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MONETARY THEORY REVERSED: VIRTUAL CURRENCY ISSUANCE AND MINERS' REMUNERATION

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Monetary theory reversed: Virtual currency issuance and miners' remuneration

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Abstract

This study analyzes the macroeconomic implications of virtual currency issuance. It builds on a standard cash-in-advance model extended with (i) 'virtual' goods, sold against virtual currency, and (ii) miners, the agents providing payment services. The main finding is that virtual currency growth may have effects opposite to those predicted by monetary theory when miners are rewarded with newly created coins. Declining currency issuance, as in Bitcoin, raises the price of virtual goods, which counteracts the traditional impact of a reduced inflation tax. The paper also shows how fiat money growth affects the welfare effects of virtual currency creation.

Keywords: Cash-in-advance, virtual currency, fiat money, money supply.

JEL-Code: E41, E42, E51.

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Résumé non-technique

L'émergence de monnaies virtuelles, dont la plus célèbre est le bitcoin, a attiré beaucoup d'attention au cours des dernières années.[1] Le bitcoin diffère d'une monnaie nationale ayant cours légal et n'est pas régulé par une autorité monétaire.[2] Un algorithme informatique détermine l'émission de bitcoins, qui se produit à un rythme décroissant. Le nombre de bitcoins émis est réduit de moitié environ tous les quatre ans et le total des bitcoins en circulation ne dépassera jamais 21 millions. Les utilisateurs n'ont pas besoin de passer par une institution financière lorsqu'ils vendent ou achètent des articles avec des bitcoins. Les transactions avec des bitcoins sont traitées par des agents privés, les *mineurs*, qui sont rémunérés avec les frais de transaction et les bitcoins nouvellement émis. Au fur et à mesure que l'émission de monnaie virtuelle décroît, les frais de transaction devront augmenter pour que les mineurs continuent à fournir les services de paiement liés aux opérations avec les bitcoins.

Cette étude analyse les conséquences d'une diminution de l'émission de monnaie virtuelle et de l'augmentation des frais de transaction qui s'en suit. Elle s'appuie sur un modèle macroéconomique standard en y incorporant (i) un bien supplémentaire, dit 'virtuel', vendu en monnaie virtuelle et (ii) des mineurs. Les biens virtuels résultent de la combinaison de biens intermédiaires et de services de paiement fournis par les mineurs. Les mineurs valident les transactions en monnaie virtuelle et sont payés avec de la monnaie virtuelle nouvellement créée et avec des frais de transaction, inclus dans les opérations avec la monnaie virtuelle. L'approche adoptée ici est que les consommateurs achètent les biens virtuels sur une plate-forme associant les biens physiques (intermédiaires) produits par les entreprises

[1]Les monnaies virtuelles appartiennent à la famille des monnaies numériques, incluant toute monnaie stockée et transférée électroniquement. Il n'existe pas de définition unique de "monnaie virtuelle", car ces systèmes de paiement évoluent continuellement (ECB, 2012; IMF, 2016). Dans cette étude, le terme "monnaie virtuelle" désigne toute monnaie numérique acceptée comme paiement pour des biens réels, comme les cryptomonnaies (par exemple le bitcoin). En outre, ce document adopte la convention utilisant un B majuscule (Bitcoin) pour se référer au système de paiement et un b minuscule pour se référer à l'unité de compte (bitcoin). Finalement, qualifier les monnaies virtuelles de "monnaie" est sujet à débat. Certains auteurs soutiennent, en effet, qu'elles ne remplissent qu'imparfaitement les fonctions principales d'une monnaie: unité de compte, réserve de valeur et intermédiaire des échanges (p.ex. Yermack, 2014). Dans le contexte de l'Union Européenne, les "monnaies virtuelles" ne sont d'ailleurs pas considérées comme des monnaies (voir la définition de monnaie dans l'article 2 de la directive 2014/62/UE) et certaines institutions, telles que l'Autorité bancaire européenne, les qualifient de "représentations numériques d'une valeur" (EBA, 2014, p.11, paragraphe 19).

[2]En principe, le cours légal s'applique à la monnaie en circulation sur un territoire et signifie que cette monnaie ne peut être refusée en règlement d'une dette. En zone euro, l'euro est la monnaie unique et les billets et pièces mis à disposition par les banques centrales de l'Eurosystème (monnaie fiduciaire) doivent, de manière générale, être acceptés comme moyen de paiement. Par contre, la monnaie scripturale, comme les dépôts bancaires dans des comptes courants, n'a pas de cours légal (p.ex. un créancier n'est pas tenu à accepter un règlement par transfert bancaire).

intermédiaires aux services de paiement fournis par des mineurs.

L'analyse se concentre sur les conséquences d'une baisse de la croissance monétaire telle que prévue dans un système de monnaie virtuelle, comme le Bitcoin, où le taux d'émission de nouvelles pièces est initialement élevé et diminue progressivement jusqu'à atteindre zéro. La principale conclusion de cette étude est que la croissance de la monnaie virtuelle, lorsque les mineurs sont payés avec de la monnaie virtuelle nouvellement créée, peut avoir des effets opposés à ceux prédits par la théorie monétaire standard. Une baisse du taux d'émission de la monnaie virtuelle agit à travers deux canaux. Un premier effet passe par une réduction de la taxe d'inflation émanant de la monnaie virtuelle, ce qui stimule la demande de biens virtuels (sans affecter les prix relatifs des biens). C'est le seul canal par lequel le taux d'émission affecte l'économie lorsque les mineurs sont uniquement rémunérés avec les frais de transaction. Cet effet standard passant par la taxe d'inflation est contrecarré par le *mécanisme de récompense*, qui apparaît lorsque les mineurs sont également rémunérés avec de la monnaie virtuelle nouvellement émise. A travers ce nouveau mécanisme, une baisse de l'émission de monnaie virtuelle augmente les frais de transaction, ce qui résulte en une hausse du prix relatif des biens virtuels (par rapport aux prix des autres biens de consommation) et en une réduction de leur demande. Des extensions du modèle considèrent les effets de taxes de consommation ainsi que d'une sécurité imparfaite du système de paiement virtuel (la sécurité s'entend comme la proportion d'encaisses de monnaie virtuelle garanties). Ces éléments - taxes de consommation et sécurité imparfaite - renforcent le mécanisme de récompense qui peut dominer l'effet traditionnel lié à la taxe d'inflation.

Un autre résultat majeur est que les effets de l'émission de monnaie virtuelle sur le bien-être dépendent de la croissance de la monnaie nationale, c'est-à-dire la monnaie émise par l'autorité monétaire nationale. Le bien-être affiche une forme de cloche lorsque l'émission de monnaie virtuelle décroît: il s'améliore d'abord à des taux d'émission élevés, mais se détériore lorsque cette croissance atteint des taux faibles. Plus la croissance de la monnaie nationale est élevée, moins la baisse de croissance de la monnaie virtuelle sera bénéfique pour le bien-être. Enfin, le modèle est calibré et simulé pour des systèmes de paiement virtuel différents (la proportion de monnaie virtuelle nouvellement émise rémunérant les mineurs varie selon les systèmes).

1 Introduction

The expansion of the Internet triggered the emergence of virtual currencies, of which the most famous, Bitcoin, attracted a lot of attention recently.[3] Bitcoin differs from a national currency with legal tender status.[4] It is not controlled by a monetary authority and an exogenous rule determines the continuous issuance of bitcoins, which occurs at a decreasing rate. The number of bitcoins issued is halved approximatively every four years and will never exceed 21 millions (see figure 1). Bitcoin users do not need payment services provided by a financial institution when selling or buying items with bitcoins. Transactions with bitcoins are processed by private agents, *miners*, who are remunerated with newly issued currency and transaction fees. One question raised by the Bitcoin payment system concerns the consequences of the progressive fading out of newly created coins, which constitute, for now, the biggest part of miners' income, and, more importantly, of the eventually associated increase in transactions fees required to keep miners recording the transactions. Indeed, the original Bitcoin document mentions that "once a predetermined number of coins have entered circulation, the incentive [for miners to process transactions] can transition entirely to transaction fees" (Nakamoto, 2008, p.4).

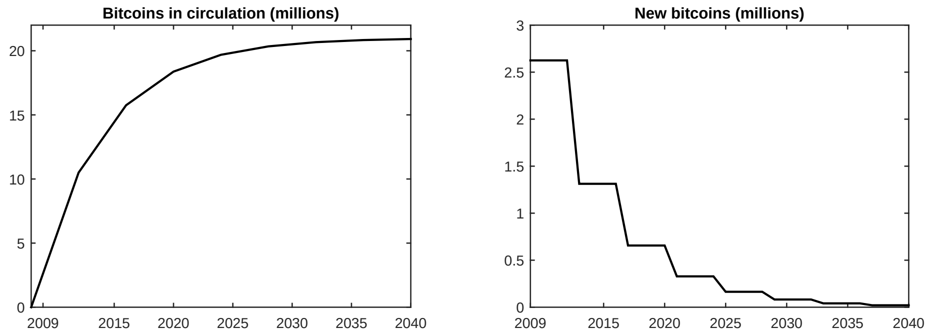
This paper investigates the theoretical effects of lower virtual currency issuance and of the implied rise in fees. No research so far addresses the macroeconomic consequences of such a mechanism, which is, however, essential for the Bitcoin debate. Bitcoin supporters praise the transparency of its issuance rule, which contrasts with the unknown future evolution of "government-manipulated" fiat money. Moreover, the shrinking growth in bitcoins endangers the currently low transactions fees, another beloved aspect of this currency. Understanding the implications of lower issuance is also relevant for other cryptocurrencies sharing similar features as Bitcoin. Indeed, since Bitcoin is open-source, it inspired many alternatives, though Bitcoin remains the most important cryptocurrency, representing more than one third of the total market capitalization of all cryptocurrencies.[5] Finally, central

[3] Virtual currencies belong to the broader family of digital currencies, which can be defined as representing any currency that is stored and transferred electronically. There is no single definition of 'virtual currency', as these payment systems evolve continuously (ECB, 2012; IMF, 2016). The term virtual currency is meant here to include any digital currency accepted as payment for real goods, like cryptocurrencies (e.g. Bitcoin). Moreover, this paper adopts the convention in the computer science literature using capital-B Bitcoin to refer to the system and lower-b bitcoin to refer to the unit of account. Note that whether virtual currencies are "money" is open to discussion (see e.g. Yermack, 2014). The European Union does not consider them "money" (which is defined in article 2 of the directive 2014/62/UE) and some institutions, like the European Banking Authority, describe them as a "digital representation of value" (EBA, 2014, p.11, point 19).

[4] Generally speaking, for a means of payment to be legal tender the law must require that it be accepted for the payment of debts. The euro is the single currency of the euro area countries and coins and banknotes made available by the European System of Central Banks must be accepted as a means of payment in the euro area. In contrast, deposits in bank accounts are not legal tender (e.g. a creditor is not obliged to accept a payment with a bank transfer).

[5] There exist several hundred alternatives, so-called 'Altcoins' (like Ethereum, Ripple or Litecoin) derived

Figure 1: Predetermined Bitcoin evolution



Source: blockchain.info.

banks are more and more interested by virtual currency payment systems and the present paper also tries to understand how fiat money growth interacts with the effects of virtual currency issuance.

This paper builds on a cash-credit model to analyze the macroeconomic effects of a reduction in virtual currency issuance (Lucas and Stokey, 1983, 1987). The model is characterized by the introduction of (i) an additional good, called 'virtual good', sold against an alternative currency to legal tender money and (ii) miners. It is kept simple otherwise, ignoring e.g. labor-leisure choices, to focus on the effects of virtual currency issuance. Households need to hold fiat money, understood here as the national currency, to purchase cash goods and virtual currency to buy virtual goods. Final goods are obtained on a one-to-one basis from intermediate goods, except virtual goods, which result from the combination of intermediate goods with payment services. This allows to introduce miners, who validate the virtual transactions and who are paid with newly created virtual money and transaction fees, i.e. the value of payment services. The approach taken here is that consumers purchase virtual goods on a platform, which associates the (intermediate) physical goods produced by intermediate firms to the payment services provided by miners. This is consistent with the fact that consumers cannot choose the miners that will process their transactions.[6]

The analysis focuses on the implications of a virtual currency system, like Bitcoin, where the issuance rate of new coins is initially high and progressively decreases until it reaches zero. The main finding is that virtual currency growth may have effects opposite to those

from Bitcoin. Each introduces a change compared to Bitcoin, supposed to be an improvement over it. Declining growth in "base money" issuance can be found in other virtual currencies (e.g. Ethereum). The Bitcoin dominance stands at 36% (February 2^{nd}, 2018) and was never below 70% before March 2017, see https://coinmarketcap.com.

[6] A similar modeling is, for instance, assumed in Aiyagari et al. (1998), where one unit of credit good results from the combination of one unit of intermediate good and some units of credit services.

predicted by standard theory when miners are paid with newly created coins. When they are not, lower virtual currency issuance reduces the inflation tax on virtual money, which raises the demand for virtual goods without affecting relative prices. This standard inflation tax effect is counteracted by what is called the reward mechanism, active when newly issued virtual currency accrues to miners. Through this novel channel, lower growth raises transaction fees, which leads to a rise in the relative price of virtual goods and depresses the demand. Model extensions consider the effects of consumption taxes and of an insecure virtual currency system, in the sense that ownership of virtual coins is not fully guaranteed. The presence of these elements raises the demand for virtual currency, as more virtual money is needed to purchase a given quantity of virtual goods, and strengthens the reward mechanism, which may even dominate the inflation tax effect.

Another major finding is that the welfare effects of virtual currency issuance depend on fiat money growth. Welfare displays an inverted U-shape with decreasing virtual currency creation: welfare improves initially at high issuance rates, but deteriorates as virtual currency growth becomes smaller and smaller. The higher fiat money growth, the higher the threshold level of virtual currency growth (above which lower issuance raises welfare). The uniform tax theorem of Atkinson and Stiglitz (1976) provides some intuition for this result (see Chari et al., 1996). In short, in a simple two-goods barter economy with standard utility functions, optimal consumption taxes are set at the same rate such that one distortion compensates the other, but optimal rates may differ as the model becomes more complex. In the present model, there is an optimal wedge between money growth rates reflecting the specific features of the goods.[7] The model is also calibrated to illustrate the results for various virtual payment system designs, differing in the share of new virtual coins used to remunerate miners.

The economic literature on Bitcoin and similar virtual currencies is in its early stages, but the interest in this topic is growing (see the discussion in Böhme et al., 2015).[8] None of these studies examines the consequences of virtual currency issuance, though related issues have been addressed by several strands of the macroeconomic literature. Chari et al. (1996) dis-

[7]The welfare results explained here hold in the more likely situation that national currency growth is not too low. Below a certain level of fiat money growth, which happens to be negative in the calibration, any reduction in virtual currency issuance is welfare improving (in line with the intuition based on the Atkinson-Stiglitz theorem).

[8]Biais et al. (2017) analyze the strategic behavior of miners in a game-theoretical framework, while Houy (2015) studies the economics of Bitcoin transaction fees and (Chiu and Wong, 2015) look at frictions involved with e-money systems such as Bitcoin. Hendrickson and Luther (2017) and Hendrickson et al. (2016) employ a monetary model to examine the conditions under which a government can ban an alternative currency like Bitcoin because it favors illegal activities. Fernández-Villaverde and Sanches (2016) explore the theoretical conditions under which multiple competing cryptocurrencies can co-exist. There also exist studies on other types of digital currencies, like Barrdear and Kumhof (2016), who quantitatively analyze the implications of a hypothetical introduction of central bank digital currency, a system that does not (yet) exist.

cuss optimal inflation in cash-credit models and, in particular, the conditions under which the Friedman rule is optimal (i.e. a zero nominal interest rate implying a negative inflation rate). Cooley and Hansen (1991) quantitatively evaluate the welfare costs of inflation in a cash-credit model. These are single-currency studies. Lucas (1982) and Svensson (1985) develop two-currency frameworks but do not focus on welfare, while Guidotti and Vegh (1993) look at optimal inflation in a model where foreign currency can be used to buy non-traded goods.[9] However, few models consider the role of costly payment services. Gillman (1993) constructs a cash-credit model where buying credit goods is time-costly for the consumer, who implicitly acts as a 'banker'. Aiyagari et al. (1998) demonstrate that higher inflation makes cash goods more expensive and thereby raises the demand for credit goods, which stimulates credit services necessary to buy these goods. Unlike the latter two studies, the present paper highlights a novel channel of currency issuance, which through miners' remuneration affects relative prices and demand. Moreover, the model features two currencies and shows how fiat money influences the welfare effects of virtual currency issuance.

The rest of the paper is organized as follows. Section 2 presents the model and section 3 discusses the steady state results of a reduction in virtual currency issuance. The model is calibrated and simulated in section 4. Additional model features are introduced in section 5. Section 6 concludes.

2 The Model

This section develops a cash-credit model with an additional sector producing 'virtual' goods that can only be bought with virtual currency. The household derives utility from three goods: cash, credit and virtual goods.[10] A traditional cash-in-advance (CIA) constraint obliges the representative agent to buy some goods – cash goods – with cash only, while other goods – credit goods – can be purchased on credit. An additional CIA constraint implies that virtual goods can only be acquired with virtual currency, which is exogenously created. Intermediate firms produce intermediate goods using capital and labor. The virtual good results from a combination, in fixed proportions, of intermediate goods and payment services provided by private agents, the miners. These are rewarded with newly issued virtual currency and transactions fees. As usual, the other final goods are obtained on a one-to-one basis from intermediate goods.

[9]Guidotti and Vegh (1993) consider a small open economy where purchasing goods requires shopping time, while Lucas (1982) and Svensson (1985) develop two-country models with cash-in-advance constraints on domestic and foreign goods. The latter two studies operate in a stochastic environment, while the other papers mentioned here (as well as the present one) do not feature uncertainty.

[10]As explained in the text, cash and virtual goods depend on money holdings accumulated over the *previous* period and thus credit goods are introduced to assure the model's forward-looking dynamics (but this does not affect the main conclusions of the paper). Introducing leisure in the utility function would be an alternative to credit goods but complicates the analysis.

2.1 Households

There is a single infinitely-lived representative household (of size 1) deriving utility from three types of consumption goods (cash consumption goods, c_1, credit consumption goods, c_2 and virtual consumption goods, c_3). Every period, the household is endowed with one unit of time (per capita time endowment) and devotes a fixed proportion, h, to labor and the rest to leisure. The household maximizes expected utility over an infinite horizon

$$\sum_{t=0}^{\infty} \beta^t \left[\psi_1 \ln(c_{1,t}) + \psi_2 \ln(c_{2,t}) + \psi_3 \ln(c_{3,t})\right] \tag{1}$$

where $\psi_3 = 1 - \psi_1 - \psi_2$. The household enters period t with nominal money balances, composed of fiat money, M_t, and virtual currency, V_t. Fiat money is intended here as the currency issued by the central bank (the terms *fiat*, *national* and *government* are used interchangeably to denote this type of money). The household's purchases are subject to two cash-in-advance constraints. Purchases of cash goods (respectively virtual goods) are limited by the holdings of government money (respectively virtual currency holdings) carried over from the previous period:

$$P_t\, c_{1,t} \leq M_t$$

$$P_t^*\, c_{3,t} \leq V_t$$

As is usual in the literature, the analysis focuses on cases where both constraints hold with equality.[11]

The nominal income of the household comprises wage income, $P_t w_t h$, interest income from capital holdings, $P_t r_t K_t$, and lump sum transfers, Ω_t. These transfers are made up of newly issued fiat (and eventually virtual) money and are defined further below. The household's budget constraint, expressed in fiat money, requires that, each period, the income and the money carried over from the previous period finances purchases of goods during the period plus bond and money holdings for the next period

$$M_{t+1} + Q_t V_{t+1} = M_t + Q_t\, V_t + P_t w_t h + P_t r_t K_t - P_t I_t - P_t(c_{1,t} + c_{2,t}) - Q_t P_t^* c_{3,t} + P_t \Omega_t$$

where K_t is capital, r_t is the real interest rate associated with capital, w_t is the real wage rate, I_t is investment, P_t is the unit price of cash and credit goods in fiat money, P_t^* is the unit price of virtual goods in virtual currency and Q_t is the nominal exchange rate i.e. the price of virtual currency in fiat money. Superscript * identifies prices expressed in virtual currency. The budget constraint already incorporates two no-arbitrage conditions regarding

[11] For instance, it can be shown that the first constraint binds when the nominal interest factor is larger than one: $1 + R_t \equiv (1 + r_t - \delta)(1 + \pi_{t-1}) > 1\ ,\quad \forall t$. The intuition is that when the gross return from investing in capital is larger than from holding cash (gross return of one), then money is held just to satisfy the consumption of cash goods. In the steady state, the conditions for binding cash-in-advance constraints boil down to both money growth rates exceeding $\beta - 1$, with $\beta < 1$.

labor and capital income, i.e. each of these factors earns the same income across firms and sectors.

From now onwards, the model is presented in real terms, starting with the budget constraint

$$m_{t+1}(1+\pi_t) + q_t v_{t+1}(1+\pi_t^*) = m_t + q_t\, v_t + w_t h + r_t K_t - I_t \\ -(c_{1,t} + c_{2,t}) - q_t\, c_{3,t} + \Omega_t \tag{2}$$

where real holdings of fiat money and virtual currency are respectively defined as $m_t \equiv \frac{M_t}{P_t}$ and $v_t \equiv \frac{V_t}{P_t^*}$. The inflation rates of both moneys are $\pi_t \equiv \frac{P_{t+1}}{P_t} - 1$ and $\pi_t^* \equiv \frac{P_{t+1}^*}{P_t^*} - 1$ and the real exchange rate is $q_t \equiv \frac{Q_t P_t^*}{P_t}$, the price of virtual goods in terms of 'real' goods (i.e. the other final goods, which are priced in national currency). The cash-in-advance constraints become

$$c_{1,t} = m_t \tag{3}$$

$$c_{3,t} = v_t \tag{4}$$

Capital accumulates according to the standard law of motion

$$K_{t+1} = (1-\delta)K_t + I_t \tag{5}$$

where δ is the depreciation rate of capital.
The household maximizes (1) with respect to $c_{1,t}, c_{2,t}, c_{3,t}, K_{t+1}, m_{t+1}, v_{t+1}$ subject to (2)-(5). Combining the first-order conditions of the household maximization problem, yields:

$$\frac{c_{2,t+1}}{c_{2,t}} = \beta\,(1 + r_{t+1} - \delta), \tag{6}$$

$$\frac{\psi_1}{\psi_2}\frac{c_{2,t}}{c_{1,t}} = (1 + r_t - \delta)(1 + \pi_{t-1}) \tag{7}$$

$$\frac{\psi_3}{\psi_2}\frac{c_{2,t}}{c_{3,t}} = (1 + r_t - \delta)\, q_{t-1}\, (1 + \pi_{t-1}^*) \tag{8}$$

Equation (6) is the Euler equation, while equations (7) and (8) state that the mix of consumption goods should be chosen so that the marginal rate of substitution equals the relative price.

2.2 Production

The production side of the economy consists of intermediate firms, final firms and miners. Intermediate goods are transformed into cash, credit and investment goods on a one-to-one basis, while virtual goods result from the combination of intermediate goods with payment services provided by miners. A representative intermediate firm (z) operates under perfect competition, implying that all goods sell at the same price. Moreover, cash, credit and investment goods are sold to consumers at the price of intermediate goods (from which they are directly derived). The output of the intermediate firm, Y_z, can be decomposed as follows

$$Y_{z,t} = c_{1,t} + c_{2,t} + I_t + Y_{v,t} \tag{9}$$

where Y_v are intermediate goods used in the production of virtual goods. The intermediate firm uses a constant returns to scale technology with labor and capital as inputs

$$Y_{z,t} = A_z\, K_t^{\alpha}\, h_{z,t}^{1-\alpha} \tag{10}$$

where α is the capital share in production, K capital, while A_z is total factor productivity and h_z labor in the intermediate firm. The demands for capital and labor of the profit-maximizing z-firm are

$$r_t = \alpha\, \frac{Y_{z,t}}{K_t} \tag{11}$$

$$w_t = (1-\alpha)\, \frac{Y_{z,t}}{h_{z,t}} \tag{12}$$

The representative (final) virtual firm can be seen as the platform on which consumers can purchase virtual goods, which are a combination of intermediate goods and payment services. The perspective adopted here is that consumers use digital wallets to purchase goods, whose price includes 'transactions fees', i.e. the value of payment services that are part of miners' income. Thus the representative virtual firm produces virtual goods using intermediate goods, Y_v, and payment services, Y_x, and operates under perfect competition.

It is assumed that one unit of virtual good output (Y^*) is composed of one unit of physical good associated to $1/\gamma_x$ units of mining services, where γ_x represents the required units of mining services for one unit of output. Similarly, but in a different context, Aiyagari et al. (1998) assume that one unit of credit good results from the combination of one unit of intermediate good and some units of credit services. The total production cost amounts to $\frac{1}{q_t}Y_{v,t} + p_{x,t}Y_{x,t}$, where $1/q_t$ is the relative price of intermediate goods sold to the final virtual firm, $1/q_t \equiv P_t/(Q_t P_t^*)$, and $p_{x,t}$ is the relative price of payment services, $p_{x,t} \equiv P_{x,t}/P_t^*$. The final firm minimizes the cost subject to the constraint

$$Y_t^* = \min\left\{Y_{v,t},\, \frac{Y_{x,t}}{\gamma_x}\right\}$$

Combining the optimal input quantities $Y_{v,t} = Y_t^*$ and $Y_{x,t} = \gamma_x Y_t^*$ leads to

$$Y_{x,t} = \gamma_x Y_{v,t} \tag{13}$$

The zero-profit condition yields

$$\frac{1}{q_t} + \gamma_x p_{x,t} = 1 \tag{14}$$

As observed e.g. with the Bitcoin system, when the buyer of a good sends one unit of virtual currency through the network (i.e. the 'inputs'), the seller of the intermediate good receives $1/q_t$ (i.e. the 'outputs') and the difference between the two represents the 'fee', $\gamma_x p_{x,t}$.

As mentioned above, the agents providing the payment services necessary to process virtual transactions, miners, are paid with 'transaction fees', i.e. the value of their mining services. An original feature of a virtual currency payment system like Bitcoin is that miners are also granted newly issued virtual money, $P_t^* \Gamma_{v,t}$. Miners are thought of private agents who may work alone or together with other miners in 'mining pools' (and then share their revenues). Though they require computers and special software to run their activity, they are free to work from home without having to rent buildings, factories, warehouses etc. It is therefore assumed here that miners do not need to rent capital and produce payment services according to $Y_{x,t} = A_x\, h_{x,t}$, where A_x is productivity and $h_{x,t}$ is time spent validating transactions.

The zero profit condition yields the representative miner's real wage

$$w_{x,t} = \frac{1}{h_{x,t}} \left(p_{x,t} Y_{x,t} + \theta\, \Gamma_{v,t} \right) \tag{15}$$

where $\Gamma_{v,t}$ is newly issued virtual currency in real terms, of which a share $\theta \in [0, 1]$ accrues to the miner and the rest is retained by the developer of the system (and transferred lump sum to households). The first term in equation (15) corresponds to the value of mining services (transaction fees) and the second term to income from newly issued virtual currency (both per unit time of labor).

2.3 Monetary policy

Denote by g_m the growth of the nominal money supply, M^s. Each period, the monetary authority provides lump sum monetary transfers to the household, $P_t\, \Gamma_{m,t}$, where $\Gamma_{m,t} \equiv g_{m,t}\, m_t^s$ and $m_t^s \equiv M_t^s / P_t$. The nominal money supply evolves according to $M_{t+1}^s = M_t^s + P_t \Gamma_{m,t}$, and, in real terms:

$$m_{t+1}^s = \frac{1 + g_{m,t}}{1 + \pi_t} m_t^s \tag{16}$$

2.4 Virtual currency rule

The nominal virtual currency supply, V^s, evolves according to $V_{t+1}^s = V_t^s + P_t^* \Gamma_{v,t}$, where $\Gamma_{v,t} \equiv g_{v,t}\, v_t^s$, $v_t^s \equiv V_t^s / P_t^*$ and g_v is the growth rate of nominal virtual currency supply. In real terms, the dynamics of virtual money are described by

$$v_{t+1}^s = \frac{1 + g_{v,t}}{1 + \pi_t^*}\, v_t^s \tag{17}$$

One feature of Bitcoin and other cryptocurrencies is that they do not envisage a decrease in the number of virtual coins in circulation. Thus virtual currency growth is assumed non-negative throughout this study, $g_v \geq 0$.

2.5 Government

Government revenue, T_t, is transferred to the household as a lump sum and financed through the printing of fiat money. The government budget constraint is $T_t = g_{m,t}\, m_t^s$.

2.6 Market clearing conditions

The equilibrium conditions for government money and virtual currency imply

$$m_t^s = m_t \tag{18}$$

$$v_t^s = v_t \tag{19}$$

The no-arbitrage condition for labor income implies that miners are paid the same real wage as other workers

$$w_t = q_t w_{x,t} \tag{20}$$

The equilibrium conditions in the labor and virtual goods markets are characterized by

$$h = h_{r,t} + h_{x,t} \tag{21}$$

$$Y_{v,t} = c_{3,t} \tag{22}$$

Furthermore, $T_{v,t}$ represents newly issued virtual currency that does not serve as a reward for miners and which accrues lump sum to the households

$$T_{v,t} = (1 - \theta_t)\, g_{v,t}\, v_t^s \tag{23}$$

When $\theta = 1$, all the newly issued virtual money accrues to miners and when $\theta = 0$ it goes directly to households. Total lump sum transfers to households equal $\Omega_t = T_t + q_t T_{v,t}$.

The economy's resource constraint is

$$Y_{z,t} + q_t\, p_{x,t}\, Y_{x,t} = c_{1,t} + c_{2,t} + I_t + q_t\, c_{3,t} \tag{24}$$

Equation (24) states that all output is consumed or invested. Finally, equations (6)-(12), (16), (17), (20)-(24) characterize the economy.

3 Steady state analysis

This section provides analytical steady state results on the implications of virtual money growth. It first analyzes the effects on relative prices, sectoral labor activity and consumption before looking at the welfare impact.

Effect on relative prices

Before looking at the impact of virtual money issuance on the relative prices of mining services, p_x, and virtual goods, q, it is useful to derive some intermediate results. In the steady state, the Euler equation (6) implies that the interest rate equals $r = \frac{1}{\beta} - 1 + \delta$. It follows that the output-labor ratio in the intermediate firm is constant, as observed from rearranging equation (10), $\frac{Y_z}{h_z} = A_z(K/h_z)^{\alpha}$ with $\frac{K}{h_z} = (A_z\alpha/r)^{\frac{1}{1-\alpha}}$ resulting from equation (11). The real exchange rate, q, can be obtained from equation (14)

$$q = \frac{1}{1 - \gamma_x p_x} \tag{25}$$

Virtual currency growth, g_v, affects q through p_x. As explained below, when lower growth triggers a rise in transaction fees, represented by the price of mining services, p_x, the real exchange rate, q, increases.

Combining equations (4), (13), (22) as well as $\Gamma_v \equiv g_v v$ with equation (15) yields the following expression for the representative miner's wage

$$w_x = \frac{A_x}{\gamma_x}(\gamma_x p_x + \theta g_v)$$

The relative price of mining services can be obtained by inserting the above expressions for q and w_x in the no-arbitrage condition (20):

$$p_x = \frac{1}{\gamma_x} \frac{(1-\alpha)e - \theta g_v}{(1-\alpha)e + 1} \tag{26}$$

where e is a combination of parameters, $e \equiv \frac{Y_z}{h_z}\frac{\gamma_x}{A_x}$. From the above equation, it is straightforward to see that a decrease in g_v has no effect on p_x when $\theta = 0$, but raises p_x when $\theta > 0$. This latter case applies to currencies like Bitcoin, since a fading out of revenues from newly created virtual currency must be compensated by higher 'transaction fees'. Indeed, Bitcoin foresees that transaction fees will constitute the only incentive for miners to validate transactions once a predetermined number of coins has been attained (Nakamoto, 2008, p.4). The proposition below summarizes this effect:

Proposition 1 *Following a decline in virtual currency issuance, the price of mining services*

(i) remains unchanged when miners are paid with fees only, i.e., $\frac{\partial p_x}{\partial g_v} = 0$, if $\theta = 0$

(ii) increases when miners are also rewarded with newly created currency, i.e., $\frac{\partial p_x}{\partial g_v} < 0$, if $\theta > 0$

The proof of Proposition 1 follows from equation (26). When $\theta = 0$, a change in g_v affects neither p_x nor the real exchange rate, q, see equation (25). However, when $\theta > 0$, a reduction in virtual currency issuance reduces miners' revenue from newly issued coins and thus fees need to increase, which raises the real exchange rate, i.e. the price of final virtual goods relative to other final goods.

Effect on the employment ratio

In the steady state, $\pi_m = g_m$ and $\pi_v = g_v$ (see equations (16) and (17)). The miner-to-worker employment ratio, defined as $n \equiv h_x/h_z$, can be written as

$$n = \frac{a\,(1-b)\,e}{a + (1+g_v)\,q} \tag{27}$$

where $a, b > 0$ are parameter combinations (see Appendix A). Thus the impact of g_v on n is described by

$$\frac{dn}{dg_v} = \underbrace{\frac{\partial n}{\partial g_v}}_{<0} + \underbrace{\frac{\partial n}{\partial q}\,\frac{\partial q}{\partial p_x}\,\frac{\partial p_x}{\partial g_v}}_{\geq 0} \tag{28}$$

Equation (28) indicates that virtual currency issuance affects labor reallocation through two channels. The first channel, called *inflation tax effect*, is represented by the first term. A decrease in virtual money growth, g_v, reduces the inflation tax and increases the demand for virtual goods. This stimulates the production of virtual goods and raises the attractiveness of the mining activity (increase in n). The second term of equation (28) depends on $\frac{\partial p_x}{\partial g_v}$ and constitutes an additional force called the *reward mechanism*. This channel is absent when miners' remuneration is based on fees only ($\theta = 0$, see Proposition 1). However, when newly issued virtual currency accrues to miners ($\theta > 0$), additional price effects come into play (as $\frac{\partial p_x}{\partial g_v} > 0$). Indeed, a lower issuance of virtual money means less rewards for miners, driving up the price of mining services and rendering virtual goods more expensive (higher p_x and q). This depresses the demand for virtual goods and makes the mining activity less attractive. It is worth noting that remunerating miners with newly created money acts like a subsidy to the virtual sector and introduces a distortion in the reallocation of labor that is reduced by a lower virtual currency issuance. To sum up, the inflation tax effect and the reward mechanism act in opposite directions.

Equation (28) can be expressed as follows

$$\frac{dn}{dg_v} = -\frac{n\,q}{a + (1+g_v)\,q}\,\frac{1-\theta}{1+\theta\,g_v} \tag{29}$$

This result leads to the next proposition, with proof located in Appendix A:

Proposition 2 *As a result of lower virtual currency creation, the miner-to-worker employment ratio*

(i) increases, because the reward mechanism only mitigates the inflation tax effect when no or not all newly issued virtual currency accrues to miners, i.e., $\frac{\partial n}{\partial g_v} < 0$, if $0 \leq \theta < 1$

(ii) remains unchanged, since the reward mechanism exactly compensates the inflation tax effect when all newly created virtual money goes to miners, i.e., $\frac{\partial n}{\partial g_v} = 0$, if $\theta = 1$

It is worth noting that the size of θ measures the distortion in the reallocation of labor induced by the reward mechanism. The higher θ the larger the miner-to-worker ratio.

Effect on consumption

It is possible to show that c_3 depends on n, as explained in Appendix B

$$c_3 = \frac{A_x}{\gamma_x}\, h \left(1 - \frac{1}{1+n}\right) \tag{30}$$

Equation (30) offers the following insight. When $0 \leq \theta < 1$, a reduction in virtual money issuance attracts workers to the mining activity raising thereby the production of virtual goods. However, when $\theta = 1$, reward mechanism offsets the inflation tax effect and c_3 remains unchanged.

From equation (7), it can be observed that the consumption of cash goods moves proportionally with the consumption of credit goods c_2

$$c_1 = \frac{\psi_1}{\psi_2}\, \frac{1}{(1+r-\delta)(1+g_m)}\; c_2 \tag{31}$$

Appendix B explains how c_2 can be written as a function of n. The reaction of c_2 to a change in g_v depends negatively on $\frac{\partial n}{\partial g_v}$

$$\frac{\partial c_2}{\partial g_v} = -\,\vartheta \cdot \frac{\partial n}{\partial g_v} \tag{32}$$

where $\vartheta \equiv \frac{\psi_2(1+r-\delta)(1+g_m)}{\psi_1+\psi_2(1+r-\delta)(1+g_m)} \frac{Y_z}{1+n}(1-b+\frac{1}{e}) > 0$.

The implications of virtual money issuance on the consumption of the three goods are summarized in the proposition below, with proof in Appendix B.

Proposition 3 *Lower virtual currency creation induces*

(i) a reduction in the consumption of cash and credit goods, c_1 and c_2, and an increase in the consumption of virtual goods, c_3, when not all newly created virtual money accrues to miners, i.e., $\frac{\partial c_1}{\partial g_v} > 0, \frac{\partial c_2}{\partial g_v} > 0, \frac{\partial c_3}{\partial g_v} < 0,$ if $0 \leq \theta < 1$

(ii) no change in the three consumption goods when all newly issued coins go to miners, i.e., $\frac{\partial c_1}{\partial g_v} = \frac{\partial c_2}{\partial g_v} = \frac{\partial c_3}{\partial g_v} = 0,$ if $\theta = 1$

Effect on welfare

Inserting c_1 and c_3, written as functions of c_2, in equation (1) yields the following formulation for steady state welfare, W:

$$W = \frac{1}{1-\beta}\, [\chi + \ln\, c_2 - \psi_3 \ln\,(1+g_v) - \psi_3 \ln q\,] \tag{33}$$

where χ is a collection of parameters (see Appendix C for details). The effect of virtual money growth on welfare is depicted by:

$$\frac{\partial W}{\partial g_v} = \frac{1}{1-\beta}\left[\; \underbrace{\underbrace{-\,\xi\,\frac{\partial n}{\partial g_v}}_{\substack{\text{indirect}\\ \text{effects}}}}_{\geq 0} \;\; \underbrace{\underbrace{-\,\frac{\psi_3}{1+g_v}}_{\substack{\text{direct effect of the}\\ \text{inflation tax } (<0)}} \;+\; \underbrace{\frac{\psi_3\,\theta}{1+\theta\, g_v}}_{\substack{\text{direct effect of the}\\ \text{reward mech. } (\geq 0)}}}_{\leq 0}\;\right] \tag{34}$$

where $\zeta \equiv \vartheta/c_2 > 0$. The inflation tax effect acts directly on welfare through the second term of equation (34) and indirectly through the first term. Similarly, the reward mechanism affects welfare directly through the third term and indirectly through labor reallocation appearing in the first term. One way to understand equation (34) is to interpret the first term as a quantity effect, while the second term describes a tax effect and the third term a price effect. As shown in equation (34), the indirect effects (first term) and the direct effects (combination of the second and third terms) work in opposite direction. Before stating the conditions determining which of the two dominate, it is useful to define two threshold values $(\bar{g}_m, \bar{g}_v)$ and to discuss two special cases (when $\theta = 0$ and when $\theta = 1$).

Define

$$X \equiv \frac{(\psi_1 + \psi_2)(1 + g_m)}{\psi_1 + \psi_2(1 + r - \delta)(1 + g_m)} \, \frac{1 + e\left(1 - \alpha\frac{\delta}{r}\right)}{1 + e(1 - \alpha)} \tag{35}$$

Then $\bar{g}_v \equiv \frac{X-1}{1-\theta X}$ and $X > 1 \Leftrightarrow g_m > \bar{g}_m$, where

$$\bar{g}_m \equiv -\frac{(1 - \delta/r)\{\psi_1 \alpha\, e + \psi_2[\alpha\, e - r(1 + e(1 - \alpha))]\}}{\psi_1[1 + e(1 - \alpha\delta/r)] + \psi_2(1 - \delta/r)[e\alpha - r(1 + e(1 - \alpha))]}$$

The first polar case is when newly created virtual currency does not compensate miners, $\theta = 0$. The reward mechanism is then absent and a decrease in virtual currency issuance has a positive welfare effect through a decrease in the inflation tax (second term of equation (34)) and a negative effect by stimulating mining activity (first term). As explained below, the effects on welfare depend on the difference between growth in fiat money and virtual currency, g_m and g_v, respectively. The intuition for this finding can be traced back to the public finance result that in a non-monetary economy the optimal policy is to tax all consumption goods at the same rate.[12] This result is discussed in Chari et al. (1996) in the context of a cash-credit good model. They show that, in a simple two-goods barter economy, the optimal consumption taxes must be the same (one distortion "offsetting" the other), but the tax wedge becomes positive as the model gets more complex (e.g. preferences over the goods differ). In the present model, there is an optimal wedge between g_m and g_v (as the three consumption goods have different features). In particular, when fiat money growth is not too low ($g_m > \bar{g}_m$) there exists an optimal growth rate of virtual currency, $\bar{g}_v$, above which any lowering of g_v increases welfare. When fiat money growth is below a certain threshold ($g_m \leq \bar{g}_m$), which turns out to be negative in the numerical analysis in section 4, any decrease in virtual money issuance raises welfare (because $\bar{g}_v$ is negative and $g_v \geq 0$). Generally speaking, the analysis stops here for currency schemes where new coins are not

[12]This finding is known as the Atkinson-Stiglitz theorem (Atkinson and Stiglitz, 1976): If the utility function is separable in leisure and the sub-utility function over consumption goods is homothetic, then it is optimal to tax consumption goods at the same rate.

used as a remuneration, like 'non-mineable' virtual currencies (e.g. Ripple) or local currencies.

The original cryptocurrency scheme, Bitcoin, stipulates that miners are remunerated with newly issued coins, which introduces a new, additional channel by which virtual money creation affects the economy. When $\theta > 0$, less virtual money creation generates higher transaction fees and raises the price of virtual goods. This renders the mining activity less attractive and lessens the direct beneficial welfare effects from a reduced inflation tax. The second special case arises when $\theta = 1$, as the reward mechanism exactly compensates the inflation tax effect and there is no welfare effect of a decrease in virtual currency issuance. The rise in the price of virtual goods offsets the reduction in the inflation tax and there are no labor flows.

The next proposition, with proof in Appendix C, explains how virtual currency issuance affects welfare.

Proposition 4 *Given the threshold values $\bar{g}_m, \bar{g}_v$ and $\bar{\theta} \equiv \frac{1}{X}$, the following findings hold.*

a. In a not too low fiat money growth environment, i.e. $g_m > \bar{g}_m$, lower virtual currency issuance:

a1. deteriorates welfare if $g_v < \bar{g}_v$ and improves welfare if $g_v > \bar{g}_v$ when $0 \leq \theta < \bar{\theta}$

a2. decreases welfare when $\bar{\theta} \leq \theta < 1$

a3. leaves welfare unchanged when $\theta = 1$

b. In a low fiat money growth environment, i.e. $g_m \leq \bar{g}_m$, reduced virtual currency creation:

b1. enhances welfare when $0 \leq \theta < 1$

b2. does not modify welfare when $\theta = 1$

As mentioned above, the reward mechanism introduces a distortion attracting workers to the mining activity. First consider the case where $0 \leq \theta < \bar{\theta}(< 1)$. The reward mechanism is not too strong and a reduction in virtual currency issuance enhances welfare when g_v is sufficiently large, as in the special case $\theta = 0$ discussed above. It can be remarked that a higher θ raises the threshold $\bar{g}_v$, making the condition for a welfare improvement harder to meet. Indeed, the reward mechanism becomes more important and a decrease in g_v implies larger foregone earnings from mining, while the benefits from a lower inflation tax remain the same (second term of equation (28)). When $0 < \bar{\theta} < \theta(< 1)$, the reward mechanism is so important that lower virtual currency issuance induces an unambiguous welfare decrease. In this case, a welfare improvement requires a sufficiently negative g_v, which is not envisaged by a virtual currency scheme like Bitcoin. In terms of equation (34), the indirect effects (first term of the equation) dominate the combined direct effects (second and third terms of equation (34)). In the case where $\theta = 1$, the reward mechanism exactly compensates the inflation tax effect, directly and indirectly, and there will be no effect on welfare.

Finally consider the situation of low fiat money growth, $g_m < \bar{g}_m$. When $\theta < 1$, a reduction in virtual currency issuance never deteriorates welfare, because the inflation tax on fiat money is so low that the direct impact of the virtual inflation tax dominates the other effects. When $\theta = 1$, the reward mechanism again exactly compensates the inflation tax effect.

4 Numerical analysis

This section illustrates different cases discussed in the previous section. First, it presents the parameterization of the model and then it discusses the numerical results.

Calibration

The model is calibrated for three payment system designs, differing only in the share of transaction fees in mining revenue implying a specific θ associated to each design. Virtual currency aspects are calibrated using data on Bitcoin.

Deep parameters. The initial steady state is calibrated to the year 2013, which is the year when Bitcoin adoption rose considerably. One period is one quarter and the parameters of the model are calibrated on a range of real-world data. The subsequent parameters are set according to standard values used in the literature. The time preference parameter is set to $\beta = 0.99264$ (which corresponds to an annual discount rate of 3%), the quarterly capital depreciation rate equals $\delta = 0.0125$, the capital share in production $\alpha = 0.33$ and total hours are fixed to $h = 0.2$.
Technology and preferences. Parameters A_x and ψ_3 are used respectively to match the ratio of mining revenues to GDP, $MY \equiv (qw_xh_x)/Y$ where $Y \equiv Y_z + qp_xY_x$ and the ratio of virtual good consumption to GDP, $C3Y \equiv qc_3/Y$. These indicators are computed as follows from available data on Bitcoin. The consumption value of virtual goods, $C3$, is measured by the estimated transaction value with bitcoins (in USD), while mining revenues, M, comprise the total value (in USD) of coinbase block rewards and transaction fees paid to miners (Blockchain.info, 2017). These numbers are world aggregates and not available by country. The US's share of bitcoin downloads (Sourceforge, 2017) can be used to approximate country values for $C3$ and M.[13] GDP is taken from the World Development Indicators (WDI, 2017). Since the creation of Bitcoin, the US are among the top nations in terms of downloads. In 2013, US bitcoin downloads reached 23% of the total, which yields $C3Y = 0.025\%$

[13] A country's share in aggregate transactions made with bitcoins may be different from its share in aggregate mining revenues. The fact that a country may have a higher share of virtual currency transactions than its share of mining services is left for consideration in future research.

and $MY = 0.078 * 10^{-6}$.

Productivity in the intermediate sector, A_z, is fixed to 1, while parameter γ_x is set to $1/1600$, which can be roughly interpreted as the inverse of the number of transactions per block (this value only affects the initial level of p_x). The preference parameter ψ_2 helps to match the credit-to-cash good consumption ratio, c_2/c_1, computed from survey data as in Cooley and Hansen (1991). Bagnall et al. (2016) report that for the US the share of payment transactions conducted by cash, debit and credit correspond to 46%, 26% and 19%, respectively and thus c_2/c_1 equals 0.19/(0.46+0.26)=26.4%. Finally, ψ_1 can be calculated from $\psi_1 + \psi_2 + \psi_3 = 1$ (with ψ_3 used to target $C3Y$).

Table 1: Parameter values

Deep parameters		Technology		Preferences		Money growth (annual)	
β	0.99264	A_r	1	ψ_1	0.79283	g_m	1%
δ	0.0125	A_x	0.054*	ψ_2	0.20716	g_v	12.5%
α	0.33	γ_x	1/1600	ψ_3	1-ψ_1-ψ_2		
h	0.2						

*The value of A_x corresponds to design 1 (A_x equals 0.055 in design 2 and 0.056 in design 3). Changes in ψ_1 and ψ_2 (and thus ψ_3) are small such that the values reported in the table hold across the three designs (other parameters are unaffected). Parameter θ is shown in table 2.

Money growth. The quarterly growth rate of fiat money is 0.249% corresponding to an annual inflation rate of 1%, roughly the average figure for major economies over recent years (1.3% in the US and 0.7% in the euro area over the period 2013-2016). The growth rate of virtual currency is set to $g_v = 0.02988$, which is the average over the four quarters in 2013 and represents an annual inflation rate of 12.5%. The resulting threshold level for the inflation rate of fiat money corresponds to an annual rate of almost -2% (for the three designs, see table 2). The numerical analysis below focuses on examples related to the more plausible situation where $g_m > \bar{g}_m$, i.e. part a of Proposition 4.

Table 2: Different payment system designs

Design	Fees-to-Mining revenue	implied θ	Cases of Proposition 4		implied critical values $\bar{\theta}$	$\bar{g}_m$ (ann.)	$\bar{g}_v$ (ann.)
1	100%	0	case a1:	$\theta = 0$	0.9941	-1.97%	2.39%
2	33%	0.671	case a1:	$0 < \theta < \bar{\theta}$	0.9942	-1.92%	7.37%
3	0.5%	0.997	case a2:	$0 < \bar{\theta} \leq \theta$	0.9943	-1.89%	-30.8%

Calibration of θ (table 2). The welfare effects of a decrease in virtual currency issuance are illustrated for three payment system designs. The payment systems are distinct in terms of the share of newly issued money attributed to miners (θ), which is calibrated as follows.[14] It is assumed that the share of fees to mining revenues, $FeeM \equiv p_x Y_x/(w_x h_x)$, is the only difference across designs leading to $\theta = MY/C3Y(1 - FeeM)/g_v$. Table 2 highlights this aspect of the three designs. In design 1, fees constitute all the mining revenue, which corresponds to $\theta = 0$ in case a1 of Proposition 4. Design 2 also considers case a1 but with θ strictly positive ($0 < \theta < \bar{\theta}$), where one third of the mining revenue is made up of fees, implying $\theta > 0.67$. Design 3 relates to case a2 ($0 < \bar{\theta} < \theta$) with fees representing less than 1% of mining revenue.

Steady state numerical results

Figure 2 shows the effects - on the main variables of interest - of a decrease in virtual currency issuance from $g_v = 2.98\%$, i.e. 12.5% in annual terms, to 0% (the x-axis is in reverse order, from high to low g_v values). Each row corresponds to a different design of the virtual payment system.

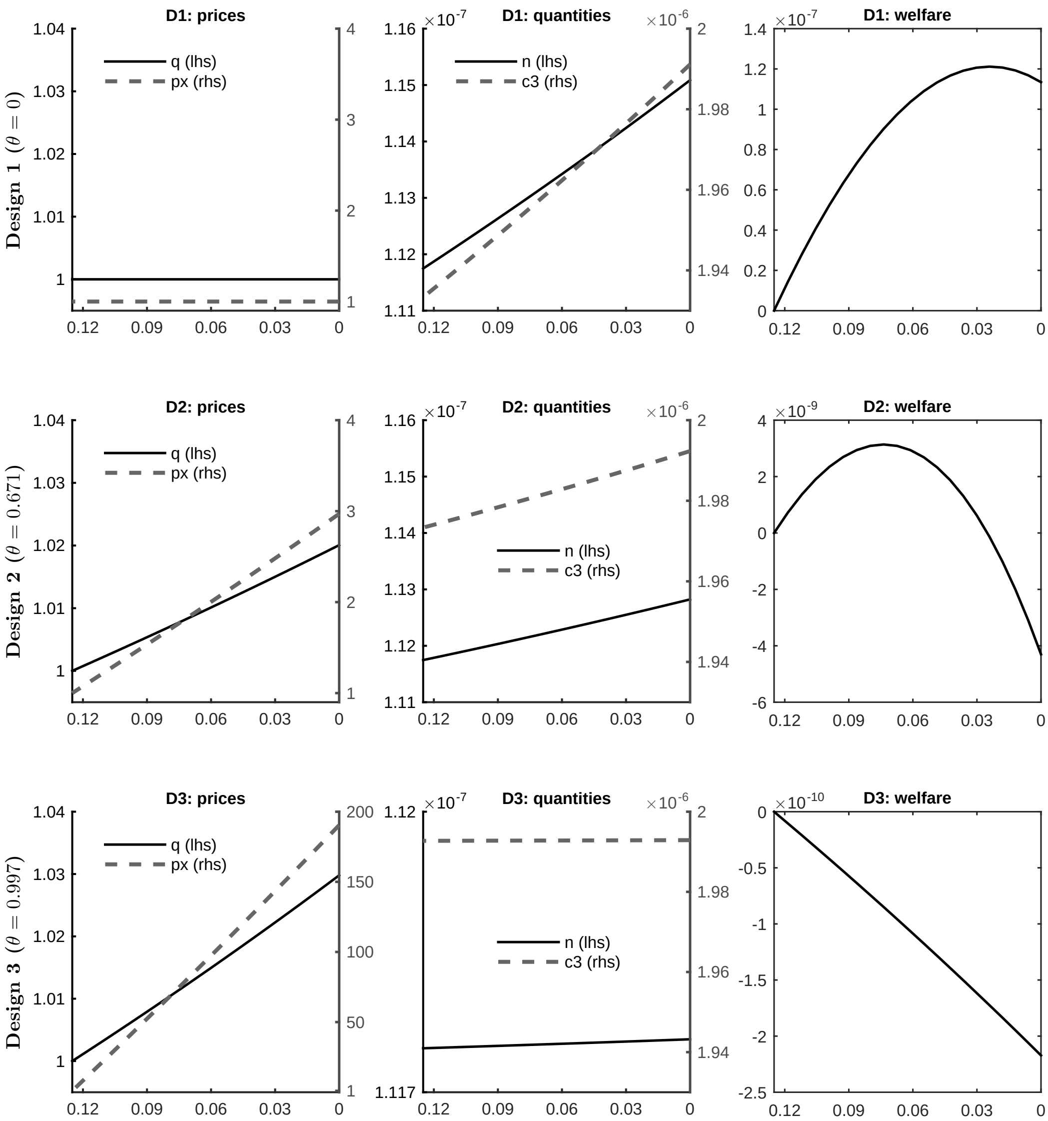
In design 1 ($\theta = 0$), the reward mechanism is absent and only the inflation tax effect is at work. A decrease in virtual money growth does not have any effect on relative prices, p_x and q, but the reduction in the inflation tax raises the demand for virtual goods (c_3). This stimulates the production of virtual goods, rendering mining activity more attractive (as indicated by the increase in n). The last plot shows that welfare rises up to a certain value of g_v (2.4% in annual terms) and then decreases slightly.[15]

In design 2 ($\theta < \bar{\theta}$), the reward mechanism comes into play. As virtual currency creation fades out, transaction fees become more and more important in miners' revenues, making virtual goods more expensive (between the initial and the final g_v, p_x almost triples while q rises by 2%). Compared to the previous design, the employment ratio n is initially higher but increases less because of the loss in terms of foregone earnings from newly issued coins. Higher prices of virtual goods generates a lower increase in virtual consumption (c_3) and the peak in the welfare change occurs at a higher value of g_v than in the previous setting

[14] Note that, though other targets remain similar across designs, the general equilibrium effects imply that the values of other parameters than θ can differ. This can be the case for $A_x, \psi_1, \psi_2, \psi_3$, while $\beta, \delta, \alpha, h, A_r, \gamma_x, g_m$ and g_v are constant across settings.

[15] The figure shows the consumption equivalent welfare change, which is the percentage change in consumption that an individual would require to be as well off as under a virtual currency issuance of $g_v = 2.98\%$. The consumption equivalent welfare change, μ, solves $W(c_1', c_2', c_3') = W((1+\mu)c_1^0, (1+\mu)c_2^0, (1+\mu)c_3^0)$, which implies $\mu = exp(W' - W^0) - 1$ with W^0 and W' being steady state welfare for initial g_v and lower g_v values, respectively. Formally, $W^0 \equiv W(c_1^0, c_2^0, c_3^0)$ where c_1^0, c_2^0 and c_3^0 are the consumption levels when $g_v = 2.98\%$, while $W' \equiv W(c_1', c_2', c_3')$ with c_1', c_2' and c_3' are the consumption levels for lower values of g_v.

Figure 2: steady state effects of a decrease in virtual currency issuance
for three payment system designs (*x-axis in descending order*)



The figure shows the effects of a decrease in the quarterly growth rate from $g_v = 2.98\%$ (12.5% in annual terms) to 0. The x-axis displays the annualized virtual currency growth rate in reverse order. Prices p_x and q are normalized to 1 for the initial g_v. n is the employment ratio (h_x/h_z) and c_3 is consumption of virtual goods. Finally, the last plot of each row is the consumption equivalent welfare change (normalized to 0 for the initial g_v).

(the threshold value for g_v is 7.4% in annual terms).

Finally, in design 3 ($\theta > \bar{\theta}$), the reward mechanism almost fully compensates the effects of a reduction in the inflation tax. Compared to the previous settings, labor allocation across activities and consumption of virtual goods remain almost unchanged, while welfare deteriorates (though welfare changes are smaller). Obviously, a design where $\theta = 1$ would imply that both effects exactly compensate each other and virtual currency issuance only affects p_x and q.[16]

5 Further analysis

This section presents model extensions considering consumption taxes and security of the virtual currency system. It also takes a look at the model-based predictions for real and nominal exchange rates under changes in issuance of a Bitcoin-like virtual currency issuance.

5.1 Additional model features

The subsequent paragraphs discuss the effects of virtual currency growth in the presence of (i) consumption taxes and (ii) imperfect security of the virtual currency system, intended as the degree to which ownership is guaranteed.

Consider first consumption taxes. There are discussions on whether virtual currencies should be regarded as products or currencies. In this respect, the European Court of Justice (ECJ) published a decision on October 22^{nd}, 2015, stating that 'transactions to exchange a traditional currency for the Bitcoin virtual currency or vice versa' should be exempt from taxes (ECJ, 2015). The judgment further states that the Bitcoin virtual currency 'cannot be characterized as tangible property' and 'has no purpose other than to be a means of payment'. This decision also avoids concerns about the double taxation of Bitcoin transactions, i.e. merchants being taxed once when they sell goods and services for virtual currencies and a second time when they convert virtual currencies to fiat currencies. The focus here is not on taxation of transactions across currencies but on commodity taxes.

A second aspect discussed here is the security of the virtual payment system. Virtual currency owners may be unable to use their virtual currencies because they lost or forgot the key/password to access their account, because they died without passing on their password or because the hard-drive containing the key failed (see e.g. Sparkes, 2015). In any case, the non-accessible coins remain idle, i.e. visible in the system but non-spendable. This also explains why the number of bitcoins in circulation always exceeds the effective, spendable supply of bitcoins. Moreover, virtual currency accounts are also prone to hacker attacks or fraud. A hacker may steal the secret key of an account to spend another person's bitcoins.

[16] Figures for a design where $\theta = 1$ can be obtained upon request.

A famous example are the February 2014 attacks against the then biggest Bitcoin exchange platform, Mt. Gox, which led to the closing of the website and filing for bankruptcy protection. About 750'000 bitcoins belonging to customers and 100'000 belonging to Mt. Gox, worth about 500 million USD, were stolen.

Let τ represent the consumption tax on cash and credit goods and τ^v the tax on virtual goods. A share $\sigma \in [0, 1)$ of virtual currency is assumed to be non-spendable on virtual good consumption. Thus σ is an indicator of the safety of the virtual currency system, a low σ indicating a high degree of system security. Introducing τ, τ^v and σ in the model implies that equations (2), (3) and (4) change in the following way

$$
\begin{aligned}
m_{t+1}(1+\pi_t) + q_t v_{t+1}(1+\pi_t^*) = m_t + q_t\,(1-\sigma_t)v_t + w_t h + r_t K_t - I_t \\
-(1+\tau_t)(c_{1,t}+c_{2,t}) - (1+\tau_t^v)q_t\, c_{3,t} + \Omega_t
\end{aligned}
\tag{36}
$$

$$c_{1,t}(1+\tau_t) \leq m_t \tag{37}$$

$$c_{3,t}(1+\tau_t^v) \leq (1-\sigma_t)v_t \tag{38}$$

Equation (36) shows that only $(1-\sigma_t)\,v_t$ of real balances in virtual currency are carried over from the previous period in the budget constraint and equation (38) can be used to purchase virtual goods (for simplicity, $\sigma_t\,v_t$ is rebated lump sum to households and included in the term Ω_t).
In equilibrium, combining $\Gamma_{v,t} \equiv g_{v,t}v_t^s$ with equations (19), (22) and (38) gives

$$\Gamma_{v,t} \;=\; g_{v,t}Y_{v,t}(1+\tau_t^v)/(1-\sigma_t)$$

Using this expression together with equation (13) in equation (15) yields the representative miner's wage

$$w_{x,t} = \frac{1}{\gamma_x}\frac{Y_{x,t}}{h_{x,t}}\left(\gamma_x p_{x,t} + \Theta_t\, g_{v,t}\right) \tag{39}$$

with

$$\Theta_t \equiv \theta\,\frac{1+\tau_t^v}{1-\sigma_t} \tag{40}$$

Equation (40) indicates that θ still determines whether miners are rewarded with newly issued virtual money ($\theta > 0$) or not ($\theta = 0$). However, when $\theta > 0$, the strength of the reward mechanism, represented by Θ, depends also on the taxation of virtual good consumption, τ^v, as well as on the security of the system, σ.

Equations and propositions of section 3 still carry through if θ is interpreted as Θ. Thus, the impact of a virtual currency growth on the price of mining services is derived from the steady state equation for p_x, which is equivalent to (26) but with Θ instead of θ

$$\frac{\partial p_x}{\partial g_v} = -\frac{1}{\gamma_x}\,\frac{\Theta}{1+e(1-\alpha)}$$

The higher τ^v and/or σ, the stronger the reward mechanism and the larger the reaction of p_x to a change in g_v. An increase in τ^v (and/or σ) raises the demand of virtual currency as more virtual money is needed to purchase a given quantity of virtual goods. The response of the employment ratio is analogous to equation (29) with Θ playing the role of θ[17]

$$\frac{dn}{dg_v} = -\frac{n\,q}{a\,b + (1+g_v)\,q}\,\frac{1-\Theta}{1+\Theta\,g_v}$$

This equation shows that the reward mechanism may not only mitigate the inflation tax effect ($\Theta < 1$) or exactly compensate it ($\Theta = 1$), but can now even dominate it (when $\Theta > 1$) implying $\frac{dn}{dg_v} > 0$. In this latter case, the loss in terms of foregone earnings is so large that a decrease in g_v discourages mining.

The effects on welfare when $\Theta > 1$ can be seen from the next equation

$$\left.\frac{\partial W}{\partial g_v}\right|_{\Theta>1} = \frac{1}{1-\beta}\left[\underbrace{-\,\xi\,\frac{\partial n}{\partial g_v}}_{\substack{\text{indirect effects}\\ <0}}\ \underbrace{-\,\frac{\psi_3}{1+g_v} + \frac{\psi_3\,\Theta}{1+\Theta\,g_v}}_{\substack{\text{direct effects}\\ >0}}\right] \tag{41}$$

When $\Theta > 1$, the reward mechanism dominates the inflation tax effect. In the above expression, there is a switch in sign, compared to the situation when $\Theta < 1$, in the first term representing the indirect effects as well as in the combination of the second and third terms. This means that a decrease in g_v improves welfare through indirect effects, but deteriorates it through direct effects as the strong increase in the price of virtual goods (third term) dominates the direct effect of the inflation tax (the second term of (34)). It is worth mentioning that the new thresholds $\overline{\Theta}, \widetilde{g}_v$ and $\widetilde{g}_m$ are based on $\widetilde{X} \equiv \Lambda(1-\sigma)X$, in an analogous way than $\bar{\theta}, \bar{g}_v$ and $\bar{g}_m$ depend on X in section 3, see Appendix D for details. Of course, in the absence of taxes ($\tau = \tau^v = 0$) and with a secure system ($\sigma = 0$), the model is back to the benchmark setting, with $\widetilde{X} = X$, $\overline{\Theta} = \bar{\theta}$, $\widetilde{g}_v = \bar{g}_v$, $\widetilde{g}_m = \bar{g}_m$.

The conditions that determine which effect dominates are stated in Proposition 5 and can be summarized as follows. In a not too low fiat-money growth environment, $g_m > \widetilde{g}_m$, the distortion induced by the reward system is so strong that lower virtual money issuance always improves welfare. The indirect effects implied by labor reallocation (first term of equation (34)) are dominant as mining activity is relatively attractive (favored by the high inflation tax on fiat money). In a low fiat-money growth environment ($g_m \leq \widetilde{g}_m$), two cases need to be distinguished. When Θ is moderate ($1 < \Theta \leq \overline{\Theta}$), mining activity is relatively low and there are few intersectoral labor flows: direct effects dominate and welfare decreases. However, when Θ is large ($\Theta > \overline{\Theta} > 1$), the loss in foregone earnings from new coins is

[17]Parameter $\widetilde{a} \equiv \Lambda(1-\sigma)a$, where a is defined in Appendix A and $\Lambda \equiv \frac{1+\tau}{1+\tau^v}$.

important and the indirect effects dominate if $g_v > \widetilde{g}_v$. The proposition below, with proof in Appendix 5, focuses on the welfare implications when $\Theta > 1$.

Proposition 5 *In addition to cases a1-a3 and b1-b2 from Proposition 4, virtual currency issuance affects welfare in the following manner when* $\Theta > 1$.

- a. *For* $g_m > \widetilde{g}_m$, *reduced virtual currency issuance*
  - a4. *improves welfare when* $\Theta > 1$
- b. *For* $g_m < \widetilde{g}_m$, *lower virtual currency creation*
  - b3. *deteriorates welfare when* $1 < \Theta \leq \overline{\Theta}$
  - b4. *reduces welfare if* $g_v < \widetilde{g}_v$ *and enhances welfare if* $g_v > \widetilde{g}_v$ *when* $1 < \overline{\Theta} < \Theta$

(Note that case b3 applies for $g_m = \widetilde{g}_m$ *whenever* $\Theta > 1$*)*

At last, table 3 summarizes the implications of a reduction in virtual currency issuance for the variables of interest in all possible cases. The results in section 3 when $\tau = \tau^v = \sigma = 0$ correspond to the cases where $\Theta \leq 1$ interpreting $\Theta, \overline{\Theta}, \widetilde{g}_v$ and $\widetilde{g}_m$ as $\theta, \bar{\theta}, \bar{g}_v$ and $\bar{g}_m$ respectively.

Table 3: Summary of the effects of a decrease in virtual currency growth

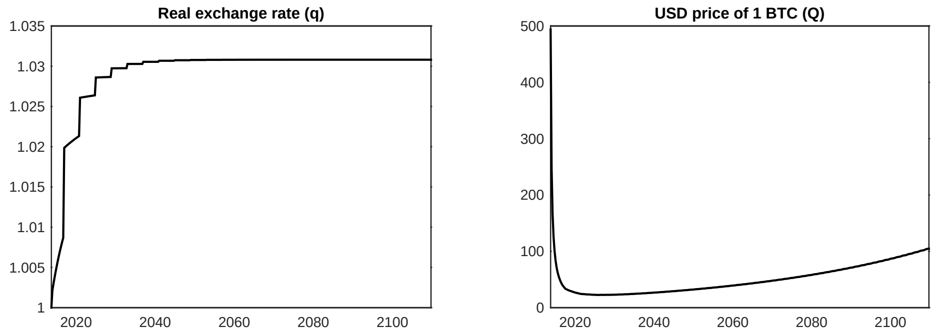
	Cases			p_x, q	n, h_x, c_3	h_z, c_1, c_2	W
$g_v \searrow$	(a) $g_m > \widetilde{g}_m$	(a1)	$0 \leq \Theta < \overline{\Theta}$	$+^*$	+	-	+ if $g_v > \widetilde{g}_v$ - if $g_v < \widetilde{g}_v$
		(a2)	$\overline{\Theta} \leq \Theta < 1$	+	+	-	-
		(a3)	$\Theta = 1$	+	0	0	0
		(a4)	$\Theta > 1$	+	-	+	+
	(b) $g_m \leq \widetilde{g}_m$	(b1)	$0 \leq \Theta < 1$	$+^*$	+	-	+
		(b2)	$\Theta = 1$	+	0	0	0
		(b3)**	$1 < \Theta \leq \overline{\Theta}$	+	-	+	-
		(b4)**	$1 < \overline{\Theta} < \Theta$	+	-	+	- if $g_v < \widetilde{g}_v$ + if $g_v > \widetilde{g}_v$

The table indicates whether a reduction ($\searrow$) in virtual money growth, g_v, decreases ("-"), increases ("+") or has no impact ("0") on a variable. $\Theta \equiv \theta(1+\tau^v)/(1-\sigma)$, c_1, c_2, c_3 are the consumptions of cash, credit and virtual goods, respectively, p_x is the price of mining services, q is the real exchange rate, h_x is mining employment, h_z is employment in intermediate production, $n \equiv h_x/h_z$, and W is welfare. *In cases (a1) and (b1), the effect on p_x and q is nil when $\Theta = \theta = 0$. **When $g_m = \widetilde{g}_m$, case (b3) applies for any value of $\Theta > 1$. In the benchmark model presented in section 3, where $\tau = \tau^v = \sigma = 0$, only cases (a1) - (a3) and (b1) - (b2) arise and $\Theta = \theta$, $\overline{\Theta} = \bar{\theta}$, $\widetilde{g}_v = \bar{g}_v$ and $\widetilde{g}_m = \bar{g}_m$.

5.2 Dynamic simulation

This subsection illustrates the price effects of a virtual money supply that increases at a decreasing rate. To do so, the model is applied to Bitcoin, but it is important to stress that the aim is not to forecast the price of Bitcoin nor to analyze the determinants of Bitcoin price formation (for such a study, see Ciaian et al., 2016). Indeed, the price of Bitcoin is volatile and determined by factors that are not accounted for in this model.[18]

Figure 3: Model-based evolution of the real (q) and the nominal (Q) exchange rates



The simulation starts from an initial steady state in 2013. A conservative approach is taken in setting zero taxation and $\sigma = 0.03$, which together with $\phi = 1$ implies $\Theta = 1.0309$ (note that these values do not qualitatively affect the results discussed here). The rest of the calibration follows section 4 with quarterly growth rates of virtual currency and fiat money of 2.988% and 0.249%, respectively (12.5% and 1% on annual basis). One period is one quarter and the model is simulated for 600 periods. During the transition to the new steady state, g_m and g_v evolve. Between 2014 and 2017, fiat money issuance progressively increases to reach a rate of 0.496% and remains at that level afterward, consistent with the 2% annual inflation target common to major central banks. The growth rate g_v replicates the evolution of virtual money supply, V^s, such that newly issued virtual coins are halved approximately every 4 years to reach a predetermined total number of approximately 21 million around 2140 (see figure 1 of section 1).

Figure 3 displays the effects of money growth on real and nominal exchange rates (q and Q, respectively). The left panel shows that virtual money growth affects q, which is the case whenever miners are also paid , see Proposition 1 in section 3. As g_v declines, q increases

[18] For instance, the monthly average USD market price of one bitcoin climbed from 16 dollars on January, 2013, to more than 800 dollars one year later and then dropped to less than 500 dollars four months afterwards (average across major bitcoin exchanges, see blockchain.info). A variety of factors drive the price of Bitcoin, like news about security breaches and hacker attacks, which affect Bitcoin adoption.

and eventually stabilizes after 2040 as virtual money growth approaches zero (fiat money growth has no impact on q). The right panel displays the nominal value of virtual currency against national currency, i.e. the Bitcoin price in US dollars, which can be written $Q = q\frac{M^s}{V^s}\frac{v}{m}$ (see section 2.1 and Guidotti, 1993, p.119). The evolution of Q can be explained by the changes that g_m and g_v induce in M^s and V^s, respectively. The price of Bitcoin falls from an initial level of \$ 493.9 (which is its daily average price over the last quarter in 2013, see blockchain.info) to hit a low of \$ 22.6 in the mid-2020s and increases afterward as M^s starts growing faster than V^s (virtual money becomes relatively scarcer). Finally, the above projection (qualitatively) reproduces the observed decline in the Bitcoin price between 2013-q4 and 2015-q2 (from \$ 493.9 to \$ 236.3) but not its increase since 2015-q2 (above \$ 10000 since December 2017). Many factors ignored in this study can explain changes in Bitcoin prices, including speculation (Bolt and van Oordt, 2016) or Bitcoin adoption costs associated to information search (Ciaian et al., 2016).

6 Conclusion

This paper analyzes the macroeconomic implications of declining virtual currency issuance. A standard monetary model is extended with the introduction of virtual goods produced by combining intermediate goods and payment services provided by 'miners', the private agents validating 'virtual' transactions. Two effects are at work when virtual currency growth declines: an inflation tax effect and a reward mechanism. The first effect implies that lower virtual money growth decreases the inflation tax on virtual money, which raises the demand for virtual goods and stimulates virtual sector activity. The second mechanism is specific to virtual currencies, like Bitcoin, where miners get paid with newly created virtual currency. When virtual currency issuance decreases, higher transaction fees are needed to compensate miners. This leads to an increase in the price of virtual goods and to a reduction in the demand for these goods, which counteracts the effects of a reduced inflation tax. A further analysis shows that the security of the system as well as virtual commodity taxes affect this second mechanism. The study provides the conditions under which steady state welfare improves.

The present analysis can be extended in several directions. Two of them are discussed here. The first consists in studying the potential benefits of adopting a distributed ledger technology, like the blockchain. Bitcoin has attracted a lot of attention because of this disruptive technology, that is claimed to significantly reduce the costs associated with financial services. Businesses and financial institutions deciding to adopt a distributed ledger technology could lower costs associated with keeping two separate records (which implies reconciling them at each step of contract execution). A possible extension could model inefficient payment services and evaluate how a technology like the blockchain tackles these inefficiencies. Another issue ignored by the present study relates to the international implications of introducing a virtual currency. In the case of 'mineable' virtual currencies, users and miners may be located in a different country. A two-country model could examine the interactions of two national currencies with a common virtual money. Future work should address these questions.

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Appendix

A Proof of Proposition 2

Equation (7) together with equation (8) leads to

$$n = \frac{\psi_3}{\psi_2} \frac{1}{1+g_v} \frac{1}{1+r-\delta} \frac{1}{q} \frac{c_2/h_z}{c_3/h_x} \tag{A.1}$$

since $\pi_m = g_m$ and $\pi_v = g_v$ in the steady state (see equations (16) and (17)). Using equation (9) with $\frac{c_3}{h_x} = \frac{A_x}{\gamma_x}$, obtained from equation (22), yields

$$\frac{c_2}{h_z} = \frac{\psi_2(1+r-\delta)(1+g_m)}{\psi_1+\psi_2(1+r-\delta)(1+g_m)} \frac{Y_z}{h_z} \left[1-b-\frac{n}{e}\right] \tag{A.2}$$

Equation (27) results from combining the above expressions for $\frac{c_2}{h_z}$ and $\frac{c_3}{h_x}$ with equation (A.1). It is then possible to derive equation (29), where $a \equiv \frac{\psi_3(1+g_m)}{\psi_1+\psi_2(1+r-\delta)(1+g_m)}$ and $b \equiv \delta\frac{\alpha}{r}$. From equation (29), it is straightforward to verify Proposition 2. ■

B Proof of Proposition 3

Using equation (21), intermediate sector employment can be formulated as

$$h_z = \frac{h}{1+n} \tag{B.1}$$

Combining equations (13), (22) and $Y_x = A_x h_x$ yields

$$c_3 = \frac{A_x}{\gamma_x} h_x \tag{B.2}$$

Inserting $h_x = h - h_z$ and equation (B.1) in equation (B.2) results in equation (30). It can be observed that virtual currency growth influences the consumption of credit goods, c_2, through n. With the help of equation (B.1), $c_2 = \frac{c_2}{h_z}\frac{h}{1+n}$ where $\frac{c_2}{h_z}$ depends on n as shown in equation (A.2). c_2 is inversely related to n, see also equation (32), and positively affects c_1, as evidenced in equation (31). Finally, the proof of Proposition 3 follows from Proposition 2, since the three consumption goods can be written as a function of n. ■

C Proof of Proposition 4

The consumption of virtual goods can be expressed as a function of c_2, which is obtained by combining equations (7) and (8)

$$c_3 = \frac{\psi_3}{\psi_2} \frac{1}{1+r-\delta} \frac{1}{1+g_v} \frac{c_2}{q} \tag{C.1}$$

Inserting this relationship and equation (31) in equation (1), leads to steady state welfare, W, as formulated in (33), where $\chi \equiv \psi_1 \ln(\psi_1/\psi_2) + \psi_3 \ln(\psi_3/\psi_2) - \psi_1 \ln(1+g_m) - (\psi_1 + \psi_3)\ln(1+r-\delta)$. Deriving (33) with respect to g_v gives equation (34), where ξ is obtained by using equation (A.2), $\xi \equiv \vartheta/c_2 = \frac{1+e(1-b)}{[1-b(1+n)][1+n(1+e)]}$.
After some rearrangements, equation (34) leads to

$$\frac{\partial W}{\partial g_v} \lesseqqgtr 0 \quad \Leftrightarrow \quad (\theta-1)\, g_v\, (1-\theta X) \lesseqqgtr (\theta-1)\,(X-1), \tag{C.2}$$

Part a of Proposition 4 considers that $g_m > \bar{g}_m$, implying $X > 1$. Under case a1, when $0 \leq \theta < \bar{\theta}$, it is straightforward to check that a decrease in g_v improves (deteriorates) welfare for values above (below) $\bar{g}_v$, i.e. $\frac{\partial W}{\partial g_v} < 0 \Leftrightarrow g_v > \bar{g}_v$ and $\frac{\partial W}{\partial g_v} > 0 \Leftrightarrow g_v < \bar{g}_v$. Under case a2, $\frac{\partial W}{\partial g_v} > 0$ as $\bar{g}_v < 0$. Indeed, with $g_v \geq 0$, it is impossible to have $\frac{\partial W}{\partial g_v} < 0$ which requires $g_v < \bar{g}_v < 0$. Case a3, where $\theta = 1$, is verified by inspecting equation (34), where the second and third terms cancel each other out, while the first term is zero, see equation (29).
Part b of Proposition 4 deals with the situation where $g_m \leq \bar{g}_m$, implying $X \leq 1$ and thus $\bar{g}_v \leq 0$. Under case b1, $\frac{\partial W}{\partial g_v} \leq 0$ is always true. Since g_v is assumed to be non-negative, it is impossible to have $\frac{\partial W}{\partial g_v} > 0$, because this requires $g_v < \bar{g}_v \leq 0$. Case b2 is analogous to case a3. ■

D Proof of Proposition 5

Rearranging equation (34) for $\Theta > 1$ leads to

$$\frac{\partial W}{\partial g_v} \lesseqqgtr 0 \quad \Leftrightarrow \quad g_v\,(1-\Theta\,\widetilde{X}) \lesseqqgtr (\widetilde{X}-1), \tag{D.1}$$

Note that $\widetilde{X} \equiv \Lambda(1-\sigma)\,X$, where X is defined in equation (35) and $\Lambda \equiv \frac{1+\tau}{1+\tau^v}$. Moreover, $\overline{\Theta} \equiv 1/\widetilde{X}$ while $\widetilde{g}_v \equiv \frac{\widetilde{X}-1}{1-\Theta\,\widetilde{X}}$ and

$$\widetilde{g}_m \equiv -1 + \frac{\psi_1[1+e(1-\alpha)]}{(\psi_1+\psi_2)[1+e(1-\alpha\delta/r)]\Lambda(1-\sigma) - \psi_2(1+r-\delta)[1+e(1-\alpha)]}$$

Under case (a4), $\widetilde{X} > 1$, since $\widetilde{g}_m > g_m$, and g_v is always larger than $\widetilde{g}_v$ implying that $\frac{\partial W}{\partial g_v} > 0$ is always true. Under case (b3), $\widetilde{g}_v < 0$ and thus $g_v > \widetilde{g}_v$ leading to $\frac{\partial W}{\partial g_v} < 0$. Indeed, $\frac{\partial W}{\partial g_v} > 0$ requires $g_v < 0$, which is excluded by assumption (see section 2.4). Moreover, when $g_m = \widetilde{g}_m$, $\widetilde{X} = 1$ and thus $\frac{\partial W}{\partial g_v} < 0$. Under case (b4), $\widetilde{g}_v > 0$ for $\Theta > \overline{\Theta}$. This implies $\frac{\partial W}{\partial g_v} < 0 (> 0)$ when $g_v > \widetilde{g}_v (< \widetilde{g}_v)$. ■

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