alpha)(1-\mu)}{\mu(1-\phi)^{1-\alpha}} \\
&\Leftrightarrow \qquad \phi < \phi_k^{AS} \text{ and } \chi \geq \underline{\chi}_k(\phi) > 0 \\
&\qquad\ \ \text{or } \ \phi > \phi_k^{AS} \text{ and } \chi \leq \underline{\chi}_k(\phi) < 0
\end{aligned}
$$

We rule out the second possibility because this would violate Definition 1. This proofs Proposition 3.

C Proof of Proposition 4

We use equation (17) $m = k/(1-\phi)$ and the steady state expression for k from Proposition 2. We also use the definition for $\underline{\chi}_m(\phi)$ provided in Proposition 4. Moreover, $\beta, \alpha, \mu \in (0,1)$ and we have restrictions on ϕ and χ characterized in Definition 1. To simplify the notation below, we also define $D \equiv 1 + \frac{1}{\beta} + \frac{1}{1-\phi}$. It is then straightforward to write

$$
\begin{aligned}
\frac{dm}{d\phi} \geq 0 \quad &\Leftrightarrow \quad \frac{dk}{d\phi}(1-\phi) + k \geq 0 \\
&\Leftrightarrow \quad \frac{\sigma^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \frac{\partial \sigma}{\partial \phi}(1-\phi) + \sigma^{\frac{1}{1-\alpha}} \geq 0 \\
&\Leftrightarrow \quad \frac{\chi \mu \alpha}{D(1-\phi)^{\alpha}(1-\alpha)\sigma} - \frac{1}{D(1-\phi)(1-\alpha)} + 1 \geq 0 \\
&\Leftrightarrow \quad \frac{\chi \mu \alpha}{(1-\phi)^{\alpha}} \geq \left((1-\alpha)(1-\mu) + \frac{\chi \mu}{(1-\phi)^{\alpha}}\right) \frac{1 - D(1-\phi)(1-\alpha)}{D(1-\phi)} \\
&\Leftrightarrow \quad \chi \left(\frac{1+\beta}{\beta}\right) \geq \frac{(1-\alpha)(1-\mu)}{\mu(1-\phi)^{1-\alpha}} \left(\alpha - \frac{(1+\beta)(1-\phi)(1-\alpha)}{\beta}\right) \\
&\Leftrightarrow \quad \chi \geq \underline{\chi}_m(\phi)
\end{aligned}
$$

This proofs Proposition 4.

D Proof of Proposition 5

We use equation (18) $U = \tilde{U} - \beta \ln(1-\phi) + [1+\beta(1+\alpha)] \ln k$ and the steady state expression for k from Proposition 2. We also use the definitions for Λ, ϕ_U^{AS} and $\underline{\chi}_U(\phi)$ provided in Proposition 5. Moreover, $\beta, \alpha, \mu \in (0,1)$ and we have restrictions on ϕ and χ characterized in Definition 1. To simplify the notation below, we also define $D \equiv 1 + \frac{1}{\beta} + \frac{1}{1-\phi}$ as in Appendix C. It is then straightforward to write

$$
\begin{aligned}
\frac{dU}{d\phi} \geq 0 \quad \Leftrightarrow \quad & \Lambda \frac{dk}{d\phi}(1-\phi) + k \geq 0 \\
\Leftrightarrow \quad & \Lambda \, \frac{\sigma^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \, \frac{\partial \sigma}{\partial \phi}(1-\phi) + \sigma^{\frac{1}{1-\alpha}} \geq 0 \\
\Leftrightarrow \quad & \frac{\chi \mu \alpha \Lambda}{D(1-\phi)^{\alpha}(1-\alpha)\sigma} - \frac{\Lambda}{D(1-\phi)(1-\alpha)} + 1 \geq 0 \\
\Leftrightarrow \quad & \frac{\chi \mu \alpha \Lambda}{(1-\phi)^{\alpha}} \geq \left((1-\alpha)(1-\mu) + \frac{\chi \mu}{(1-\phi)^{\alpha}} \right) \frac{\Lambda - D(1-\phi)(1-\alpha)}{D(1-\phi)} \\
\Leftrightarrow \quad & \chi \left(\frac{((\Lambda-\alpha)(1-\phi)+1)(1+\alpha(\Lambda-1)) - \Lambda}{1-\phi} \right) \geq \\
& \frac{(1-\alpha)(1-\mu)}{\mu(1-\phi)^{1-\alpha}} \left(\Lambda(1-(1-\phi)(1-\alpha)) - (1-\alpha)(1-\alpha(1-\phi)) \right) \\
\Leftrightarrow \quad & \quad \phi < \phi_U^{AS} \text{ and } \chi \geq \underline{\chi}_U(\phi) \\
& \text{or } \; \phi > \phi_U^{AS} \text{ and } \chi \leq \underline{\chi}_U(\phi)
\end{aligned}
$$

To show we can rule out the second possibility, we must look at what happens when $\phi_U^{AS} < \phi \leq 1$. We know that the denominator of $\underline{\chi}_U(\phi)$ is negative when $\phi > \phi_U^{AS}$. To show that the numerator is positive, we compute $B(\phi) \equiv \Lambda(1-(1-\phi)(1-\alpha)) - (1-\alpha)(1-\alpha(1-\phi))$ at $\phi = \phi_U^{AS}$. It gives $B(\phi_U^{AS}) = (2\alpha + 1/\beta) - (\alpha + 1/\beta) \times \frac{(1-\alpha)^2}{1+\alpha(\alpha+1/\beta)} > 0$. Moreover, we immediately see that the $\partial B(\phi)/\partial \phi = (1-\alpha)(1+1/\beta) > 0$. Then $B(\phi)$ and hence the numerator of $\underline{\chi}_U(\phi)$ are positive when $\phi_U^{AS} < \phi \leq 1$. As a result, $\underline{\chi}_U(\phi) \leq 0$ when $\phi_U^{AS} < \phi \leq 1$, which rules out the second possibility above because this would violate Definition 1. This proofs Proposition 5.

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