CAHIER D'ÉTUDES
WORKING PAPER

N° 147

LED: AN ESTIMATED DSGE MODEL OF THE LUXEMBOURG ECONOMY FOR POLICY ANALYSIS

ALBAN MOURA

AUGUST 2020

BANQUE CENTRALE DU LUXEMBOURG

EUROSYSTÈME

LED: AN ESTIMATED DSGE MODEL OF THE LUXEMBOURG ECONOMY FOR POLICY ANALYSIS

ALBAN MOURA

Abstract. This paper outlines a new estimated dynamic stochastic general equilibrium (DSGE) model of the Luxembourg economy named LED, for Luxembourg Estimated DSGE. The paper provides a thorough discussion of the model structure, explains how LED is solved and estimated, and shows how it can be used to study important properties of the Luxembourg economy. The empirical results are encouraging: parameter estimates take reasonable values, the model fits the data well, and its implications regarding the determinants of economic growth and cyclical fluctuations in Luxembourg are plausible.

JEL Codes: C11, C32, E32, E37.

Keywords: DSGE models, open-economy macroeconomics, Bayesian inference, policy analysis, Luxembourg.

July 2020. Banque centrale du Luxembourg, Département Économie et Recherche, 2 boulevard Royal, L-2983 Luxembourg (e-mail: alban.moura@bcl.lu). For helpful comments and suggestions, I thank (without implicating) the BCL Executive Board, as well as Yves Eschette, Patrick Fève, Paolo Guarda, Olivier Pierrard, and Jean-Pierre Schoder.

Résumé Non Technique

Cet article présente un modèle d'équilibre général dynamique et stochastique (DSGE, en anglais) de l'économie luxembourgeoise, dénommé LED pour *Luxembourg Estimated DSGE*. L'article expose la structure du modèle, les étapes nécessaires à sa résolution et à son estimation, et les résultats de l'estimation. Il considère également quelques applications économiques relatives à la dynamique macroéconomique du Luxembourg.

Contrairement à leurs prédécesseurs, dont la simplicité limitait les performances empiriques, les modèles DSGE actuels affichent une complexité suffisante pour reproduire de manière adéquate les propriétés statistiques des données macroéconomiques. De plus, cette amélioration de l'adéquation avec les données a été obtenue sans trop compromettre la nature structurelle des modèles, qui restent basés sur la théorie microéconomique et prennent en compte les anticipations des agents économiques. Ainsi, les modèles DSGE modernes sont moins sensibles à la célèbre *critique de Lucas* que les modèles macro-économétriques traditionnels et offrent un cadre attractif pour l'analyse de la politique économique. Tout ceci explique la croissance du nombre de modèles DSGE développés dans des institutions telles que les banques centrales au cours des années passées.

LED partage la structure générale des modèles de type New Keynesian utilisés dans les autres banques centrales. Il prend aussi en compte trois spécificités de l'économie luxembourgeoise : le contenu en importations élevé de la production domestique, le rôle majeur des travailleurs transfrontaliers dans le marché du travail domestique, et l'appartenance à la zone euro impliquant une évolution exogène de la politique monétaire pour le Luxembourg. LED inclue également plusieurs instruments de politique fiscale, notamment sous forme de dépenses publiques et de taxes sur la consommation et les revenus. Finalement, il convient de noter que, si le secteur financier n'apparait pas de manière explicite dans LED, il reste pris en compte implicitement : puisque les services financiers représentent environ 30% du PIB luxembourgeois et 50% des exportations, tout modèle capable de reproduire les agrégats macroéconomiques du pays doit nécessairement capturer en partie le comportement du secteur financier.

LED est estimé, à partir de techniques Bayésiennes, sur un ensemble de données concernant l'économie luxembourgeoise, la zone euro et le reste du monde pour la période 1995Q1-2019Q4. Les résultats de l'estimation indiquent que LED parvient à reproduire les caractéristiques importantes des données : les paramètres estimés prennent des valeurs raisonnables et le modèle capture bien les relations statistiques entre les variables dans l'échantillon. Ces résultats suggèrent que LED constitue un outil empirique intéressant pour l'analyse économique à la BCL. La fin du papier présente quelques applications potentielles, par exemple l'étude des effets des chocs d'offre, de demande et des chocs extérieurs sur

l'économie luxembourgeoise, l'identification des chocs et frictions importantes pour la dynamique économique du Luxembourg, et l'interprétation des développements économiques passés.

1. Introduction

This paper presents an estimated dynamic stochastic general equilibrium (DSGE) model of the Luxembourg economy named LED, for Luxembourg Estimated DSGE. The paper reviews the structure of the model, the various steps involved in the solution and estimation process, the estimation results, and the outcomes of selected economic applications focusing on macroeconomic dynamics in Luxembourg.

While the first vintages of DSGE models, including the famous model of real business cycles (RBC), were of limited empirical use given their restrictive assumptions, the current generation of DSGE models pioneered by Smets and Wouters (2003, 2007) and Christiano, Eichenbaum, and Evans (2005) has reached a degree of complexity sufficient to adequately reproduce the salient properties of macroeconomic time series. This improvement in fit has been obtained without compromising too much on the structural nature of the models, which remain based on sound microeconomic principles and provide an explicit treatment of expectations. As a result, DSGE models are less vulnerable to the Lucas critique than more traditional macroeconometric models and, therefore, constitute an attractive framework for policy analysis. These appealing properties explain the surge in the number of large-scale DSGE models developed at policy-making institutions, including central banks.[1] The main objective of this work is to provide the BCL with a similar tool.[2]

LED borrows most of its structure from other policy-oriented New Keynesian DSGE models, for instance the New Area Wide Model (NAWM) developed at the European Central Bank (ECB) by Coenen, Christoffel, and Warne (2008). Its backbone is a stochastic growth model with optimizing households and firms, augmented by a set of real and nominal frictions designed to generate realistic dynamics at short to medium horizons. Real frictions

[1]Examples include the International Monetary Fund (Bayoumi, Faruqee, Laxton, Karam, Rebucci, Lee, Hunt, and Tchakarov, 2004), the Federal Reserve Board (Erceg, Guerrieri, and Gust, 2006), the Sveriges Riksbank (Adolfson, Laseen, Linde, and Villani, 2007), the European Central Bank (Coenen, Christoffel, and Warne, 2008), the European Commission (Ratto, Roeger, and in't Veld, 2008), the Bank of Spain (Burriel, Fernandez-Villaverde, and Rubio-Ramirez, 2010), the Bank of England (Burgess, Fernandez-Corugedo, Groth, Harrison, Monti, Theodoridis, and Waldron, 2013), the Bank of Finland (Kilponen, Orjasniemi, Ripatti, and Verona, 2016), and the Sveriges Riksbank (Adolfson, Laseen, Christiano, Trabandt, and Walentin, 2013).

[2]Two DSGE models are already in use at the BCL Economics and Research Department: LOLA (Pierrard and Sneessens, 2009; Marchiori and Pierrard, 2012, 2015), a calibrated overlapping generations model focusing on long-run policy and demographic issues, and LU-EAGLE (Moura and Lambrias, 2018; Garcia Sanchez and Moura, 2019), a calibrated multi-country model of Luxembourg within the euro area and the global economy tailored to short-run analysis. LED's structure makes it very close to LU-EAGLE, with the important difference that the foreign block in LED is modeled more parsimoniously. Another major difference is that LED is estimated from the data, whereas the size of LU-EAGLE prevents estimation. A closed-economy DSGE model is also used at the BCL Financial Stability Department to evaluate macro-prudential policy measures (Sangaré, 2019).

include habit formation in consumption and a set of adjustment costs (to investment, exports, imports, and labor demand), while nominal frictions include price and wage rigidities. In addition, LED introduces three elements required to reproduce some important characteristics of Luxembourg's economy: the high import content of domestic production, captured by production functions featuring an imperfect substitution between domestic and imported inputs; the significant share of Luxembourg's workforce accounted for by cross-border workers, who spend most of their income outside the country; and the euro-area membership, implying that nominal interest rates are set by a monetary authority reacting to union-wide aggregates in which the weight of Luxembourg is close to zero. The model also includes a domestic fiscal authority determining the level of public expenditures and the design of the tax system. Finally, LED features various shocks to technology, preferences, markups, foreign demand, and policy, that generate the dynamic behavior of the model.

It is important to stress that Luxembourg's financial industry does not appear as a separate sector in LED. Rather, it is implicitly included in the domestic production sector: since financial services represent about 30% of Luxembourg's GDP and 50% of its exports, any model that reproduces the behavior of aggregate variables in the country will necessarily capture some properties of the financial sector. At the same time, this representation is obviously limited since it focuses on the contribution of financial firms to production and employment, at the expense of other important dimensions related to macroprudential oversight. While developing a richer specification with an explicit financial sector seems an important task for future research, it felt natural to begin with a more standard model that could be used as a future reference.

LED is estimated from Luxembourg data using Bayesian likelihood methods. The Bayesian approach has a straightforward logic: the researcher has some initial beliefs about the values potentially taken by the model parameters, and these beliefs are updated in view of the sample information contained in the data. More formally, Bayesian estimation assigns values to the model parameters by maximizing the likelihood function, which measures the proximity between the model and the data, subject to a penalty term formalizing the initial beliefs and called the prior distribution (see, e.g., Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide, 2016, for a thorough presentation). This approach offers several advantages for the estimation of DSGE models, including the proper treatment of presample information, the ability to deal with misspecification and identification issues in a transparent fashion, and the immediate construction of probability distributions for the model's parameters and implications (moments, impulse response functions, forecasts. . .). In the case of LED, estimation is based on a set of 18 quarterly macroeconomic time series, designed to contain information about domestic demand (GDP and its subcomponents), international trade (imports and exports), a number of deflators, the labor market (resident and cross-border employment, wages), and relevant foreign variables from the euro area and the rest of the world. The estimation sample runs from 1995Q1 to 2019Q4.

The empirical results suggest that LED is quite successful at reproducing the characteristics of the Luxembourg economy contained in the data. In particular, the parameter estimates are reasonable and the model is able to fit the autocovariance structure of the data fairly well. These findings suggest that LED could constitute an interesting empirical tool for economic and policy analysis at the BCL. The paper ends with a number of selected applications illustrating the type of insights that could be gained from LED. These applications include analyzing the effects of supply, demand, and foreign shocks on the macroeconomic outlook in Luxembourg, identifying the empirically relevant shocks and frictions in the Luxembourg economy, and interpreting historical developments in terms of individual contributions originating from the model's structural shocks.

The rest of the paper is organized as follows. The presentation of LED is divided in two parts, with Section 2 providing a non-technical overview and Section 3 describing the model in detail. Section 4 presents the estimation strategy and discusses the empirical results. Finally, Section 5 reports the outcomes of selected applications of the model, while Section 6 concludes and discusses potential extensions.

2. Outline of the Model

LED is a medium-scale DSGE model with 152 equations. These can be split into 116 economic relationships defining the behavior of endogenous variables, the laws of motion of 18 exogenous stochastic processes, and 18 equations relating model variables to potential observables. This relatively large number of equations warrants a brief, non-technical outline before digging into the details, in order to highlight both the model's structure and its core economic mechanisms. This is the purpose of this section.

As mentioned in the Introduction, the backbone of LED is a stochastic growth model with optimizing households and firms. Starting with the former, the domestic economy is populated by a continuum of infinitely-lived households who consume final goods, save in domestic and foreign assets, accumulate the economy's capital stock, supply labor services to the production sector, and set their nominal wages. As a result, in the model households' choices are key to the equilibrium dynamics of consumption, investment, labor, and wages.

Two economic forces shape these choices. First, there is a tradeoff between consumption and saving, or equivalently between current and future consumption. In particular, a rise in the returns to saving (for instance through higher interest rates) increases households incentives to cut consumption in order to save more. Conversely, a rise in the households' desire to consume entails a drop in the saving rate for given interest rates. In the model, the presence of both consumption habits and investment adjustment costs helps this tradeoff generate plausible dynamics by smoothing the movements of consumption and saving. The second key tradeoff for households is between consumption and leisure. Households dislike working, but have to supply labor services to firms to earn wages, thereby increasing their income and ultimately their consumption. Therefore, in LED variations in consumption translate into movements in desired hours worked, which in turn affect equilibrium wages in a smooth fashion due to wage rigidity.

The representation of the labor market follows most of the DSGE literature by assuming imperfect competition between households. This setup is useful to introduce sticky wages in the model, since nominal rigidities cannot exist in perfectly competitive markets.[3] In LED, wage rigidity originates from Calvo frictions, with only a subset of households able to reoptimize their wage at each period. In addition, LED incorporates an element highly specific to Luxembourg's labor market: cross-border workers. These are agents who work in Luxembourg but live and spend most of their income outside the country. As in Moura and Lambrias (2018), they are introduced in the model by assuming that the labor services supplied by resident and cross-border workers are sufficiently differentiated that domestic firms need both as inputs for production.

Turning to firms, LED features a two-stage production structure and multiple sectors. Domestic intermediate-good producers constitute the basic production units in the model. They use capital and labor services, rented from resident households (as well as from cross-border workers when it comes to labor), to manufacture their goods.[4] Total factor productivity exhibits a stochastic upward trend, which induces long-run growth of per-capita income in the model. Intermediate-good producers set their prices in monopolistically competitive markets, subject to Calvo frictions. This is a source of price rigidity, as producers cannot instantaneously pass on variations in their marginal costs into prices. Overall, in the model developments in domestic producer prices are determined by the pricing behavior of forward-looking firms subject to nominal frictions.

There are four final goods in the model, used respectively for private consumption, investment, public consumption, and exports. Only the last one can be traded internationally, whereas the three others are just for domestic use. These goods are produced by competitive retailers as combinations of domestic and imported intermediate inputs. Thus, retailers are just a model device representing how final agents substitute between domestic and foreign goods and services. Given this production structure, inflation at the sectoral level depends on both domestic and foreign producer prices (as well as on the exchange rate). Both the share of imported inputs and the elasticity of substitution between domestic and foreign inputs are sector specific. The model also embeds adjustment costs that lower the price elasticity of inputs in the short run, reflecting frictions in reorganizing production.

[3] In a competitive environment, any seller stuck with a price above the new market price would instantaneously lose all customers.

[4] All firms in the model can be thought of as producing both goods and services, even though only the generic term of "goods" is used below.

LED also devotes specific attention to the modeling of public policy. As regards fiscal policy, the model introduces a domestic fiscal authority, akin to a government, that consumes part of final output and raises revenues from taxes levied on the private sector. The model considers a rich set of tax instruments, including a VAT-like tax on consumption expenditures, taxes on labor and capital income, and social contributions. As regards monetary policy, LED is a model of a small open economy within a currency union: short-term policy rates are set by a monetary authority akin to the European Central Bank, based on a Taylor-type rule defined in terms of euro-area aggregates.

Finally, LED adopts a parsimonious specification of the foreign block aimed at maximizing the model's ability to generate coherent economic stories.[5] This objective underlies the choice to represent the rest of the euro area using a semi-structural New-Keynesian model, which allows to interpret economic developments in the monetary union and to identify their impact on the domestic economy. This semi-structural model has three equations: a Phillips curve linking inflation with the output gap in the euro area, an IS curve defining the output gap as a function of the real interest rate in the euro area, and the aforementioned Taylor-like rule for the common monetary policy. Given the small size of the country, the weight of the Luxembourg economy in the euro-area aggregates defined by this model is taken to be zero. Demand from outside the euro area and the price of oil are represented as exogenous stochastic processes.

3. Model

This section describes the model in details, starting with households and firms, then turning to public policy and finally to the foreign block.

3.1. **Households.** Resident households have an infinite horizon and make the final consumption and investment decisions. They accumulate physical capital, whose services they rent out to firms, and also buy and sell nominal bonds. Finally, households act as wage setters in the monopolistic market for labor services.

3.1.1. *Preferences.* The typical resident household $h \in [0, 1]$ has lifetime preferences given by

$$U_{h,0} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\epsilon_t^{PREF} \ln \left(C_{h,t} - \kappa C_{t-1} \right) - \frac{1}{1+\zeta} N_{h,t}^{1+\zeta} \right],$$

where E_0 is the expectation operator conditional on date-0 information, $\beta \in [0, 1)$ is the discount rate, $\kappa \in [0, 1)$ is a measure of (external) consumption habits, and $\zeta > 0$ is the inverse elasticity of labor supply. $C_{h,t}$ and $N_{h,t}$ denote household h's consumption and

[5]Most central bank DSGE models represent foreign variables as independent stochastic processes. The closest approach to the one implemented here is the structural VAR representation adopted in the Riksbank model RAMSES (Adolfson, Laseen, Christiano, Trabandt, and Walentin, 2013).

labor supply, and C_t is aggregate consumption. Finally, ϵ_t^{PREF} a preference shock evolving according to

$$\ln \epsilon_t^{PREF} = \rho_{\epsilon^{PREF}} \ln \epsilon_{t-1}^{PREF} + \eta_t^{\epsilon^{PREF}},$$

with $\rho_{\epsilon^{PREF}} \in (0,1)$ and $\eta_t^{\epsilon^{PREF}}$ an identically and independently distributed (iid) normal shock. This preference shock plays a similar role to the 'risk premium' shock considered in, e.g., Smets and Wouters (2007) and Coenen, Christoffel, and Warne (2008).

3.1.2. *Budget constraint.* Household h's expenditures at date t are

$$\left(1+\tau_t^C\right)\left(P_{C,t}C_{h,t} + P_{I,t}I_{h,t}\right) + \left[(1-\Gamma_{B,t})R_t^{EA}\right]^{-1} B_{h,t} + \Phi_{h,t} + T_t.$$

Consumption and investment expenditures are subject to a value-added tax τ_t^C. $B_{h,t}$ denotes household h's position in risk-less nominal euro bonds paying a gross return R_t^{EA}. $\Gamma_{B,t}$ is an external financial premium given by

$$\Gamma_{B,t} = \gamma_B \left[\exp\left(\frac{B_t}{P_{GDP,t}GDP_t} - \overline{B}_Y\right) - 1\right],$$

with $\gamma_B > 0$ and $P_{GDP,t}GDP_t$ denoting nominal GDP (the product of the GDP deflator, P_{GDP}, with real GDP, GDP). In equilibrium, no lending-borrowing occurs between domestic agents, so $\overline{B}_Y$ parametrizes the average external asset position of the economy. The external premium cost implies that domestic agents have to pay an increasing return on their international debt, which helps ensure the model has a well-defined steady state and determinate dynamics (Schmitt-Grohe and Uribe, 2003). Finally, $\Phi_{h,t}$ represents the net cost from household h's participation in state-contingent security markets, ensuring that all households choose identical allocations in spite of heterogeneous wage rates, and T_t is a lump-sum government tax.

Date-t income is measured by

$$(1-\tau_t^N-\tau_t^{W_h})W_{h,t}N_{h,t} + (1-\tau_t^K)[R_{K,t}u_{h,t} - \Gamma_{u,t}P_{I,t}]K_{h,t-1} + \tau_t^K\delta P_{I,t}K_{h,t-1}$$
$$+(1-\tau_t^D)D_{h,t} + B_{h,t-1}.$$

Here, $W_{h,t}$ denotes the nominal wage rate received by household h, $u_{h,t}$ measures capital utilization, $K_{h,t-1}$ is the stock of capital owned by household h at date t (and thus formed at date $t-1$), and $D_{h,t}$ refers to the profits (or dividends) intermediate firms pay to households. Utilization is normalized to unity in steady state and the associated cost $\Gamma_{u,t}$ is given by

$$\Gamma_{u,t} = \gamma_{u1}(u_{h,t}-1) + \frac{\gamma_{u2}}{2}(u_{h,t}-1)^2,$$

with $\gamma_{u1}, \gamma_{u2} \geq 0$. The fiscal authority sets distortionary taxes on labor income (rate τ_t^N), capital income (rate τ_t^K), and profits (rate τ_t^D), while $\tau_t^{W_h}$ is an additional tax on labor income akin to a social security contribution. Capital taxation is net of utilization costs and partly corrected from physical depreciation ($\delta \in [0,1]$ is the depreciation rate).

3.1.3. *Capital accumulation.* The law of motion of household h's capital stock is

$$K_{h,t} = (1-\delta)K_{h,t-1} + \epsilon_t^I \left(1-\Gamma_{I,t}\right) I_{h,t}.$$

Here, ϵ_t^I is a technology process shifting the efficiency of new investment goods over time. Estimated DSGE models often find that investment shocks account for a significant share of aggregate fluctuations in both the US (Justiniano, Primiceri, and Tambalotti, 2011; Moura, 2018) and the euro area (Smets and Wouters, 2003; Coenen, Christoffel, and Warne, 2008). This efficiency shock evolves according to

$$\ln \epsilon_t^I = \rho_{\epsilon^I} \ln \epsilon_{t-1}^I + \eta_t^{\epsilon^I},$$

with $\rho_{\epsilon^I} \in (0,1)$ and $\eta_t^{\epsilon^I}$ an iid normal shock. $\Gamma_{I,t}$ is a dynamic investment adjustment cost given by

$$\Gamma_{I,t} = \frac{\gamma_I}{2}\left(\frac{I_{h,t}}{I_{h,t-1}} - g_z\right)^2,$$

with $\gamma_I \geq 0$.

3.1.4. *Utility maximization.* Taking into account that all households choose identical allocations in equilibrium, the optimal choices for C_t, u_t, I_t, K_t, and B_t are characterized by

$$\begin{aligned}
\Lambda_t &= \frac{\epsilon_t^{PREF}}{(1+\tau_t^C)(C_t - \kappa C_{t-1})},\\
R_t^K &= P_{I,t}\Gamma'_{u,t},\\
\frac{(1+\tau_t^C)P_{I,t}}{P_{C,t}} &= Q_t\epsilon_t^I\left(1-\Gamma_{I,t}-\Gamma'_{I,t}\frac{I_t}{I_{t-1}}\right) + \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}\epsilon_{t+1}^I\Gamma'_{I,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2,\\
Q_t &= \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-\delta)Q_{t+1} + (1-\tau_{t+1}^K)\frac{R_{t+1}^K}{P_{C,t+1}}u_{t+1} + \frac{P_{I,t+1}}{P_{C,t+1}}\left(\delta\tau_{t+1}^K - [1-\tau_{t+1}^K]\Gamma_{u,t+1}\right)\right],\\
1 &= \beta(1-\Gamma_{B,t})R_t^{EA}E_t\frac{\Lambda_{t+1}}{\Lambda_t\Pi_{C,t+1}},
\end{aligned}$$

where $\Pi_{C,t} = P_{C,t}/P_{C,t-1}$ is consumption price inflation.

3.1.5. *Wage setting.* As originally proposed by Erceg, Henderson, and Levin (2000), the model for the labor market postulates that each resident household h supplies its labor service $N_{h,t}$ under monopolistic competition and sets the corresponding nominal wage rate $W_{h,t}$. Individual wages adjust sluggishly due to staggered wage contracts à la Calvo (1983): for any given household, the probability of optimally resetting its wage at any given period is $1-\xi_W$, with $\xi_W \in [0,1)$. Wages that are not reoptimized adjust according to the following rule of thumb:

$$W_{h,t} = g_{z,t}\Pi_{C,t-1}^{\chi_W}\overline{\Pi}^{1-\chi_W}W_{h,t-1},$$

where $\overline{\Pi}$ denotes steady-state inflation. It follows that nominal wages are indexed to productivity developments and a geometric average of past and average consumer price inflation rates, with $\chi_W \in [0,1]$ measuring the weight on past inflation.[6]

A resident household reoptimizing at date t maximizes expected lifetime utility subject to the budget constraint and to demand for its specific labor service derived in Section 3.2.2. The optimal reset wage $\widetilde{W}_{R,t}$ is characterized by

$$E_t \sum_{k=0}^{\infty} (\beta \xi_W)^k \Bigg[\Lambda_{t+k} \left(1 - \tau^N_{t+k} - \tau^{W_h}_{t+k}\right) \frac{\widetilde{W}_{R,t}}{P_{C,t+k}} \left(\frac{z_{t+k}}{z_t}\right) \left(\frac{P_{C,t+k-1}}{P_{C,t-1}}\right)^{\chi_W} \overline{\Pi}^{(1-\chi_W)k} \\ - \frac{\theta_{W,t}}{\theta_{W,t}-1} N^{\zeta}_{h,t+k} \Bigg] N_{h,t+k} = 0.$$

It is the same for all resident households reoptimizing at date t. As a result, the aggregate wage index for resident households $W_{R,t}$ evolves according to

$$W_{R,t} = \left[\xi_W \left(g_{z,t} \Pi^{\chi_W}_{C,t-1} \overline{\Pi}^{1-\chi_W} W_{R,t-1} \right)^{1-\theta_{W,t}} + (1-\xi_W) \widetilde{W}^{1-\theta_{W,t}}_{R,t} \right]^{\frac{1}{1-\theta_{W,t}}}.$$

3.1.6. *Foreign Workers.* As discussed in Section 3.2.2, labor supplied by foreign workers is a required input for Luxembourg intermediate firms. This defines an endogenous demand function for foreign labor in the model.

Turning to supply, foreign workers solve a maximization program similar to that of resident households, resulting in the wage equation:

$$W_{F,t} = \left[\xi_W \left(g_{z,t} \Pi^{\chi_W}_{C,t-1} \overline{\Pi}^{1-\chi_W} W_{F,t-1} \right)^{1-\theta_{W,t}} + (1-\xi_W) \widetilde{W}^{1-\theta_{W,t}}_{F,t} \right]^{\frac{1}{1-\theta_{W,t}}}.$$

Because the marginal utility of consumption of foreign workers is difficult to characterize endogenously, the optimal reset wage $\widetilde{W}_{F,t}$ is not explicitly modeled. Instead, its law of motion is taken to be

$$\widetilde{W}_{F,t} = \epsilon^{\widetilde{W}_F}_t \widetilde{W}_{R,t}.$$

This assumption ensures that the wage rates for resident households and foreign workers share common long-run dynamics, while allowing for short-term divergences captured by the stationary process $\epsilon^{\widetilde{W}_F}_t$. The latter evolves according to

$$\ln \epsilon^{\widetilde{W}_F}_t = \rho_{\epsilon^{\widetilde{W}_F}} \ln \epsilon^{\widetilde{W}_F}_{t-1} + \eta^{\epsilon^{\widetilde{W}_F}}_t,$$

[6]Following standard terminology, "indexation" refers here to the direct effect of consumer inflation from the previous quarter on current price and wage developments. This should not be confused with Luxembourg's legal indexation mechanism that automatically adjusts wages, pensions, and social benefits to past inflation. The legal mechanism operates according to a threshold system, which triggers only when the cumulative increase in consumer prices since the last adjustment reaches 2.5%. DSGE models cannot capture this kind of non-linearity and instead assume that past inflation feed backs to current wages every period.

with $\rho_{\epsilon^{\widetilde{W}_F}} \in (0,1)$ and $\eta_t^{\epsilon^{\widetilde{W}_F}}$ an iid normal shock.[7]

Finally, in line with the existing institutional arrangements, foreign workers pay labor income taxes and social security contributions to the domestic fiscal authority.

3.2. **Firms.** There are three types of firms in the model. First, competitive retailers produce the final goods by combining domestic and imported inputs. Second, monopolistically competitive producers manufacture the domestic intermediate inputs. Third, exporters serve foreign markets. This section describes the first two types, while the behavior of exporters is discussed in Section 3.3.

3.2.1. *Final good production.* There are four final goods in the model, used respectively for private consumption, investment, public consumption, and exports. The first three cannot be traded across regions, while the last one is sold abroad by exporters. These goods are produced by competitive retail firms as different combinations of domestic and imported intermediate inputs.

Technology. The final consumption good is produced in quantity Q_t^C according to a constant-elasticity-of-substitution (CES) production function combining a basket of domestic intermediate goods, H_t^C, and a basket of imported foreign goods, IM_t^C:

$$Q_t^C = \left[\nu_C^{\frac{1}{\mu_C}} \left([1-\Gamma_{HC,t}] H_t^C \right)^{\frac{\mu_C - 1}{\mu_C}} + (1-\nu_C)^{\frac{1}{\mu_C}} \left([1-\Gamma_{IMC,t}] IM_t^C \right)^{\frac{\mu_C - 1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}} .$$

Here, $\mu_C > 0$ measures the elasticity of substitution between domestic and imported inputs in the production of consumption, while $\nu_C \in [0,1]$ measures the relative weight of domestic inputs.

The consumption-producing firm incurs adjustment costs $\Gamma_{HC,t}$ and $\Gamma_{IMC,t}$ when varying the use of its domestic and imported inputs, equal to

$$\Gamma_{HC,t} = \frac{\gamma_{HC}}{2} \left(\frac{H_t^C}{H_{t-1}^C} - g_z \right)^2, \qquad \Gamma_{IMC,t} = \frac{\gamma_{IMC}}{2} \left(\epsilon_t^{IM} \frac{IM_t^C}{IM_{t-1}^C} - g_z \right)^2,$$

with γ_{HC}, $\gamma_{IMC} > 0$ and $g_z > 0$ the average growth rate of the economy. These (external) costs lower the price elasticity of inputs in the short run, reflecting frictions in reorganizing production. The disturbance ϵ_t^{IM} shifts the import cost over time and constitutes an import demand shock. It evolves according to

$$\ln \epsilon_t^{IM} = \rho_{\epsilon^{IM}} \ln \epsilon_{t-1}^{IM} + \eta_t^{\epsilon^{IM}},$$

with $\rho_{\epsilon^{IM}} \in (0,1)$ and $\eta_t^{\epsilon^{IM}}$ an iid normal shock.

[7] Since the wage of cross-border workers is not used as observable to estimate LED, this shock is shut down in the current version of the model.

Cost minimization. Letting $P_{H,t}$ and $P_{IM,t}$ denote the price indexes corresponding to the bundles of domestic and imported inputs, cost minimization yields the standard demand system

$$H_t^C = \nu_C \left(\frac{P_{H,t}}{P_{C,t}\Gamma^\dagger_{HC,t}} \right)^{-\mu_C} \frac{Q_t^C}{1-\Gamma_{HC,t}}, \quad IM_t^C = (1-\nu_C) \left(\frac{P_{IM,t}}{P_{C,t}\Gamma^\dagger_{IMC,t}} \right)^{-\mu_C} \frac{Q_t^C}{1-\Gamma_{IMC,t}},$$

where

$$\Gamma^\dagger_{HC,t} = 1 - \Gamma_{HC,t} - \Gamma'_{HC,t}\frac{H_t^C}{H_{t-1}^C}, \qquad \Gamma^\dagger_{IMC,t} = 1 - \Gamma_{IMC,t} - \Gamma'_{IMC,t}\frac{\epsilon_t^{IM} IM_t^C}{IM_{t-1}^C}.$$

The associated price index for the final consumption good is

$$P_{C,t} = \left[\nu_C \left(\frac{P_{H,t}}{\Gamma^\dagger_{HC,t}} \right)^{1-\mu_C} + (1-\nu_C) \left(\frac{P_{IM,t}}{\Gamma^\dagger_{IMC,t}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}}.$$

Production of the final goods used for investment, public consumption, and exports is modeled in a similar fashion. All production functions are of the CES form and combine domestic and imported intermediate goods. Both the elasticities of substitution (μ_I, μ_G, μ_X) and the CES weights (ν_I, ν_G, ν_X) are sector specific, while the import demand shock ϵ_t^{IM} affects all sectors symmetrically. The input demand functions and the sectoral price indexes are derived just as for the consumption sector.

Demand for domestic and imported intermediate inputs. The baskets of domestic and imported inputs in the consumption sector verify

$$H_t^C = \left[\int_0^1 \left(H_{f,t}^C\right)^{\frac{\theta_{H,t}-1}{\theta_{H,t}}} df \right]^{\frac{\theta_{H,t}}{\theta_{H,t}-1}}, \qquad IM_t^C = \left[\int_0^1 \left(IM_{f^\star,t}^C\right)^{\frac{\theta_{IM,t}-1}{\theta_{IM,t}}} df^\star \right]^{\frac{\theta_{IM,t}}{\theta_{IM,t}-1}},$$

where $H_{f,t}^C$ and $IM_{f^\star,t}^C$ respectively denote the demands for the intermediate goods produced by domestic producer f and foreign exporter $f^\star$. $\theta_{H,t} > 1$ represents the time-varying elasticity of substitution between domestic intermediate goods. It also determines the price markup in the domestic market, given by $\theta_{H,t}/(\theta_{H,t}-1) > 1$. Similarly, $\theta_{IM,t} > 1$ represents the time-varying elasticity of substitution between imported intermediate goods and determines the price markup in the market for imports. These elasticities evolve according to

$$\ln\theta_{z,t} = \rho_{\theta_z}\ln\theta_{z,t-1} + (1-\rho_{\theta_z})\ln\theta_z + \eta_t^{\theta_z},$$

with $\theta_z > 1$, $\rho_{\theta_z} \in (0,1)$, $\eta_t^{\theta_z}$ an iid normal shock, and $z = H,\ IM$. The baskets of inputs used in the production of the other final goods are defined in a similar fashion.

Letting $P_{H,f,t}$ and $P_{IM,f^\star,t}$ denote the prices in euro charged by firms f and $f^\star$, cost minimization yields

$$H_{f,t}^C = \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\theta_{H,t}} H_t^C, \qquad P_{H,t} = \left(\int_0^1 P_{H,f,t}^{1-\theta_{H,t}} df \right)^{\frac{1}{1-\theta_{H,t}}},$$

and

$$IM^C_{f^\star,t} = \left(\frac{P_{IM,f^\star,t}}{P_{IM,t}}\right)^{-\theta_{IM,t}} IM^C_t, \qquad P_{IM,t} = \left(\int_0^1 P_{IM,f^\star,t}^{1-\theta_{IM,t}} df^\star\right)^{\frac{1}{1-\theta_{IM,t}}}.$$

These equations also define the price indexes $P_{H,t}$ and $P_{IM,t}$.

Finally, aggregating across all four final-good firms yields the following demand equations for domestic and foreign intermediate goods f and $f^\star$:

$$H_{f,t} = H^C_{f,t} + H^I_{f,t} + H^G_{f,t} + H^X_{f,t} = \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\theta_{H,t}} H_t,$$

$$IM_{f^\star,t} = IM^C_{f^\star,t} + IM^I_{f^\star,t} + IM^G_{f^\star,t} + IM^X_{f^\star,t} = \left(\frac{P_{IM,f^\star,t}}{P_{IM,t}}\right)^{-\theta_{IM,t}} IM_t,$$

with $H_t = H^C_t + H^I_t + H^G_t + H^X_t$ and $IM_t = IM^C_t + IM^I_t + IM^G_t + IM^X_t$.

3.2.2. *Intermediate good production.* Monopolistically competitive domestic firms produce the differentiated intermediate goods used as inputs by the final sector. These firms face pricing frictions à la Calvo, introducing price stickiness in the model.

Technology. The typical intermediate firm $f \in [0,1]$ has access to the increasing-returns-to-scale Cobb-Douglas technology

$$Y_{f,t} = \max\left[\epsilon_t \left(K^s_{f,t}\right)^\alpha (z_t N_{f,t})^{1-\alpha} - \psi z_t, 0\right],$$

where $Y_{f,t}$ denotes output, $K^s_{f,t}$ capital services, and $N_{f,t}$ labor services, while $\alpha \in (0,1)$ defines the share of output devoted to capital payments in steady state. There are two technology disturbances, which are common across firms. First, ϵ_t is a persistent but stationary shock, that evolves according to

$$\ln \epsilon_t = \rho_\epsilon \ln \epsilon_{t-1} + \eta^\epsilon_t,$$

with $\rho_\epsilon \in (0,1)$ and η^ϵ_t an iid normal shock. Second, z_t denotes a permanent technology shock that introduces a unit root in the model and captures long-run technical progress. It evolves according to a random walk with drift:

$$g_{z,t} = \rho_{gz} g_{z,t-1} + (1-\rho_{gz}) g_z + \eta^{g_z}_t,$$

where $g_{z,t} = z_t/z_{t-1}$, $\rho_{gz} \in (0,1)$, and $\eta^{g_z}_t$ an iid normal shock.[8] As already mentioned, $g_z > 0$ is the average growth rate of the economy. Finally, $\psi > 0$ denotes a fixed cost of production, identical across firms. It is rescaled by the permanent technology shock to obtain a well-defined steady state.

Cost minimization. Intermediate firms pay a social security tax on the wage bill, denoted $\tau^{W_f}_t$. Letting $R_{K,t}$ and W_t stand for the rental cost of capital services and the aggregate wage

[8]To enforce the usual balanced-growth restrictions, the same stochastic technology trend drives foreign real variables in LED.

rate, firm f's cost-minimization problem yields the following demand functions for capital and labor services:

$$R_{K,t}K_{f,t}^s = \alpha\left(Y_{f,t} + \psi z_t\right)MC_{f,t}, \qquad \left(1+\tau_t^{W_f}\right)W_t N_{f,t} = (1-\alpha)\left(Y_{f,t}+\psi z_t\right)MC_{f,t},$$

where $MC_{f,t}$ denotes the firm's nominal marginal cost. Since all intermediate firms face the same input prices and have access to the same production function, marginal costs must be identical across firms, so that

$$MC_{f,t} = MC_t = \frac{1}{\epsilon_t z_t^{1-\alpha}}\left(\frac{R_{K,t}}{\alpha}\right)^{\alpha}\left[\frac{\left(1+\tau_t^{W_f}\right)W_t}{1-\alpha}\right]^{1-\alpha}.$$

Price setting. Each intermediate firm f sells its output under monopolistic competition and sets its own price. Individual prices adjust sluggishly due to staggered price contracts à la Calvo: for any given firm, the probability of optimally resetting its price at any given period is $1-\xi_H$, with $\xi_H \in [0,1)$. Prices that are not reoptimized adjust according to the following rule of thumb:

$$P_{H,f,t} = \Pi_{H,t-1}^{\chi_H}\overline{\Pi}^{1-\chi_H}P_{H,f,t-1},$$

so that intermediate prices are indexed to a geometric average of past intermediate-good inflation $\Pi_{H,t-1} = P_{H,t-1}/P_{H,t-2}$ and average inflation $\overline{\Pi} > 0$, with $\chi_H \in (0,1)$ measuring the weight on past inflation.

A firm that reoptimizes its price at date t maximizes the expected discounted sum of its future nominal profits:

$$E_t\sum_{k=0}^{\infty}\xi_H^k\Lambda_{t,t+k}\left(P_{H,f,t+k} - MC_{t+k}\right)H_{f,t+k},$$

where $\Lambda_{t,t+k}$ is the household's stochastic discount factor for nominal payoffs. Maximization is subject to the above rule and to the demand for domestic input f derived in Section 3.2.1. The optimal reset price $\widetilde{P}_{H,t}$ is characterized by

$$E_t\sum_{k=0}^{\infty}\xi_H^k\Lambda_{t,t+k}\left[\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_H}\overline{\Pi}^{1-\chi_H}\right)\widetilde{P}_{H,t} - \frac{\theta_{H,t+k}}{\theta_{H,t+k}-1}MC_{t+k}\right]H_{f,t+k} = 0.$$

Hence, the optimal pricing strategy equates the expected discounted sum of future revenues to a markup over the expected discounted sum of future marginal costs of production. Since both markups and marginal costs are equal across firms, all optimizing firms choose the same reset price in equilibrium. As a result, the price index for domestic intermediate goods $P_{H,t}$ evolves according to

$$P_{H,t} = \left[\xi_H\left(\Pi_{H,t-1}^{\chi_H}\overline{\Pi}^{1-\chi_H}P_{H,t-1}\right)^{1-\theta_{H,t}} + (1-\xi_H)\widetilde{P}_{H,t}^{1-\theta_{H,t}}\right]^{\frac{1}{1-\theta_{H,t}}}.$$

Demand for labor services. The index of labor services, $N_{f,t}$, combines labor varieties supplied by both resident households and foreign workers according to

$$N_{f,t} = \left[\nu_N^{\frac{1}{\mu_N}} \left([1-\Gamma_{R,t}] \, N_{R,f,t} \right)^{\frac{\mu_N - 1}{\mu_N}} + (1-\nu_N)^{\frac{1}{\mu_N}} \left([1-\Gamma_{F,t}] \, N_{F,f,t} \right)^{\frac{\mu_N - 1}{\mu_N}} \right]^{\frac{\mu_N}{\mu_N - 1}},$$

where $N_{R,f,t}$ stands for labor supplied by resident households and $N_{F,f,t}$ for labor supplied by foreign households, $\mu_N > 0$ measures the elasticity of substitution between resident and foreign labor, and $\nu_N \in [0,1]$ measures the relative weight of resident workers (Moura and Lambrias, 2018). Finally, the firm incurs adjustment costs $\Gamma_{R,t}$ and $\Gamma_{F,t}$ when varying the use of resident and foreign labor, equal to

$$\Gamma_{R,t} = \frac{\gamma_R}{2} \left(\epsilon_t^N \frac{N_{R,f,t}}{N_{R,t-1}} - 1 \right)^2, \qquad \Gamma_{F,t} = \frac{\gamma_F}{2} \left(\frac{N_{F,f,t}}{N_{F,t-1}} - 1 \right)^2,$$

with γ_R, $\gamma_F > 0$. The disturbance ϵ_t^N shifts the demand for resident labor over time and evolves according to

$$\ln \epsilon_t^N = \rho_{\epsilon^N} \ln \epsilon_{t-1}^N + \eta_t^{\epsilon^N},$$

with $\rho_{\epsilon^N} \in (0,1)$ and $\eta_t^{\epsilon^N}$ an iid normal shock.

Letting $W_{R,t}$ and $W_{F,t}$ denote the (average) wage rates received by resident households and foreign workers, the usual cost-minimization program yields the demand functions

$$N_{R,f,t} = \nu_N \left(\frac{W_{R,t}}{W_t \Gamma_{R,t}^\dagger} \right)^{-\mu_N} \frac{N_{f,t}}{1-\Gamma_{R,t}}, \qquad N_{F,f,t} = (1-\nu_N) \left(\frac{W_{F,t}}{W_t \Gamma_{F,t}^\dagger} \right)^{-\mu_N} \frac{N_{f,t}}{1-\Gamma_{F,t}},$$

with

$$\Gamma_{R,t}^\dagger = 1 - \Gamma_{R,t} - \Gamma'_{R,t} \frac{\epsilon_t^N N_{R,f,t}}{N_{R,t-1}}, \qquad \Gamma_{F,t}^\dagger = 1 - \Gamma_{F,t} - \Gamma'_{F,t} \frac{N_{F,f,t}}{N_{F,t-1}},$$

as well as the aggregate wage index

$$W_t = \left[\nu_N \left(\frac{W_{R,t}}{\Gamma_{R,t}^\dagger} \right)^{1-\mu_N} + (1-\nu_N) \left(\frac{W_{F,t}}{\Gamma_{F,t}^\dagger} \right)^{1-\mu_N} \right]^{\frac{1}{1-\mu_N}}.$$

The indexes of resident and foreign labor services verify

$$N_{R,f,t} = \left[\int_0^1 \left(N_{f,t}^h \right)^{\frac{\theta_{W,t}-1}{\theta_{W,t}}} dh \right]^{\frac{\theta_{W,t}}{\theta_{W,t}-1}}, \qquad N_{F,f,t} = \left[\int_0^1 \left(N_{f,t}^{h^\star} \right)^{\frac{\theta_{W,t}-1}{\theta_{W,t}}} dh^\star \right]^{\frac{\theta_{W,t}}{\theta_{W,t}-1}},$$

where $N_{f,t}^h$ and $N_{f,t}^{h^\star}$ respectively denote the demands for the differentiated labor services supplied by resident household h and foreign worker $h^\star$ by firm f. $\theta_{W,t} > 1$ represents the time-varying elasticity of substitution between household-specific labor services. It also determines the markup in the labor market, given by $\theta_{W,t}/(\theta_{W,t}-1) > 1$. This elasticity evolves according to

$$\ln \theta_{W,t} = \rho_{\theta_W} \ln \theta_{W,t-1} + (1-\rho_{\theta_W}) \ln \theta_W + \eta_t^{\theta_W},$$

with $\theta_W > 1$, $\rho_{\theta_W} \in (0,1)$ and $\eta_t^{\theta_W}$ an iid normal shock. Letting $W_{h,t}$ and $W_{h^\star,t}$ denote the wage rates, cost minimization yields

$$N_{f,t}^{h} = \left(\frac{W_{h,t}}{W_{R,t}}\right)^{-\theta_{W,t}} N_{R,f,t}, \qquad N_{f,t}^{h^\star} = \left(\frac{W_{h^\star,t}}{W_{F,t}}\right)^{-\theta_{W,t}} N_{F,f,t},$$

and

$$W_{R,t} = \left(\int_0^1 W_{h,t}^{1-\theta_{W,t}} dh\right)^{\frac{1}{1-\theta_{W,t}}}, \qquad W_{F,t} = \left(\int_0^1 W_{h^\star,t}^{1-\theta_{W,t}} dh^\star\right)^{\frac{1}{1-\theta_{W,t}}}.$$

Aggregating over firms, total demand for labor services supplied by resident household h is

$$N_{h,t}^{d} = \left(\frac{W_{h,t}}{W_{R,t}}\right)^{-\theta_{W,t}} \int_0^1 N_{R,f,t}\, df = \left(\frac{W_{h,t}}{W_{R,t}}\right)^{-\theta_{W,t}} N_{R,t}.$$

3.3. **International trade and asset position.** This section discusses imports, exports, foreign demand, and the economy's external asset position.

3.3.1. *Imports.* Imported inputs are produced by foreign firms whose behavior mirrors that of domestic intermediate producers. The typical foreign intermediate firm $f^\star \in [0,1]$ sells (part of) its production in the domestic market, setting its price in euro.[9] This assumption seems especially suited in a model for Luxembourg, since more than 70% of the country's imports come from the rest of the euro area. Again, there is slow adjustment in prices due to Calvo rigidities.

The optimality condition for a foreign firm that reoptimizes its price at date t is given by

$$E_t \sum_{k=0}^{\infty} \xi_{IM}^{k} \Lambda_{t,t+k}^{\star} \left[\left(\prod_{s=1}^{k} \Pi_{IM,t+s-1}^{\chi_{IM}} \overline{\Pi}^{1-\chi_{IM}} \right) \frac{\widetilde{P}_{IM,t}}{S_t^\star} - \frac{\theta_{IM,t+k}}{\theta_{IM,t+k}-1} MC_{t+k}^{\star} \right] IM_{f^\star,t+k} = 0,$$

where variables and parameters have the usual interpretation. $S_t^\star$ is the nominal effective exchange rate, expressed in units of the domestic currency, i.e. the euro, per unit of a composite foreign currency. Additionally, $MC_t^\star = (P_{O,t})^{\omega^\star} (P_t^\star)^{1-\omega^\star}$ represents the foreign firm's nominal marginal cost, expressed as a geometric average between the oil price $P_{O,t}$ (expressed in units of the composite foreign currency) and a composite foreign price level $P_t^\star$. The weight $\omega^\star \in [0,1]$ measures the share of oil in imports. Section 3.6 provides more details about $S_t^\star$ and $P_t^\star$.

Given the Calvo framework, the price index for imports $P_{IM,t}$ evolves according to

$$P_{IM,t} = \left[\xi_{IM} \left(\Pi_{IM,t-1}^{\chi_{IM}} \overline{\Pi}^{1-\chi_{IM}} P_{IM,t-1} \right)^{1-\theta_{IM,t}} + (1-\xi_{IM}) \widetilde{P}_{IM,t}^{1-\theta_{IM,t}} \right]^{\frac{1}{1-\theta_{IM,t}}}.$$

[9]Coenen, Christoffel, and Warne (2008) use the same local currency pricing assumption.

3.3.2. *Exports.* There is a continuum of domestic firms that transform the final export good into specialized intermediates supplied to foreign markets.[10] Firms in the export sector transform the final export good into specialized varieties in a "brand naming" process. The typical export firm $ex \in [0,1]$ has access to the linear technology

$$EX_{ex,t} = Q^X_{ex,t},$$

where $EX_{ex,t}$ denotes output and $Q^X_{ex,t}$ is the demand of the final export good. It follows immediately that the marginal cost in the export sector is identical across firms and given by $MC_{EX,t} = P_{X,t}$.

Export firm ex sells its output under monopolistic competition and sets its own price in euro. Again, this assumption reflects the empirical regularity that Luxembourg exports mostly to other euro-area countries. Individual prices adjust sluggishly due to staggered price contracts à la Calvo: for any given firm, the probability of optimally resetting its price at any given period is $1 - \xi_{EX}$, with $\xi_{EX} \in [0,1)$. Prices that are not reoptimized adjust according to the following rule of thumb:

$$P_{EX,ex,t} = \Pi^{\chi_{EX}}_{EX,t-1} \overline{\Pi}^{1-\chi_{EX}} P_{EX,ex,t-1},$$

so that intermediate prices are indexed to a geometric average of past export inflation $\Pi_{EX,t-1} = P_{EX,t-1}/P_{EX,t-2}$ and average inflation $\overline{\Pi}$, with $\chi_{EX} \in (0,1)$ measuring the weight on past inflation.

An export firm that reoptimizes its price at date t maximizes the expected discounted sum of its future nominal profits:

$$E_t \sum_{k=0}^{\infty} \xi^k_{EX} \Lambda_{t,t+k} \left(P_{EX,ex,t+k} - P_{X,t+k} \right) EX_{ex,t+k}.$$

Maximization is subject to the above indexation rule and to the demand for export variety ex derived in Section 3.2.1. The optimal reset export price $\widetilde{P}_{EX,t}$ is characterized by

$$E_t \sum_{k=0}^{\infty} \xi^k_{EX} \Lambda_{t,t+k} \left[\left(\prod_{s=1}^{k} \Pi^{\chi_{EX}}_{EX,t+s-1} \overline{\Pi}^{1-\chi_{EX}} \right) \widetilde{P}_{EX,t} - \frac{\theta_{EX,t+k}}{\theta_{EX,t+k} - 1} P_{X,t+k} \right] EX_{ex,t+k} = 0.$$

Moreover, all optimizing export firms choose the same reset price in equilibrium. The price index for exports $P_{EX,t}$ evolves according to

$$P_{EX,t} = \left[\xi_{EX} \left(\Pi^{\chi_{EX}}_{EX,t-1} \overline{\Pi}^{1-\chi_{EX}} P_{EX,t-1} \right)^{1-\theta_{EX,t}} + (1 - \xi_{EX}) \widetilde{P}^{1-\theta_{EX,t}}_{EX,t} \right]^{\frac{1}{1-\theta_{EX,t}}}.$$

[10] As discussed in Christiano, Trabandt, and Walentin (2011), this model device is useful to introduce price stickiness in the export sector. Indeed, the export retailer described in Section 3.2.1 cannot be a source of price rigidity because it behaves competitively.

3.3.3. *Foreign demand.* As in Coenen, Christoffel, and Warne (2008), a representative foreign retail firm purchases the specialized export goods and aggregates them into

$$EX_t = \left[\int_0^1 \left(EX_{ex,t}\right)^{\frac{\theta_{EX,t}-1}{\theta_{EX,t}}} dex\right]^{\frac{\theta_{EX,t}}{\theta_{EX,t}-1}}.$$

$\theta_{EX,t} > 1$ represents the time-varying elasticity of substitution between exported varieties. It also determines the price markup in the export market, which is given by $\theta_{EX,t}/(\theta_{EX,t}-1) > 1$. The elasticity of substitution evolves according to

$$\ln\theta_{EX,t} = \rho_{\theta_{EX}} \ln\theta_{EX,t-1} + (1-\rho_{\theta_{EX}})\ln\theta_{EX} + \eta_t^{\theta_{EX}},$$

with $\theta_{EX} > 1$, $\rho_{\theta_{EX}} \in (0,1)$ and $\eta_t^{\theta_{EX}}$ an iid normal shock.

The foreign retailer takes input prices as given and minimizes its costs subject to its production function. This yields the usual conditions

$$EX_{ex,t} = \left(\frac{P_{EX,ex,t}}{P_{EX,t}}\right)^{-\theta_{EX,t}} EX_t, \qquad P_{EX,t} = \left(\int_0^1 P_{EX,ex,t}^{1-\theta_{EX,t}} dex\right)^{\frac{1}{1-\theta_{EX,t}}}.$$

The retailer supplies the volume of EX_t that satisfies foreign demand, given by

$$EX_t = \nu_t^\star \left(\frac{P_{EX,t}/S_t^\star}{P_t^\star \Gamma_{EX,t}^\dagger}\right)^{-\mu^\star} \frac{Y_t^\star}{1-\Gamma_{EX,t}},$$

where $Y_t^\star$ is foreign output and

$$\Gamma_{EX,t} = \frac{\gamma_{EX}}{2}\left(\frac{EX_t/Y_t^\star}{EX_{t-1}/Y_{t-1}^\star} - 1\right)^2, \qquad \Gamma_{EX,t}^\dagger = 1 - \Gamma_{EX,t} - \Gamma'_{EX,t}\frac{EX_t/Y_t^\star}{EX_{t-1}/Y_{t-1}^\star}.$$

The shock $\nu_t^\star$ shifts the foreign demand for export goods. It evolves according to

$$\ln\nu_t^\star = \rho_{\nu^\star}\ln\nu_{t-1}^\star + \eta_t^{\nu^\star},$$

with $\rho_{\nu^\star} \in (0,1)$ and $\eta_t^{\nu^\star}$ an iid normal shock.

3.3.4. *Trade balance and foreign asset position.* The domestic economy's nominal trade balance is given by

$$TB_t = P_{EX,t}EX_t - P_{IM,t}IM_t.$$

Net foreign assets evolve according to

$$\left(R_t^{EA}\right)^{-1} B_t = B_{t-1} + TB_t - \left(1 - \tau_t^N - \tau_t^{W_h}\right) W_{F,t}N_{F,t}.$$

This law of motion signals that the country's net external position increases with net exports and decreases with net payments to the rest of the world (here, with labor payments to foreign workers).

3.4. **Fiscal authority.** At each period, the fiscal authority purchases an amount G_t of the final government consumption good and reimburses its outstanding debt $B_{g,t-1}$. Its income comes from distortionary and lump-sum taxes levied on the private sector, as well as new bond issuance. As discussed above, there are taxes on consumption and investment expenditures (rate τ_t^C), and on labor, capital, and business income (rates τ_t^N, τ_t^K, and τ_t^D). There are also two social-security taxes levied on household wage income and on firms' wage bills (rates $\tau_t^{W_h}$ and $\tau_t^{W_f}$). Finally, there is a lump-sum tax paid by households, T_t. Hence, the fiscal authority's budget constraint is

$$P_{G,t}G_t + B_{g,t-1} = \tau_t^C \left(P_{C,t}C_t + P_{I,t}I_t\right) + \left(\tau_t^N + \tau_t^{W_h} + \tau_t^{W_f}\right)\left(W_{R,t}N_{R,t} + W_{F,t}N_{F,t}\right)$$
$$+\tau_t^K \left[R_t^K u_t - (\Gamma_{u,t} + \delta)P_{I,t}\right] K_{t-1} + \tau_t^D D_t + T_t + \left[(1-\Gamma_{B,t})R_t^{EA}\right]^{-1} B_{g,t}.$$

Public purchases evolve exogeneously according to $G_t = z_t g_t$, where

$$\ln g_t = \rho_g \ln g_{t-1} + (1-\rho_g)\ln \overline{g} + \epsilon_{g,t},$$

with $\rho_g \in (0,1)$ and $\epsilon_{g,t}$ an iid normal shock.

As in Coenen, Christoffel, and Warne (2008), LED assumes Ricardian equivalence, so that the time path of public debt is irrelevant for private agents' decisions. In practice, this requires that the distortionary tax rates τ^C, τ^D, τ^K, τ^N, τ^{W_h}, and τ^{W_f} remain constant and that lump-sum taxes adjust to close the fiscal authority's budget constraint at each period.

3.5. **Market clearing and GDP.** In general equilibrium, relative prices adjust to equalize demand and supply in each market. Imposing market clearing thus requires measuring the demand and supply for each good, taking into account the effect of price dispersion on aggregates.

3.5.1. *Consumption, investment, and government goods.* The market-clearing conditions for the final goods used for consumption, investment, and government expenditures are

$$Q_t^C = C_t, \qquad Q_t^I = I_t + \Gamma_{u,t}K_{t-1}, \qquad Q_t^G = G_t.$$

3.5.2. *Domestic intermediate goods.* The market-clearing condition for tradable good f is

$$Y_{f,t} = H_{f,t}.$$

Integrating the left-hand side over f gives the total supply of domestic intermediate goods:

$$Y_t = \int_0^1 Y_{f,t}\, df.$$

Similarly integrating the right-hand side gives the aggregate demand for domestic intermediate goods:

$$\int_0^1 H_{f,t}\, df = s_{H,t}H_t,$$

where

$$s_{H,t} = \int_0^1 \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\theta_{H,t}} df$$

is a measure of price dispersion in the market for domestic intermediate goods. Given the optimal pricing strategy, it evolves according to

$$s_{H,t} = \xi_H \left(\frac{\Pi_{H,t}}{\Pi_{H,t-1}^{\chi_H} \overline{\Pi}^{1-\chi_H}} \right)^{\theta_{H,t}} s_{H,t-1} + (1-\xi_H) \left(\frac{\widetilde{P}_{H,t}}{P_{H,t}} \right)^{-\theta_{H,t}}.$$

It can be shown that $s_{H,t} \geq 1$, so that price dispersion entails an aggregate efficiency loss. However, $s_H = 1$ in steady state and fluctuations in $s_{H,t}$ have only second-order effects, so that they disappear in the linearized version of the model. Finally, market clearing for domestic intermediate goods requires

$$Y_t = s_{H,t} H_t.$$

3.5.3. *Exported goods.* Similar computations imply that the market-clearing condition for exports is

$$Q_t^X = s_{EX,t} EX_t,$$

where

$$s_{EX,t} = \xi_{EX} \left(\frac{\Pi_{EX,t}}{\Pi_{EX,t-1}^{\chi_{EX}} \overline{\Pi}^{1-\chi_{EX}}} \right)^{\theta_{EX,t}} s_{EX,t-1} + (1-\xi_{EX}) \left(\frac{\widetilde{P}_{EX,t}}{P_{EX,t}} \right)^{-\theta_{EX,t}}.$$

3.5.4. *Capital services.* Total demand for capital services by domestic intermediate firms verifies

$$K_t^s = \int_0^1 K_{f,t}^s \, df.$$

Market clearing for capital services then requires that demand equals the supply by households based on the physical capital stock and its utilization rate:

$$K_t^s = u_t K_{t-1}.$$

3.5.5. *Labor services.* The market-clearing condition for the differentiated labor service supplied by household h is

$$N_{h,t} = \left(\frac{W_{h,t}}{W_{R,t}} \right)^{-\theta_{W,t}} N_{R,t}.$$

Integrating both sides over h gives the aggregate condition

$$N_{H,t} = s_{W,t} N_{R,t},$$

where

$$s_{W,t} = \xi_W \left(\frac{\Pi_{W,t}}{g_{z,t} \Pi_{C,t-1}^{\chi_W} \overline{\Pi}^{1-\chi_W}} \right)^{\theta_{W,t}} s_{W,t-1} + (1-\xi_W) \left(\frac{\widetilde{W}_{R,t}}{W_{R,t}} \right)^{-\theta_{W,t}}$$

is a measure of wage dispersion. As for price dispersion, wage dispersion entails an aggregate efficiency loss that disappears in the linearized version of the model.

3.5.6. *Bonds.* Lump-sum taxes balance the government budget at each period, so that public debt is zero in equilibrium: $B_{g,t} = 0$. The supply of foreign bonds is fully elastic and matches the holdings accumulated by domestic households.

3.5.7. *Corporate profits.* Domestic intermediate producers and exporters generate profits each period. The nominal profit of intermediate firm f is given by

$$D_{f,t} = \left(P_{H,f,t} - MC_t\right) H_{f,t}.$$

Integrating over f gives total profits in the intermediate sector:

$$D_{H,t} = \left(P_{H,t} - s_{H,t} MC_t\right) H_t.$$

Nominal profits in the export sector follow from similar computations:

$$D_{X,t} = \left(P_{EX,t} - s_{EX,t} P_{X,t}\right) EX_t.$$

Finally, aggregate profits verify

$$D_t = D_{H,t} + D_{X,t}.$$

3.5.8. *Nominal and real GDP.* Nominal GDP in the domestic economy is given by

$$P_{GDP,t} GDP_t = P_{C,t} C_t + P_{I,t} I_t + P_{G,t} G_t + P_{EX,t} EX_t - P_{IM,t} IM_t.$$

Simple manipulations reveal that nominal GDP is closely linked to value added in the domestic intermediate sector, since

$$P_{GDP,t} GDP_t = P_{H,t} H_t - P_{I,t} \Gamma_{u,t} K_{t-1}.$$

Hence, the only difference between nominal GDP and domestic value added is the capital utilization cost. As a result, the price of domestic intermediates is interpreted as the GDP deflator in LED, implying

$$P_{GDP,t} = P_{H,t}.$$

Given the deflator, the equation for nominal GDP identifies real GDP.

3.6. **Foreign block.** The model is closed by statistical relationships describing the behavior of foreign variables, classified into two groups. The first group corresponds to euro-area (EA) aggregates, whose dynamics are captured with a semi-structural model. The second group corresponds to rest-of-world (RW) variables, which are purely exogenous.

The joint behavior of detrended output (y^{EA}), inflation (Π^{EA}), and interest rate (R^{EA}) in the euro area is captured by the following equations:

$$\Pi_t^{EA} - \overline{\Pi} = \rho_\pi^{EA} E_t\left(\Pi_{t+1}^{EA} - \overline{\Pi}\right) + (1-\rho_\pi^{EA})\left(\Pi_{t-1}^{EA} - \overline{\Pi}\right) + \kappa^{EA} \ln \frac{y_t^{EA}}{y^{EA}} + \epsilon_t^{as^{EA}},$$

$$\ln \frac{y_t^{EA}}{y^{EA}} = \rho_y^{EA} E_t \ln \frac{y_{t+1}^{EA}}{y^{EA}} + (1-\rho_y^{EA}) \ln \frac{y_{t-1}^{EA}}{y^{EA}} - \delta^{EA}\left[R_t^{EA} - R - E_t(\Pi_{t+1}^{EA} - \overline{\Pi})\right] + \epsilon_t^{ad^{EA}},$$

$$R_t^{EA} - R = \rho_r^{EA}(R_{t-1}^{EA} - R) + (1-\rho_r^{EA})\left[\psi_1^{EA}(\Pi_t^{EA} - \overline{\Pi}) + \psi_2^{EA} \ln \frac{y_t^{EA}}{y^{EA}}\right] + \eta_t^{mp^{EA}},$$

where ρ_π^{EA}, ρ_y^{EA}, ρ_r^{EA} are in $(0,1)$, $\kappa^{EA}, \delta^{EA}, \psi_2^{EA} > 0$, $\psi_1^{EA} > 1$, and $\eta_t^{mp^{EA}}$ is an iid normal shock. $\epsilon_t^{as^{EA}}$ and $\epsilon_t^{ad^{EA}}$ are two stochastic processes evolving according to

$$\epsilon_t^{as^{EA}} = \rho_{as}^{EA} \epsilon_{t-1}^{as^{EA}} + \eta_t^{as^{EA}},$$
$$\epsilon_t^{ad^{EA}} = \rho_{ad}^{EA} \epsilon_{t-1}^{ad^{EA}} + \eta_t^{ad^{EA}},$$

with $\rho_{as^{EA}}$ and $\rho_{ad^{EA}}$ in $(0,1)$ and $\eta_t^{as^{EA}}$ and $\eta_t^{ad^{EA}}$ two iid normal shocks.

This system constitutes a semi-structural New-Keynesian model in the spirit of Clarida, Gali, and Gertler (1999), that is able to provide an interpretation of economic developments in the euro area.[11] The first equation is a hybrid Phillips curve, positively linking inflation with the output gap in the euro area. It is disturbed by the aggregate supply shock $\epsilon_t^{as^{EA}}$. The second equation is a standard IS curve, defining the output gap as a decreasing function of the real interest rate in the euro area. It is affected by the aggregate demand shock $\epsilon_t^{ad^{EA}}$. Finally, the last equation is a Taylor-like policy rule, defining how the euro-area nominal interest rate responds to the contemporaneous inflation and output gaps. It is shifted by the monetary policy shock $\eta_t^{mp^{EA}}$. Implicitly, this framework assumes that Luxembourg is too small to have significant effects on euro-area aggregates, which do not respond to movements in domestic variables.

The composite output and price indexes affecting the demand for exports are given by

$$Y_t^\star = \left(Y_t^{EA}\right)^{\psi^\star}\left(Y_t^{RW}\right)^{1-\psi^\star}, \qquad P_t^\star = \left(P_t^{EA}\right)^{\psi^\star}\left(P_t^{RW}\right)^{1-\psi^\star},$$

where $\psi^\star \in [0,1]$ denotes the fraction of Luxembourg's external trade accounted for by other euro-area countries. Likewise, the composite nominal exchange rate verifies

$$S_t^\star = \left(S_t^{EA}\right)^{\psi^\star}\left(S_t^{RW}\right)^{1-\psi^\star},$$

where $S_t^{EA} = 1$ is the nominal exchange rate between Luxembourg's currency and the euro. In these equations, the subscript RW signals rest-of-the-world variables.

[11]Strictly speaking, a structural model is one in which all parameters have an economic interpretation. This restriction explains the qualifier of 'semi structural' used here, as the coefficients ρ_π^{EA}, ρ_y^{EA}, ρ_r^{EA}, κ^{EA}, and δ^{EA} are unlikely to be structural.

Finally, turning to rest-of-world variables, detrended output evolves according to

$$\ln y_t^{RW} = \rho_y^{RW} \ln y_{t-1}^{RW} + (1 - \rho_y^{RW}) \ln y^{RW} + \eta_t^{y^{RW}},$$

with $y^{RW} > 0$, $\rho_y^{RW} \in (0, 1)$, and $\eta_t^{y^{RW}}$ an iid normal shock. RW inflation and interest rates are not modeled in LED; instead, the real exchange rate between the domestic economy and the rest of the world is assumed to follow

$$\ln s_t^{RW} = \rho_s^{RW} \ln s_{t-1}^{RW} + (1 - \rho_s^{RW}) \ln s^{RW} + \eta_t^{s^{RW}},$$

with $s^{RW} > 0$, $\rho_s^{RW} \in (0, 1)$, and $\eta_t^{s^{RW}}$ an iid normal shock. The detrended price of oil, which affects foreign firms' marginal cost, evolves according to

$$\ln p_{O,t} = \rho_{p_O} \ln p_{O,t-1} + (1 - \rho_{p_O}) \ln p_O + \eta_t^{p_O},$$

with $p_O > 0$, $\rho_{p_O} \in (0, 1)$ and $\eta_t^{p_O}$ an iid normal shock.

4. Bayesian Estimation

LED is solved with standard linearization techniques and estimated using Bayesian methods. This section discusses the solution approach, the data, and the prior distributions used in estimation. It also presents the estimation results.

4.1. **Solution method.** LED is solved using a first-order linear approximation around its stochastic balanced growth path (BGP). The intuition behind this solution procedure is described here.

When all shocks are removed from the model, or equivalently when they are all set to zero, the real variables in LED grow at the constant rate g_z inherited from technological progress.[12] The nominal variables embed an additional trend due to steady-state inflation $\overline{\Pi}$. Economists call this trajectory a balanced growth path, since it excludes shocks and features constant growth.

The BGP of DSGE models has very useful properties. First, in general it is the only equilibrium path that can be characterized analytically. In particular, it is possible to rescale all variables by their trend to obtain a system of stationary equations, which is solved by pen and paper for the deflated steady state of the model. All other equilibrium paths, in particular those that include shocks, can only be studied numerically. Second, the economic forces at play in the model ensure that equilibrium trajectories remain close to the BGP. More precisely, while shocks can push the model economy away from the BGP for some time, equilibrium paths return to the BGP as time goes by. Because of these two properties, it has become the norm in the DSGE literature to solve models with linear approximations around the BGP (or equivalently around the steady state, when growth is ignored).

[12] While this statement is true for most real variables, including GDP and its subcomponents, it is not verified by a handful of variables such as the individual labor supply, which are bounded by definition.

From a practical perspective, LED is solved according to the following steps: (a) appropriately detrend the model variables, (b) compute the deterministic steady state of the detrended model, (c) linearize the detrended system of equations around the steady state, and (d) solve the resulting system of expectational difference equations. Steps (a) and (b) are described in a technical appendix available upon request, while steps (c) and (d) are implemented numerically using the Dynare software.

4.2. **Data and shocks.** The model is estimated from the following 18 macroeconomic variables:

- real GDP (GDP)
- consumption (C)
- investment (I)
- government consumption (G)
- exports (EX)
- imports (IM)
- resident employment (E_R)
- cross-border employment (E_F)
- consumption deflator (P_C)
- export deflator (P_{EX})
- import deflator (P_{IM})
- compensation per worker (W)
- euro-area GDP (Y^{EA})†
- euro-area inflation (Π^{EA})†
- euro-area interest rate (R^{EA})†
- effective foreign demand ($Y^\star$)†
- oil price (P_O)†
- real effective exchange rate ($s^\star$)

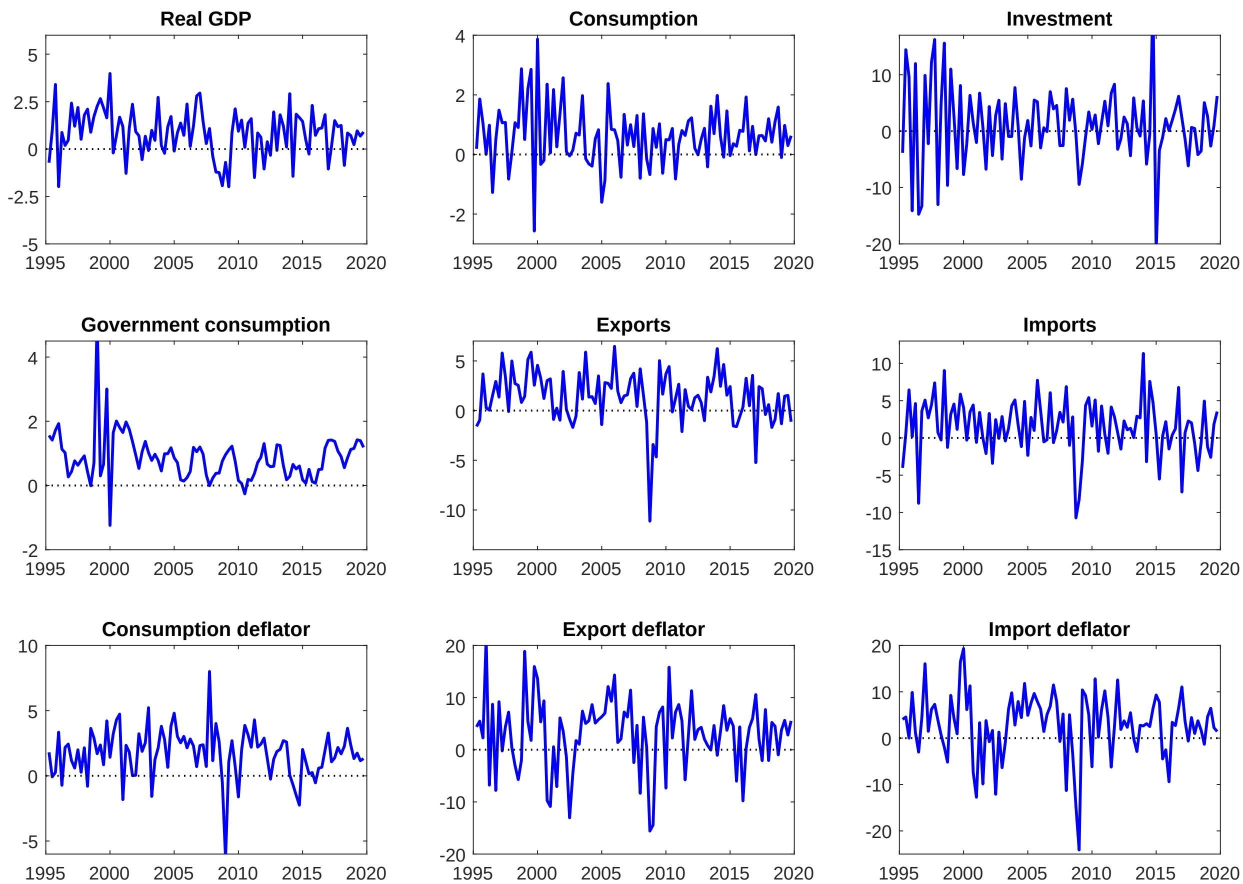
This set of observables provides important information about economic developments in Luxembourg, with data on both quantities and prices for GDP and its subcomponents.[13] In addition, knowledge of resident and cross-border employment helps identify productivity developments by documenting the behavior of the labor input, while the dynamics of wages informs about domestic production costs. Finally, the inclusion of several foreign variables strengthens the identification of international transmission mechanisms. In line with the assumption that economic developments in Luxembourg do no affect foreign variables, the parameters defining the semi-structural model for euro-area aggregates and the stochastic process for foreign output (all the variables marked with a dagger †) are kept fixed during the estimation of LED.[14]

The estimation sample runs from 1995Q1 to 2019Q4. The required time series are extracted from the ECB's Statistical Data Warehouse, which compiles data from various sources. For most of the variables related to Luxembourg (real quantities, deflators, employment, wages), the original source is the national statistical institute, STATEC. Cross-border

[13]Including the deflators for GDP, investment, and government consumption among the observables would raise an issue known as stochastic singularity. More precisely, the model predicts that some linear combinations of the price indexes hold exactly, but this restriction is not verified in the data. If additional deflators were included among the observables, this discrepancy between theory and the data would prevent the use of likelihood-based methods to estimate the models.

[14]That is, these parameters are estimated in a first step that ignores the information available in domestic observables.

FIGURE 1. Data.

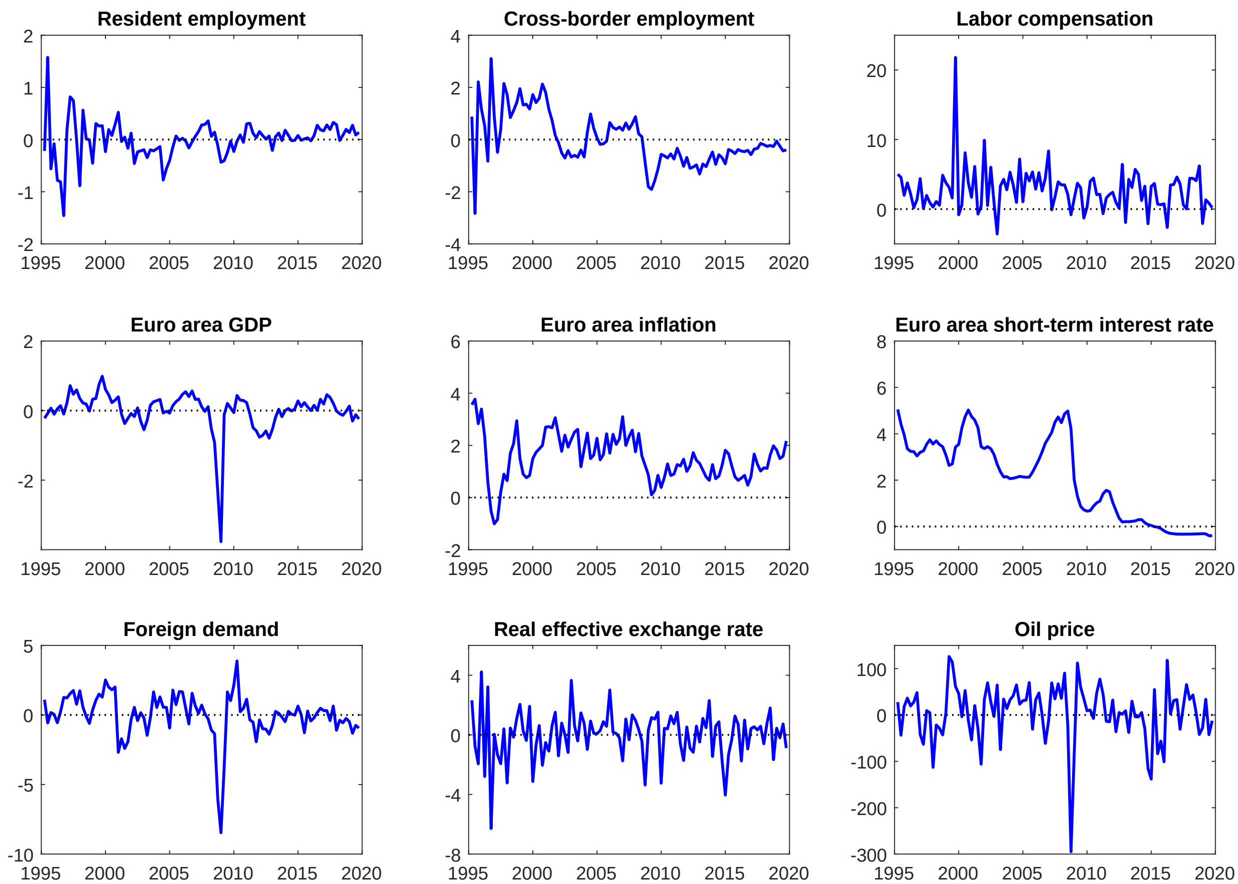


Notes. These are the (non demeaned) time series of the observed variables used in estimation. Section 4.2 details the construction of the series. Growth rates are reported in percentage terms, while inflation rates are in annualized percentage terms.

employment is constructed as the difference between total employment and resident employment, while compensation per worker is the ratio between employee compensation and the number of employees. The effective foreign demand addressed to Luxembourg is constructed by the ESCB Working Group on Forecasting as a trade-weighted average of real imports in Luxembourg's partner countries, while the real effective exchange rate is computed by the ECB as described in Schmitz, Clercq, Fidora, Lauro, and Pinheiro (2012). Regarding euro-area variables, real GDP is the chain-linked volume series computed by Eurostat, while inflation is the growth rate of the associated deflator. The short-term nominal interest rate in the euro area is from the OECD Economic Outlook before 1999 and from the Working Group on Forecasting afterward. Finally, the oil price is the Brent spot price in dollars per barrel. All series are seasonally adjusted using the Tramo-Seats software package.

Prior to estimation, the trending series for real GDP, consumption, investment, government consumption, exports, imports, the associated deflators, employment, compensation

FIGURE 2. Data.



Notes. These are the (non demeaned) time series of the observed variables used in estimation. Section 4.2 details the construction of the series. Growth rates are reported in percentage terms, while inflation and interest rates are in annualized percentage terms.

per worker, the euro-area GDP, the effective foreign demand, and the price of oil are expressed in quarter-on-quarter growth rates, approximated by the first difference of their logarithm. As in Coenen, Christoffel, and Warne (2008), additional transformations are also introduced to ensure consistency between the model's variables and the observables:

- The growth rates of GDP, consumption, investment, government consumption, imports, and exports are demeaned prior to estimation, in order to minimize the discrepancy between the model's BGP and the various trend growth rates observed in the data. The same procedure is implemented for the growth rate of real GDP in the euro area and effective demand addressed to Luxembourg. Indeed, the model allows for a single trend arising from neutral productivity growth, so that attempts to perform estimation on series exhibiting different trends would deliver biased results.

- A similar transformation is applied to the growth rates of the consumption, export, and import deflators, as well as to that of compensation per worker and euro-area prices. This is because the model's BGP implies stationary relative prices.

- Following Smets and Wouters (2003), two employment variables, E_R and E_F, are related to hours worked, N_R and N_F, by the auxiliary equations

$$\ln(E_{R,t}) = \frac{\beta}{1+\beta} E_t \ln(E_{R,t+1}) + \frac{1}{1+\beta} \ln(E_{R,t-1}) + \frac{(1-\beta\xi_E)(1-\xi_E)}{(1+\beta)\xi_E} \ln\left(\frac{N_{R,t}}{E_{R,t}}\right),$$
$$\ln(E_{F,t}) = \frac{\beta}{1+\beta} E_t \ln(E_{F,t+1}) + \frac{1}{1+\beta} \ln(E_{F,t-1}) + \frac{(1-\beta\xi_F)(1-\xi_F)}{(1+\beta)\xi_F} \ln\left(\frac{N_{F,t}}{E_{F,t}}\right),$$

  where the parameter $\xi_E \in (0,1)$ measures the sensitivity of employment with respect to hours worked. As mentioned above, the growth rates of these employment series are the observables used in estimation.

Figures 1 and 2 illustrate the time series of the transformed variables used in estimation. Two historical episodes stand out in the sample. First, the 2008-2009 Great Recession is marked by strong drops in Luxembourg exports and imports, as well as in euro-area GDP and in foreign demand addressed to Luxembourg. Given its magnitude, this recession should be especially informative about the channels through which foreign disturbances affect the Luxembourg economy. Second, there is a long period of near-zero interest rates starting in 2013, an episode which breaks the conventional monetary policy rule postulated in the model. However, the results reported in Section 4.4 suggest that the estimated euro-area Taylor rule is robust to this difficulty.

The model is estimated with the following 18 shocks:

- preference shock (ϵ^{PREF})
- temporary technology shock (ϵ)
- permanent technology shock (g_z)
- investment technology shock (ϵ^I)
- domestic price markup shock (θ_H)
- export price markup shock (θ_{EX})
- import price markup shock (θ_{IM})
- wage markup shock (θ_W)
- labor demand shock (ϵ^N)
- import demand shock (ϵ^{IM})
- foreign demand shock ($\nu^\star$)
- government consumption shock (g)
- euro-area aggr. supply shock ($\epsilon^{as^{EA}}$)
- euro-area aggr. demand shock ($\epsilon^{ad^{EA}}$)
- euro-area monetary shock ($\eta^{mp^{EA}}$)
- rest-of-world output shock (y^{RW})
- exchange rate shock (s^{RW})
- oil price shock (p_O)

These shocks correspond to the 18 observables used in estimation.[15] As described in Section 3, all shocks are modeled as first-order autoregressive processes. The only exception

[15] As all likelihood-based techniques, Bayesian estimation requires the number of shocks in the model to be at least equal to the number of observable variables.

is the monetary policy shock in the euro area, $\eta^{mp^{EA}}$, which is independently distributed over time as persistence is captured by the Taylor-rule smoothing parameter ρ_r^{EA}.

In addition, the estimated model accounts for measurement error in variables extracted from Luxembourg's quarterly national accounts, which are volatile and often subject to large revisions. More precisely, the measurement equations allow for small errors in real GDP, consumption, investment, government consumption, exports, and imports, as well as in the associated deflators.[16]

4.3. **Calibration and prior distributions.** Ignoring the standard deviations of the measurement errors, the model has 97 free parameters: 26 of them are calibrated and kept fixed during estimation, whereas the 71 others are estimated. The calibration step and the prior distributions chosen for the estimated parameters are described here.

Calibration concerns those parameters that would be poorly estimated because they have only small effects on equilibrium dynamics in the model. These include two preference parameters (the household's discount rate β and the inverse elasticity of labor supply ζ), the sensitivity of the external financial premium γ_B, two technology parameters (the Cobb-Douglas exponent on capital α and the fixed cost of production ψ), the capital depreciation coefficient δ, and the four average price and wage markups θ_H, θ_{EX}, θ_{IM}, and θ_W. The calibration assumes $\beta = 0.99$, $\zeta = 2$, $\gamma_B = 0.01$, $\alpha = 0.35$, and $\delta = 0.025$, all standard values for a quarterly model (Smets and Wouters, 2003, and Coenen, Christoffel, and Warne, 2008, use very similar values for their models of the euro area). The fixed cost ψ is calibrated so that average profits in the intermediate sector are zero. In line with Moura and Lambrias (2018), the average markups are set at 30% in both the good and labor markets. These values, together with those of all calibrated parameters, are reported in Table 1.

Other parameters are calibrated because the series used in estimation do not contain enough information about them. This is generically the case for the parameters defining the steady state of the model, which are poorly identified from growth rates and demeaned variables. Instead, it is simple to calibrate them to reproduce empirical moments estimated directly from the data, in particular sample averages. For instance, the weights in the production functions for the final consumption, investment, government consumption, and export goods are calibrated to be consistent with estimates of the import content of each sector computed from the input-output tables published by the STATEC.[17] This procedure

[16]Following Pagan (2017), measurement error applies to the *level* of the variables rather than to their growth rate. This assumption avoids introducing spurious stochastic trends when taking the model to the data.

[17]Between 2010 and 2017, the average import share in Luxembourg was 49% for private consumption, 63% for investment, 20% for public consumption, and 71% for exports. In the calibration strategy, the average share of imports in public consumption is not used and ν_G is calibrated instead to match the average import-to-GDP ratio in Luxembourg.

TABLE 1. Calibrated parameters.

Param.	Value	Description	Param.	Value	Description
β	0.99	Discount factor	ν_X	0.29	Domestic input share in EX
ζ	2.00	Labor supply elasticity	ν_N	0.60	Share of resident workers
γ_B	0.01	Foreign premium	x/y	2.01	Ratio between EX and output
α	0.35	Capital share	g/y	0.17	Ratio between G and output
ψ	1.50	Fixed production cost	τ^C	0.14	Consumption tax
δ	0.025	Depreciation rate	$\tau^D + \tau^K$	0.18	Capital income tax
θ_H	1.30	Domestic price markup	τ^N	0.25	Labor income tax
θ_{EX}	1.30	Export price markup	τ^{W^f}	0.10	Social contributions by firms
θ_{IM}	1.30	Import price markup	τ^{W^h}	0.11	Social contr. by households
θ_W	1.30	Wage markup	g_z	1.008	Productivity growth
ν_C	0.51	Domestic input share in C	$\overline{\Pi}$	1.005	Average quarterly inflation rate
ν_I	0.37	Domestic input share in I	$\psi^\star$	0.79	Share of exports going to EA
ν_G	0.91	Domestic input share in G	$\omega^\star$	0.03	Share of oil products in IM

Notes. 'C' stands for Consumption, 'I' for investment, 'G' for government consumption, 'EX' for exports, and 'IM' for imports. Also, 'EA' refers to the euro area.

results in $\nu_C = 0.51$, $\nu_I = 0.37$, $\nu_G = 0.91$, and $\nu_X = 0.29$. Likewise, the average share of cross-border workers in the Luxembourg labor market between 2010 and 2019 was 40%, implying $\nu_N = 0.60$. Similarly, the relative size of foreign variables is adjusted to ensure that exports account for 201% of GDP on average, in line with recent data.

A similar strategy is used to calibrate the parameters defining fiscal policy in the model. Government consumption accounted for 17% of GDP on average between 2010 and 2019 in Luxembourg, pinning down the value of g. As in Moura and Lambrias (2018), average tax rates are chosen to reproduce the observed structure of government revenues (see Table 2). The implied rates are $\tau^C = 14\%$ for the value-added consumption tax, $\tau^D + \tau^K = 18\%$ for the corporate income tax, $\tau^N = 25\%$ for the personal labor income tax, $\tau^{W_f} = 10\%$ for the social security contributions paid by firms, and $\tau^{W_h} = 11\%$ for the social security contributions paid by households, which are all reasonably close to the effective tax rates in Luxembourg.

The average real growth rate of the economy is calibrated at $g_z = 1.008$, implying that all real variables in the model evolve along a BGP with a trend growth rate of about 3.2% per year, while steady-state inflation is calibrated at $\overline{\Pi} = 1.005$, implying that prices increase on average by 2% each year. These two rates are in line with the behavior of real GDP and the GDP deflator in Luxembourg over the estimation sample. Finally, the calibration imposes $\psi^\star = 0.79$, so that on average 79% of Luxembourg's exports are sold to other euro-area countries, and $\omega^\star = 0.03$, so that oil products represent 3% of Luxembourg's imports. Both figures match historical averages computed from STATEC data.

Table 2 summarizes the outcome of this calibration procedure by reporting several key ratios at the steady state of the model. The table compares these model-implied values

Table 2. Key ratios in Luxembourg — Steady-state values in the model vs. data.

	Model	Data
GDP decomposition		
Private consumption	0.30	0.31
Private investment	0.19	0.19
Public consumption	0.17	0.17
Exports	2.01	2.01
Imports	1.67	1.68
Labor market		
Share of cross-border workers in total wage payments	0.40	0.40
Public finance		
Public consumption	0.17	0.17
VAT-like tax revenue	0.07	0.07
Labor income tax revenue	0.15	0.15
Capital income tax revenue	0.06	0.06
Social security contributions	0.12	0.12

Notes. Numbers represent shares in Luxembourg's nominal GDP. 'Data' refers to 2010-2019 sample averages extracted from the national accounts published by STATEC, except for labor and capital income taxes which come from the European Commission's AMECO database.

with data averages computed over the 2010-2019 period using national accounts published by STATEC, the national statistical institute. This comparison shows that LED provides a satisfactory representation of the structure of the Luxembourg economy. In particular, it reproduces the expenditure decomposition of GDP, and thus the openness of the Luxembourg economy, the significant role of cross-border workers in the labor market, and the structure of public finance.

Estimated parameters, on the other hand, govern the dynamic behavior of the model. The prior distributions for these parameters, which encapsulate the amount of *a priori* information imposed in the Bayesian estimation exercise, are reported in Tables 3 to 5. They are broadly similar to those used by, among others, Smets and Wouters (2003), Coenen, Christoffel, and Warne (2008), or Kilponen, Orjasniemi, Ripatti, and Verona (2016), who all estimate DSGE models for the euro area or euro-area countries. In addition, the priors reflect some theoretical constraints imposed on the parameters.

For instance, economic theory imposes that several parameters are bounded between 0 and 1. This restriction applies to the habit formation parameter κ, the various Calvo (ξ) and indexation (χ) coefficients governing price and wage rigidity in the model, and all autoregressive coefficients (ρ). In addition, the elasticities of substitution between domestic and imported inputs in the production functions for the final goods (μ) are estimated as

$\mu/(1+\mu)$, a rescaling that effectively bounds them between 0 and 1. The same transformation is used to rescale the elasticity of substitution between resident and cross-border workers μ_N and the price elasticity of foreign demand $\mu^\star$. All these parameters are assumed to follow standard Beta distributions. More precisely, the habit parameter has a prior mean of 0.65, in line with commonly estimated values. The Calvo coefficients fluctuate around 0.75, which corresponds to prices and wages being reoptimized on average once a year, while the indexation coefficients have a mean of 0.50. The prior distributions of the autoregressive coefficients have a common mean of 0.75. Finally, the prior distributions for the rescaled elasticities of substitution have a mean of 0.35. This choice reflects the prior assumption that domestic and imported inputs (or domestic and foreign workers) are complementary rather than substitutable.

Other parameters are bounded from below at zero. This is the case of all adjustment costs (γ), which are modeled using relatively diffuse Gamma priors centered at 1 given the lack of available *a priori* information about these parameters. Two exceptions are the adjustment cost on investment and the cost of capital utilization, γ_I and γ_{u2}, whose prior means of 2.50 and 0.20 are informed by estimation results from the DSGE literature. The standard deviations of the structural shocks and of the measurement errors should also remain positive. As usual, these parameters are assumed to follow flexible Inverse Gamma distributions.[18]

Finally, the prior distributions for the semi-structural model of the euro area reflect the available *a priori* information. As for the domestic economy, the coefficients defining the persistence of the variables (ρ_π^{EA}, ρ_y^{EA}, ρ_r^{EA}) follow Beta distributions centered at 0.75. The priors for the parameters representing the slopes of the Phillips and IS curves, κ^{EA} and δ^{EA}, are also of the Beta type, but they are centered at small values, in line with existing econometric evidence for the euro area.[19] In the monetary policy rule, the prior distribution for the inflation response coefficient is centered at 1.7, as in Smets and Wouters (2003) or Coenen, Christoffel, and Warne (2008), while the response coefficient on the output gap fluctuates around 0.50. Recall that this semi-structural model is estimated separately before the actual estimation of LED.

4.4. **Estimation results.** The right-hand panels in Tables 3 to 5 report the estimation results. In all tables, the last columns present the mode and the 90% credible posterior confidence set of each parameter. The mode is computed by maximizing the model's posterior distribution, while the confidence set is constructed using the Metropolis-Hastings algorithm

[18]Following Christiano, Trabandt, and Walentin (2011), the standard deviations of the shocks are rescaled to be of similar magnitude, in order to improve the performance of the numerical optimization algorithm used to maximize the posterior distribution.

[19]Estimation results from Smets and Wouters (2003) suggest that the slopes of the Phillips and IS curves in the euro area are 0.01 and 0.20, respectively. More recent estimates from Coenen, Christoffel, and Warne (2008) broadly confirm these values.

Table 3. Estimation results — Structural parameters

Parameter	Description	Prior distribution			Posterior distribution	
		Distribution	Mean	SD	Mode	[5%, 95%]
Standard real frictions						
κ	Consumption habits	Beta	0.65	0.05	0.85	[0.81, 0.90]
γ_I	Adj. cost: investment	Gamma	4.00	2.00	1.70	[0.72, 4.97]
γ_{u2}	Utilization cost	Gamma	0.20	0.10	0.41	[0.28, 0.55]
Production functions: Elasticities of substitution						
$\mu_C/(1+\mu_C)$	C sector	Beta	0.35	0.10	0.84	[0.76, 0.89]
$\mu_I/(1+\mu_I)$	I sector	Beta	0.35	0.10	0.34	[0.19, 0.54]
$\mu_G/(1+\mu_G)$	G sector	Beta	0.35	0.10	0.33	[0.20, 0.52]
$\mu_X/(1+\mu_X)$	EX sector	Beta	0.35	0.10	0.54	[0.45, 0.63]
Production functions: Adjustment costs						
γ_{HC}	Domestic content: C sector	Gamma	1.00	0.50	0.07	[0.06, 0.10]
γ_{HI}	Domestic content: I sector	Gamma	1.00	0.50	0.54	[0.16, 1.50]
γ_{HG}	Domestic content: G sector	Gamma	1.00	0.50	0.76	[0.28, 1.77]
γ_{HX}	Domestic content: EX sector	Gamma	1.00	0.50	2.28	[1.60, 2.60]
γ_{IMC}	Import content: C sector	Gamma	1.00	0.50	1.63	[1.17, 2.35]
γ_{IMI}	Import content: I sector	Gamma	1.00	0.50	1.11	[0.66, 1.99]
γ_{IMG}	Import content: G sector	Gamma	1.00	0.50	0.80	[0.25, 1.78]
γ_{IMX}	Import content: EX sector	Gamma	1.00	0.50	0.18	[0.09, 0.35]
Labor market						
$\mu_N/(1+\mu_N)$	Subst. elasticity: labor	Beta	0.35	0.10	0.83	[0.80, 0.90]
γ_R	Adj. cost: resident labor	Gamma	1.00	0.50	1.23	[0.64, 1.95]
γ_F	Adj. cost: foreign labor	Gamma	1.00	0.50	0.07	[0.06, 0.09]
ξ_E	Calvo: employment	Beta	0.75	0.10	0.57	[0.51, 0.64]
Nominal frictions						
ξ_H	Calvo: domestic prices	Beta	0.75	0.10	0.69	[0.59, 0.79]
χ_H	Indexation: domestic prices	Beta	0.50	0.20	0.66	[0.44, 0.86]
ξ_{IM}	Calvo: import prices	Beta	0.75	0.10	0.82	[0.77, 0.90]
χ_{IM}	Indexation: import prices	Beta	0.50	0.20	0.10	[0.06, 0.24]
ξ_{EX}	Calvo: export prices	Beta	0.25	0.10	0.25	[0.09, 0.31]
χ_{EX}	Indexation: export prices	Beta	0.50	0.20	0.15	[0.06, 0.49]
ξ_W	Calvo: wages	Beta	0.75	0.10	0.66	[0.60, 0.79]
χ_W	Indexation: wages	Beta	0.50	0.20	0.16	[0.06, 0.35]
Foreign demand						
$\mu^\star/(1+\mu^\star$	Price elasticity: foreign demand	Beta	0.35	0.10	0.48	[0.37, 0.59]
γ_{EX}	Adj. cost: foreign demand	Gamma	1.00	0.50	2.35	[1.69, 2.63]

Notes. The posterior distribution is constructed from the random-walk Metropolis-Hastings algorithm with a single chain of 200,000 draws after a burn-in period of 100,000 draws. See the notes to Table 1.

based on a Markov chain with 200,000 draws, after a burn-in sample of 100,000 draws. The acceptance ratio has the appropriate magnitude (close to 0.30) and standard diagnostics confirm convergence to a stationary posterior distribution.

In terms of point estimates, it is interesting to note that consumption habits seem larger in Luxembourg than in the euro area as a whole, whereas investment adjustment costs appear smaller. Indeed, the degree of habit formation κ is estimated above 0.80 in LED, whereas the estimates reported in Smets and Wouters (2003) and Coenen, Christoffel, and Warne (2008) for the euro area are close to 0.55. One interpretation of this finding is that LED needs strong habits to isolate household consumption from the disturbances hitting the economy, which are larger in a small and open country like Luxembourg compared to the euro area as a whole. An explanation for the smaller investment adjustment cost parameter γ_I, estimated below 1.20 in LED and between 5 and 7 in models of the euro area, is suggested by Coenen, Christoffel, and Warne (2008): households have better opportunities to smooth consumption in a small open economy since they can borrow from abroad, so that investment should be less responsive to shocks and observed fluctuations should be consistent with smaller adjustment costs.

Turning to the production side of the model, the estimated elasticity of substitution between domestic and foreign inputs is high in the consumption sector, with a posterior mode for μ_C close to 5. On the other hand, domestic and foreign inputs are strongly complementary in the production of the investment good and the government consumption good, with μ_I and μ_G estimated close to 0.50. These estimates suggest that no good local substitutes exist for foreign inputs, which makes sense for a small open economy like Luxembourg. Finally, the estimate of μ_X close to 1 suggests that the production function for exports is close to a Cobb-Douglas specification. Estimated adjustment cost parameters for domestic and foreign inputs vary substantially across the final sectors with no obvious pattern. In addition, the cost of higher capital utilization γ_{u2} is estimated close to 0.40, higher than the value reported in Coenen, Christoffel, and Warne (2008) for the euro area.

Regarding the labor market, the estimation results indicate that resident and foreign workers provide highly substitutable labor services, as the elasticity of substitution μ_N has a posterior mode just below 5. At the same time, the model needs a significant adjustment cost γ_R on resident employment to replicate the higher volatility of cross-border employment found in the data. This estimate suggests that cross-border employment constitutes the main margin of adjustment in labor input for Luxembourg firms. The estimated Calvo-style parameter for employment ξ_E is much smaller than that found by Coenen, Christoffel, and Warne (2008) for the euro area (0.57 vs. 0.85), signaling that the impact on employment from variations in average hours worked is greater in Luxembourg than in the euro area as a whole.

TABLE 4. Estimation results — Shock parameters

Parameter	Description	Prior distribution			Posterior distribution	
		Distribution	Mean	SD	Mode	[5%, 95%]
Autoregressive parameters						
ρ_ϵ	Stationary technology	Beta	0.75	0.10	0.92	[0.88, 0.93]
ρ_{gz}	Permanent technology	Normal	0.25	0.10	0.50	[0.35, 0.67]
ρ_I	Investment	Beta	0.75	0.10	0.71	[0.56, 0.83]
ρ_{θ_H}	Markup: domestic prices	Beta	0.75	0.10	0.84	[0.74, 0.92]
$\rho_{\theta_{IM}}$	Markup: import prices	Beta	0.75	0.10	0.65	[0.47, 0.82]
$\rho_{\theta_{EX}}$	Markup: export prices	Beta	0.75	0.10	0.78	[0.70, 0.85]
ρ_{θ_W}	Markup: wages	Beta	0.75	0.10	0.82	[0.75, 0.89]
ρ_{IM}	Import demand	Beta	0.75	0.10	0.80	[0.74, 0.87]
ρ_N	Labor demand	Beta	0.75	0.10	0.91	[0.81, 0.94]
$\rho_{\nu^\star}$	Foreign demand	Beta	0.75	0.10	0.85	[0.79, 0.91]
ρ_{PREF}	Preferences	Beta	0.75	0.10	0.86	[0.79, 0.92]
ρ_g	Government consumption	Beta	0.75	0.10	0.78	[0.62, 0.92]
ρ_s^{RW}	Exchange rate	Beta	0.75	0.10	0.89	[0.85, 0.93]
ρ_{p_O}	Oil price	Beta	0.75	0.10	0.91	[0.88, 0.94]
Standard deviations of innovations						
σ_ϵ	Stationary technology	Inv. Gamma	1.00	3.00	4.61	[3.96, 5.81]
σ_{gz}	Permanent technology	Inv. Gamma	1.00	3.00	4.30	[2.94, 5.19]
σ_I	Investment	Inv. Gamma	1.00	3.00	3.90	[3.31, 7.09]
σ_{θ_H}	Markup: domestic prices	Inv. Gamma	1.00	3.00	3.16	[2.23, 4.55]
$\sigma_{\theta_{IM}}$	Markup: import prices	Inv. Gamma	1.00	3.00	1.65	[1.04, 2.63]
$\sigma_{\theta_{EX}}$	Markup: export prices	Inv. Gamma	1.00	3.00	2.62	[2.09, 3.58]
σ_{θ_W}	Markup: wages	Inv. Gamma	1.00	3.00	2.22	[1.26, 4.42]
σ_{IM}	Import demand	Inv. Gamma	1.00	3.00	1.94	[1.36, 2.35]
σ_N	Labor demand	Inv. Gamma	1.00	3.00	0.96	[0.70, 1.27]
$\sigma_{\nu^\star}$	Foreign demand	Inv. Gamma	1.00	3.00	3.20	[2.14, 4.54]
σ_{PREF}	Preferences	Inv. Gamma	1.00	3.00	2.79	[2.10, 3.54]
σ_g	Government consumption	Inv. Gamma	1.00	3.00	0.50	[0.33, 3.72]
σ_s^{RW}	Exchange rate	Inv. Gamma	1.00	3.00	7.76	[6.91, 8.69]
σ_{p_O}	Oil price	Inv. Gamma	1.00	3.00	1.79	[1.62, 2.03]

Notes. See the notes to Table 3.

On the nominal side, the Calvo parameters constraining the price-setting decisions of domestic firms take moderate values. For instance, the posterior mode of ξ_H for Luxembourg is below 0.70, while the same parameter is estimated above 0.90 for the euro area by Smets and Wouters (2003) and Coenen, Christoffel, and Warne (2008). As a result, the price Phillips curve has a steeper slope in LED than in these DSGE models of the euro area, implying that domestic prices are more responsive to movements in marginal costs. The estimate is also broadly in line with the micro data studied in Lunnemann and Matha (2005),

Table 5. Estimation results — Exogenous block

Parameter	Description	Prior distribution			Posterior distribution	
		Distribution	Mean	SD	Mode	[5%, 95%]
(Semi) Structural parameters						
ρ_{π}^{EA}	Persistence: EA inflation	Beta	0.75	0.10	0.87	[0.77, 0.94]
ρ_{y}^{EA}	Persistence: EA output gap	Beta	0.75	0.10	0.59	[0.52, 0.62]
κ^{EA}	EA Phillips curve slope	Beta	0.20	0.10	0.01	[0.00, 0.02]
δ^{EA}	EA IS curve slope	Beta	0.20	0.10	0.01	[0.00, 0.05]
ρ_{r}^{EA}	EA Taylor rule: smoothing	Beta	0.75	0.10	0.94	[0.91, 0.96]
ψ_{1}^{EA}	EA Taylor rule: inflation	Normal	1.70	0.10	1.67	[1.50, 1.83]
ψ_{2}^{EA}	EA Taylor rule: output gap	Normal	0.20	0.10	0.15	[0.10, 0.24]
Autoregressive parameters for shocks						
ρ_{as}^{EA}	Permanent technology	Beta	0.75	0.10	0.74	[0.61, 0.82]
ρ_{ad}^{EA}	EA aggregate supply	Beta	0.75	0.10	0.91	[0.81, 0.94]
ρ_{y}^{RW}	EA aggregate demand	Beta	0.75	0.10	0.92	[0.87, 0.96]
Standard deviations of shock innovations						
σ_{as}^{EA}	RW output gap	Inv. Gamma	1.00	3.00	3.05	[2.22, 4.58]
σ_{ad}^{EA}	Permanent technology	Inv. Gamma	1.00	3.00	2.48	[1.79, 5.19]
σ_{mp}^{EA}	EA aggregate supply	Inv. Gamma	1.00	3.00	0.85	[0.76, 0.98]
σ_{y}^{RW}	EA aggregate demand	Inv. Gamma	1.00	3.00	4.53	[4.09, 5.14]

Notes. See the notes to Table 3.

which are consistent with Calvo parameters ranging from 0.60 to 0.75 in Luxembourg. The results also suggest that export prices are fairly flexible since the average duration between two price re-optimizations is (less than) one quarter. On the other hand, import prices seem quite sticky, with an estimate of $\xi_{IM} = 0.82$. This finding is not surprising, given that Luxembourg imports largely come from the rest of the euro area, which features very rigid prices (Coenen, Christoffel, and Warne, 2008). The estimated Calvo parameter for wages ξ_W is fully in line with the micro data discussed by Lunnemann and Wintr (2009). The estimated indexation coefficient for domestic prices χ_H is close to 0.65, higher than that reported for the euro area by Coenen, Christoffel, and Warne. On the other hand, there appears to be less persistence in wage growth in Luxembourg than in the euro area, with an estimated indexation coefficient for wages χ_W just above 0.15. This result is consistent with the empirical observation that wage changes tend to be highly synchronized in Luxembourg, as they are largely concentrated around months in which the legal indexation mechanism automatically adjusts wages, pensions, and social benefits to past inflation (see Lunnemann and Wintr, 2009, for a discussion).

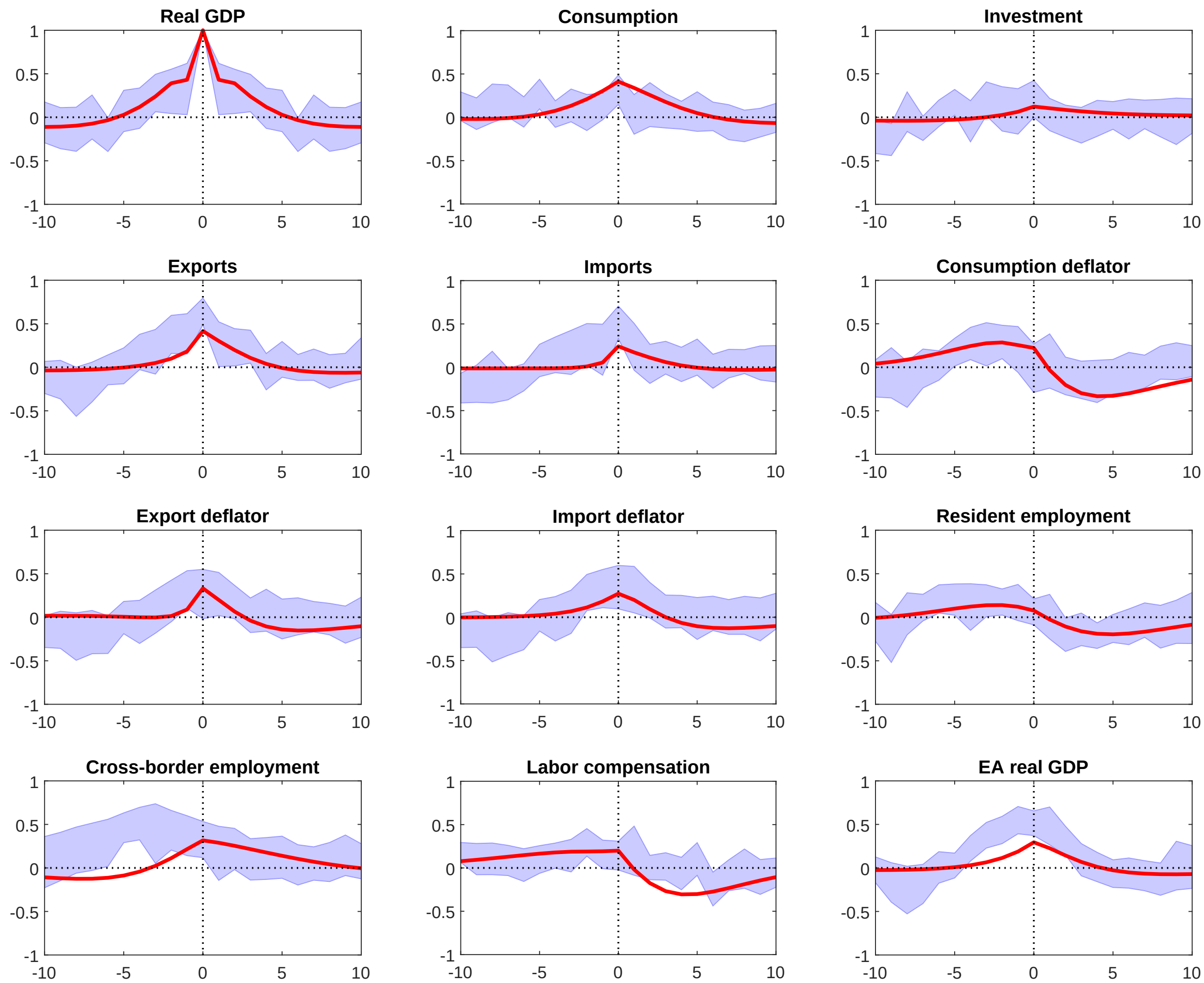
Turning to the forcing processes driving the economy, the estimation results for the semi-structural New Keynesian model of the euro area indicate that both the Phillips and IS curves in the euro area are quite flat, with κ^{EA} and δ^{EA} having their posterior modes close to zero.

Again, this finding is well in line with existing econometric evidence (Coenen, Christoffel, and Warne, 2008). The autoregressive parameters ρ_π^{EA} and ρ_y^{EA} are quite high, signaling persistent cyclical deviations in inflation and the output gap. The estimated interest-rate rule is broadly similar to those estimated in Smets and Wouters (2003) and Coenen, Christoffel, and Warne (2008), with a high degree of smoothing ρ_r^{EA}, a strong response to inflation ψ_1^{EA}, and a modest response to the output gap ψ_2^{EA}. The other structural shocks in the model have fairly persistent dynamics, even though only the government consumption features an estimated autoregressive parameter above 0.95.

4.5. **Model fit.** This section reviews the model's ability to reproduce some key properties of the data used in estimation. In particular, in Figure 3 LED's fit is evaluated by contrasting theoretical and empirical cross-correlation functions for selected observables (real GDP, consumption, investment, exports, imports, various deflators, employment, ...). In each chart, the solid red line represents the model-based cross-correlations between real GDP growth and another endogenous variable at various leads and lags, while the shaded band is a 90% Generalized Method of Moments (GMM) confidence interval centered around the empirical correlations measured in the data.

A likelihood-based estimator tries to match the entire autocovariance structure of the data; in particular, perfect fit would correspond to a situation in which all model-based correlations lie at the center of the grey bands around empirical moments. In practice, the model is a simplified representation of reality so that it cannot simultaneously fit all moments, but the figure still suggests that LED reproduces well some important features of the Luxembourg economy. First, the autocorrelation function of real GDP is adequately captured by the model, as theoretical moments always lie well within the confidence band estimated from the data. Second, the comovements between real GDP on one hand, and consumption, investment, exports, and imports are also correctly reproduced. The only discrepancies are related to the correlations between exports or imports and GDP, which are slightly underestimated around lag zero (theoretical moments at the bottom end of the confidence bands). Still, the broad picture shows that LED does a good job at replicating the correlation structure across the major components of GDP. Third, the performance is also satisfactory for prices, as the theoretical cross-correlations between real GDP and the deflators for consumption, exports, and imports are all consistent with the data. In particular, it appears that the model is able to replicate the procyclical behavior of prices in Luxembourg. Fourth, the model provides a decent fit to the data for labor-market variables, as the relationship between real GDP and both resident employment and the wage rate is correctly reproduced. Results are less satisfactory for cross-border employment, which lags real GDP growth in the data but not in the model. It is possible that a more realistic model of the labor market, for instance one based on the search and matching setup from

FIGURE 3. Selected cross-correlations with real GDP: Model vs. data.



Notes. The horizontal axis represents quarters. For quarter $j \in [-10, 10]$, the solid red line indicates the model-based cross-correlation between the variable at t and real GDP growth at $t + j$, while shaded bands represent 90% GMM confidence intervals centered around the empirical correlations.

Pierrard and Sneessens (2009) or Marchiori and Pierrard (2012, 2015), would do a better job at reproducing these dynamics.

The bottom-right panel of Figure 3 shows the relationship between real GDP growth in Luxembourg and in the euro area. It indicates that LED is able to reproduce the positive contemporaneous correlation observed in the data, with higher growth in the euro area being associated with faster economic expansion in Luxembourg. In addition, the model also replicates the negative correlation between real GDP growth in Luxembourg and the nominal interest rate in the euro area (not shown in the figure). Together, these findings suggest that LED has a rich enough structure to ensure the propagation of foreign shocks to the domestic economy.

5. Applications

This section has three parts. First, it discusses a few impulse response functions to shed light on how supply, demand, and foreign shocks affect the behavior of endogenous variables in LED. Second, it reports forecast error variance decompositions to identify the shocks that contribute the most to macroeconomic fluctuations in Luxembourg and the frictions that explain their propagation. Third, it shows how LED can help to understand economic developments in Luxembourg by providing a historical decomposition of GDP growth.

5.1. **Impulse response functions.** Figures 4 to 7 report the impulse response functions of selected model variables to four distinct shocks: a technology shock in Luxembourg, a domestic price markup shock in Luxembourg, a shock to the foreign demand addressed to Luxembourg's firms, and an interest rate shock in the euro area.[20] As in Coenen, Christoffel, and Warne (2008), this selection of shocks provides information about the effects of supply, cost-push, demand, and monetary policy shocks (with the additional feature that here the demand and monetary policy shocks originate from abroad). All figures report the mean and the 90% posterior confidence band for the impulse responses and all shocks are normalized to one standard deviation. All impulse responses are expressed as percentage deviations from LED's deterministic balanced growth path, except for those related to inflation and the interest rate, which are expressed as annualized percentage-point deviations.

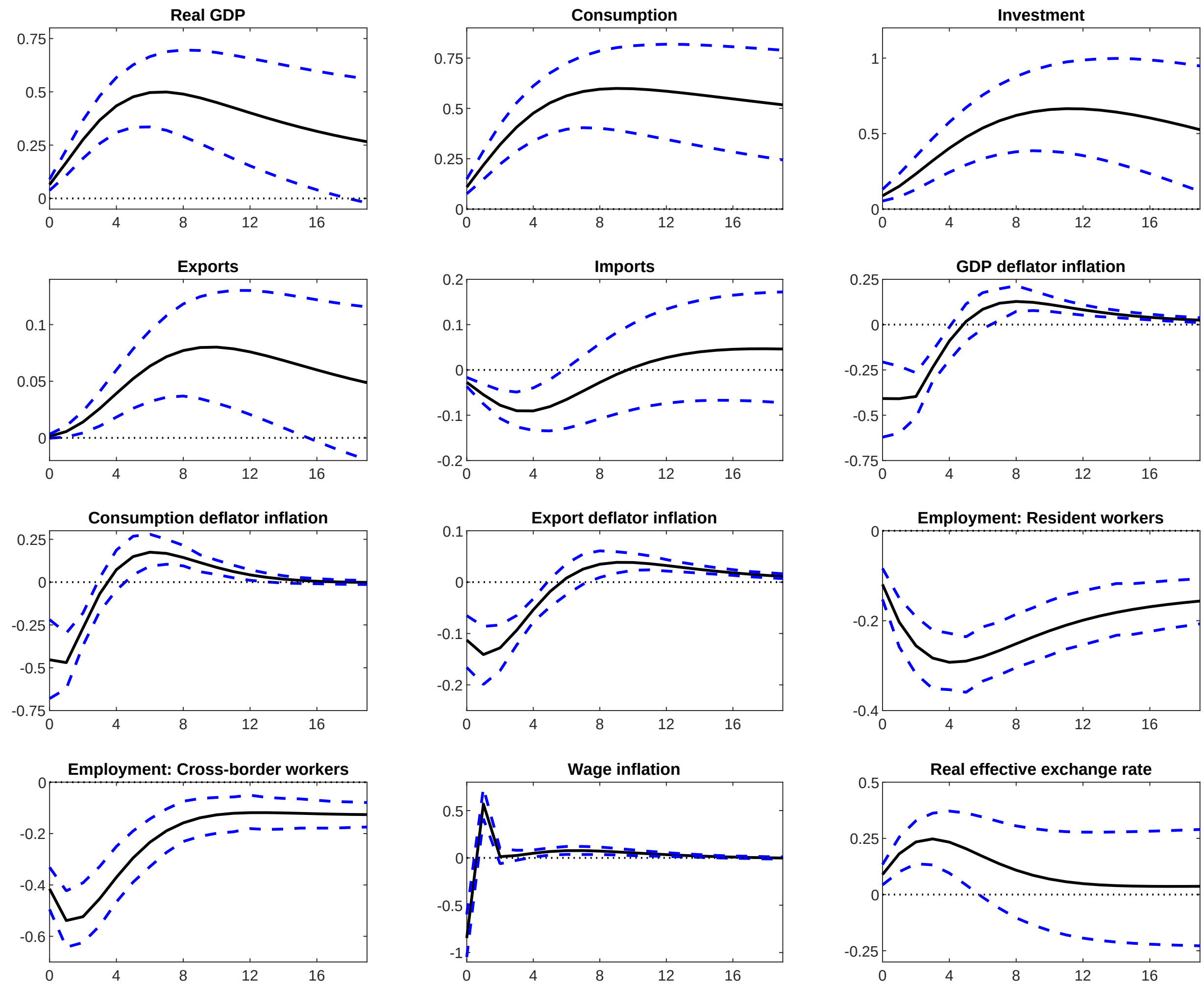
While discussing the impacts of the various shocks, it is important to keep in mind a key property of LED: in the model, as in reality, monetary policy is determined at the level of the euro area and does not react to idiosyncratic economic developments in Luxembourg. It follows that the usual logic of New Keynesian models only partly applies in LED, since the nominal interest rate is largely disconnected from movements in domestic production and prices. In particular, with no response from the nominal rate, movements in inflation translate one to one into movements in the real interest rate.

5.1.1. *Temporary technology shock.* Figure 4 reports the impulse responses to a positive temporary technology shock, whose half-life is equal to 8 quarters.[21] The effects are standard. On the one hand, higher productivity expands the domestic capacity of production, allowing output, consumption, investment, and exports to rise together. On the other hand, the fall in real marginal costs translates into lower inflation. Imports also fall, both because

[20] By definition, the impulse responses show the net effect of a shock realized at date t on a variable at date $t+j$, for $j \geq 0$. The impulse responses to the other shocks in LED are available upon request.

[21] The half-life of a random process is the time that the process needs to halve the distance from its mean after a shock, which provides a measure of persistence. For simple AR(1) processes, it is given by $\ln(0.5/\rho)$ where ρ is the autoregressive parameter.

FIGURE 4. Impulse responses to a temporary technology shock.

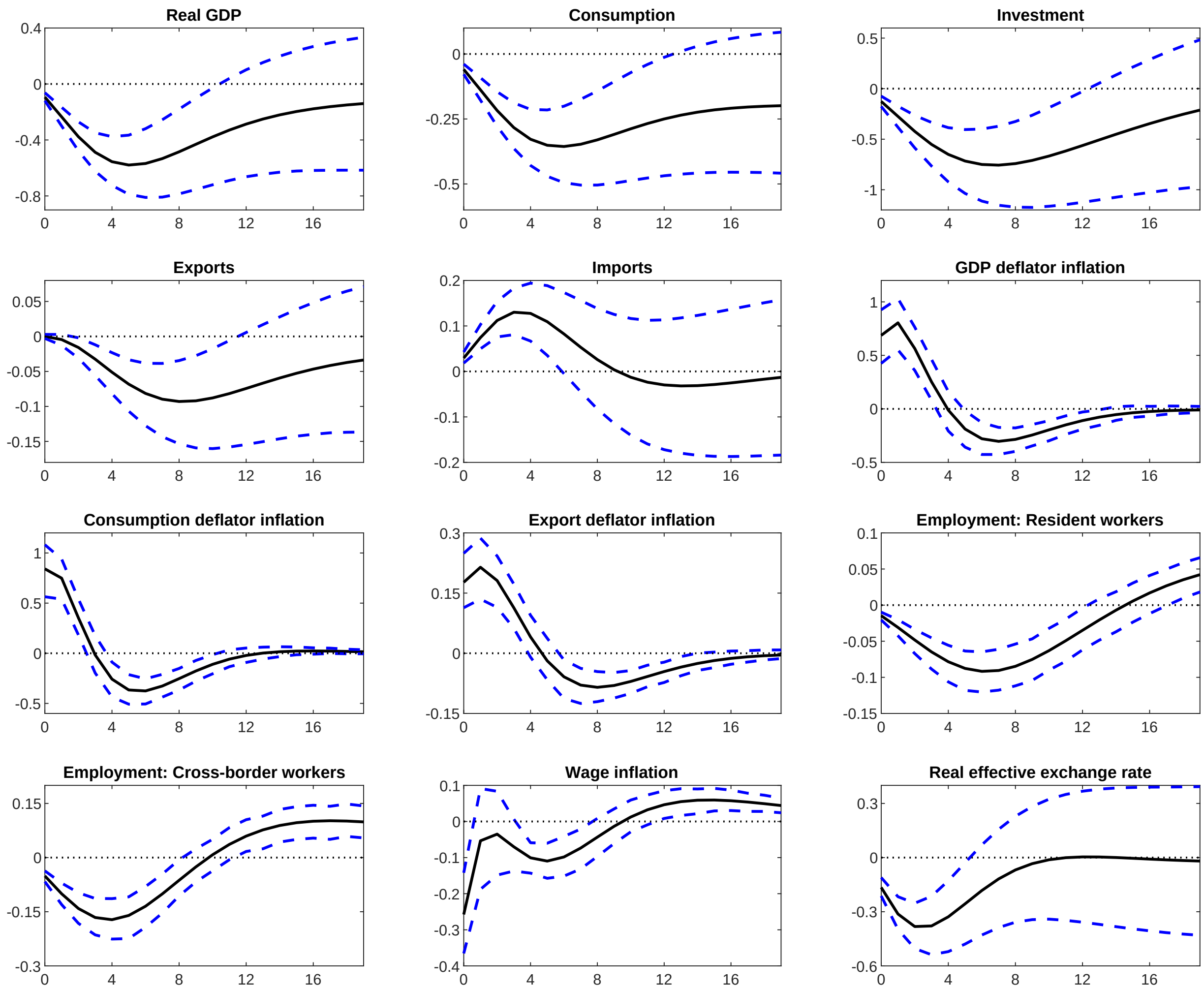


Notes. This figure shows the mean (solid black line) and the 90 percent uncertainty bands (dashed blue lines) of the impulse responses of selected variables to a temporary technology shock equal to one standard deviation. The horizontal axis represents quarters. All responses are expressed as percentage deviations from the model's balanced growth path, except for the responses of the inflation and interest rates which are reported as annualized percentage-point deviations. An increase in the real exchange rate signals a depreciation: it requires more units of domestic goods to purchase one unit of the foreign good.

domestic intermediate goods are cheaper compared to foreign varieties (this is shown by the depreciation of the real exchange rate in the bottom-right panel) and because the slow adjustment of demand limits the need for inputs. This last effect also explains the temporary decline in employment, which is larger for cross-border workers.

5.1.2. *Domestic price markup shock.* Figure 5 reports the impulse responses to a shock raising the price markup charged by domestic intermediate-good producers. The half-life of

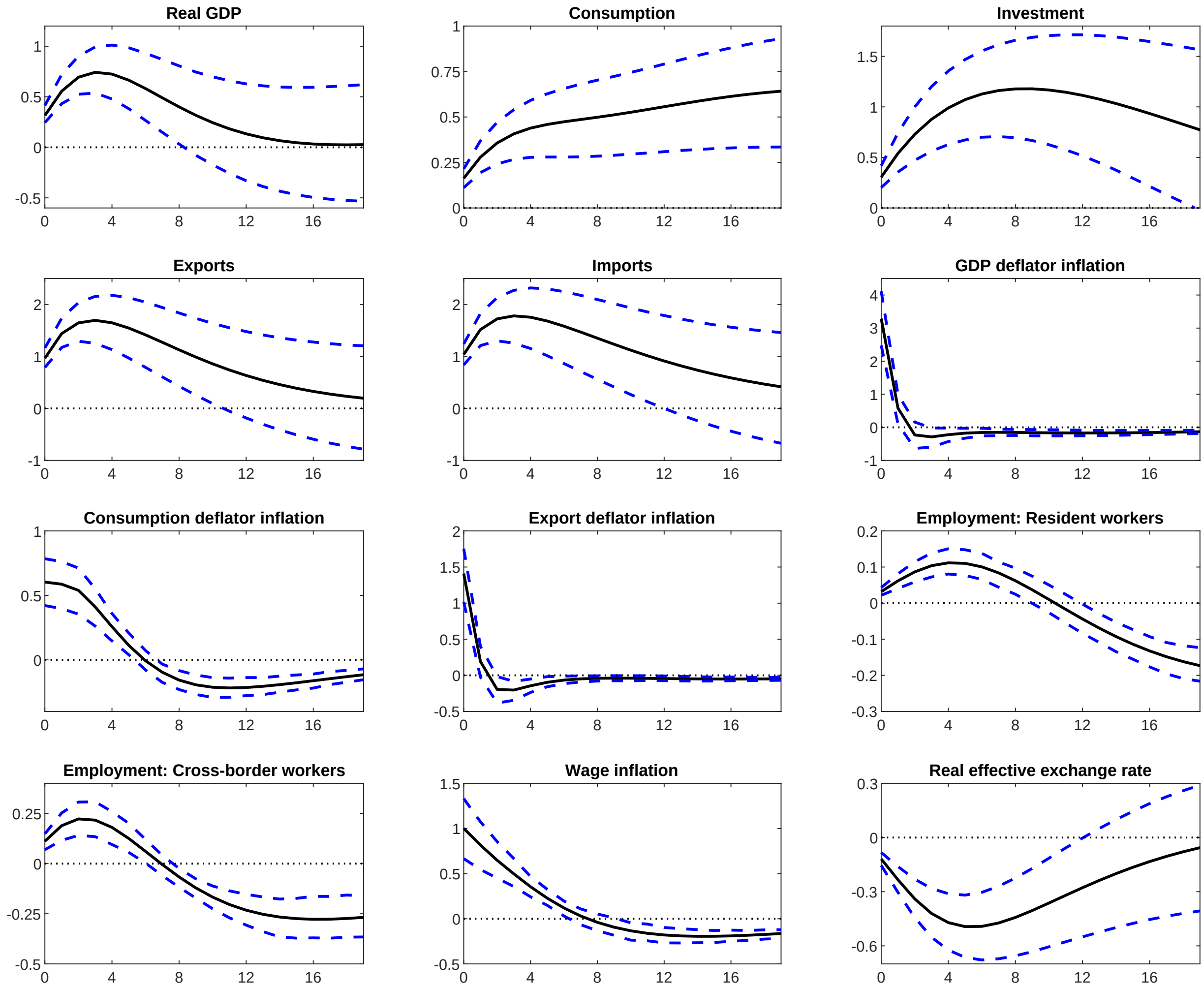
FIGURE 5. Impulse responses to a domestic price markup shock.



Notes. This figure shows the mean (solid black line) and the 90 percent uncertainty bands (dashed blue lines) of the impulse responses of selected variables to a domestic price markup shock equal to one standard deviation. See the notes to Figure 4.

the shock is equal to 4 quarters. Here also, the effects are as expected. Higher markups cause inflation to rise over the first four quarters, which weighs on consumption. Investment also falls, suggesting that the negative impact due to higher prices is more important than the associated fall in the real interest rate in Luxembourg. Domestic goods become more expensive than foreign varieties, so the real exchange rate appreciates, inducing a fall in exports and a rise in imports. The net effect on GDP is negative, causing domestic firms to cut employment over two to three years. The resulting decline in the labor force depresses wages, enhancing the negative impact of the shock on domestic demand.

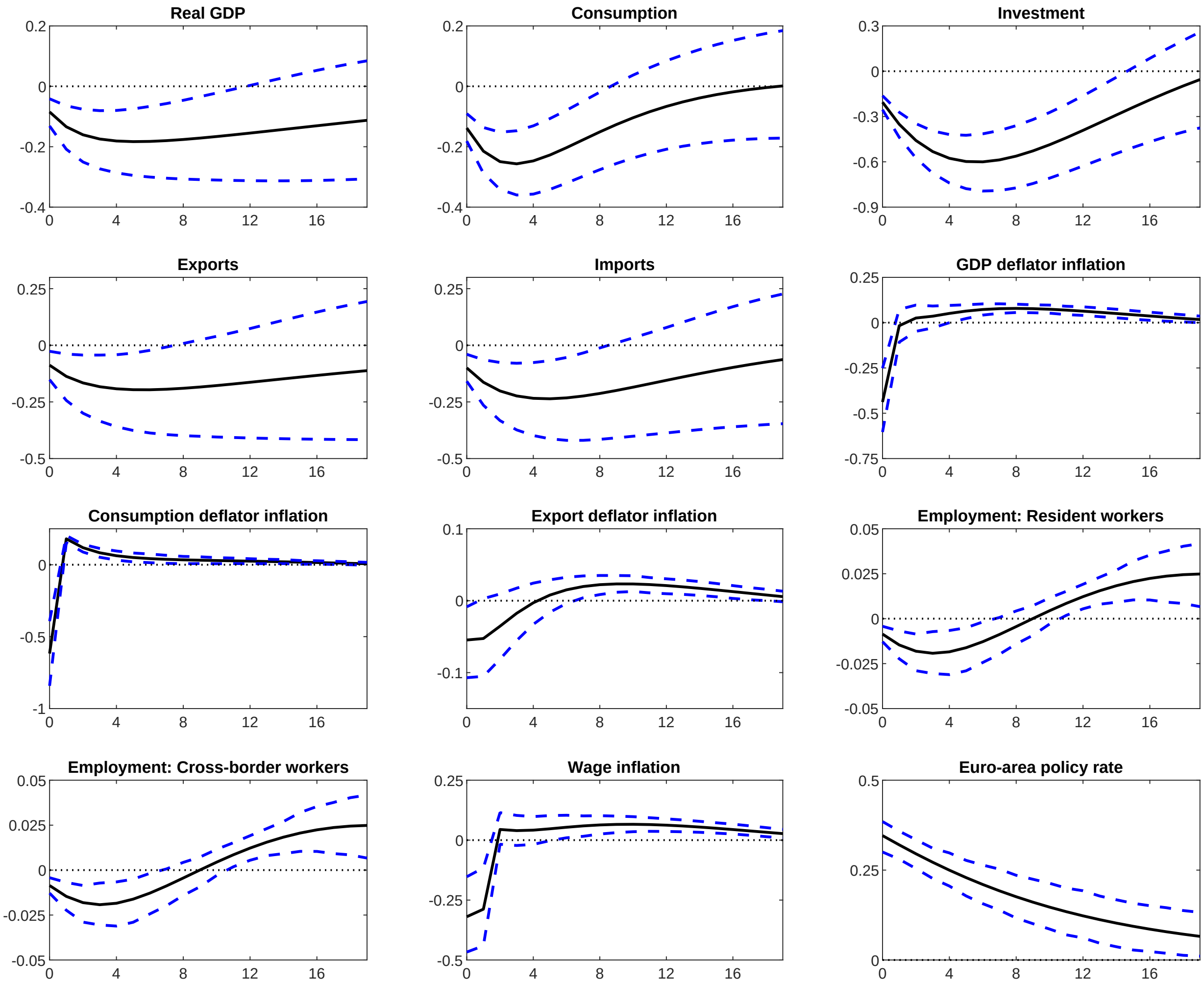
FIGURE 6. Impulse responses to a foreign demand shock.



Notes. This figure shows the mean (solid black line) and the 90 percent uncertainty bands (dashed blue lines) of the impulse responses of selected variables to a foreign demand shock equal to one standard deviation. See the notes to Figure 4.

5.1.3. *Foreign demand shock.* Figure 6 reports the impulse responses to a shock raising foreign demand addressed to Luxembourg firms. The half-life of the shock is equal to 5 quarters. As expected, the shock boosts exports and GDP over several years, but it also spills over to the domestic components of aggregate demand. Consumption, in particular, exhibits a sustained increase reflecting the large wealth effects experienced by resident households, who correctly assess the rise in their lifetime income in a forward-looking fashion. Imports and employment rise as well, since both are required inputs in the domestic production process. Domestic inflation accelerates over the first year, especially when measured by the GDP deflator. This pattern reflects higher real marginal costs, partly originating from the

FIGURE 7. Impulse responses to an interest rate shock in the euro area.



Notes. This figure shows the mean (solid black line) and the 90 percent uncertainty bands (dashed blue lines) of the impulse responses of selected variables to an interest rate shock in the euro area equal to one standard deviation. See the notes to Figure 4.

upward movement in wages. The responses of the consumption and export deflators are more limited, largely because nominal import prices barely move. Given the lack of response of the euro-area interest rate, high inflation reduces the real interest rate in Luxembourg, strengthening domestic demand further.[22] Overall, these responses highlight LED's ability to

[22]In this simulation, monetary policy in the euro area does not react to the shock because the latter only reallocates euro-area demand toward Luxembourg without changing its magnitude. If the increase in foreign demand instead originated from an area-wide demand shock, the interest rate would respond. However, additional model simulations (not reported here) suggest that the responses of domestic variables would be very similar to the ones discussed here.

generate significant movements in domestic variables in reaction to foreign shocks, a welcome property for a small-open-economy model.

5.1.4. *Interest rate shock in the euro area.* Finally, Figure 7 reports the impulse responses to a shock raising the nominal interest rate in the euro area, which is the appropriate conventional monetary policy shock for Luxembourg.[23] The estimated size of the shock is such that the interest rate increases by about 35 basis points on impact, in line with the estimate from Coenen, Christoffel, and Warne (2008). The bottom-right panel show that the half-life of the shock is close to 4 quarters. The responses are very much in line with the literature on the effects of monetary policy shocks. All components of domestic demand fall, with consumption being affected in a more persistent fashion due to the high estimated degree of habit formation. Euro-area GDP is also curtailed (not shown in the figure), resulting in lower foreign demand addressed to Luxembourg and lower exports. Facing this uniform decline in demand, domestic firms cut back their demand for labor and imports. The decline in labor demand depresses wages, putting downward pressure on domestic prices. Overall, the maximum effect on activity is reached after about a year, again mirroring estimates by Coenen, Christoffel, and Warne (2008) for the euro area.

5.2. **Which shocks and frictions are empirically important?** To assess the relative importance of the various structural shocks included in LED, the forecast error variance decompositions of real GDP growth is reported in Table 6 and the one for consumption deflator inflation is reported in Table 7. To simplify the presentation, the shocks are grouped into four categories: domestic technology (supply) shocks, domestic cost-push (markup) shocks, domestic demand shocks, and foreign shocks.[24] Both short (1 to 8-quarter) and medium to long (16 to infinity) horizons are considered.

[23] With euro-area interest rates stuck at their zero lower bound in the aftermath of he last financial crisis, the ECB implemented a variety of *unconventional* measures in recent years, including forward guidance about the path of future interest rates and large-scale asset purchases. These non-standard instruments are not considered in this version of the model. Recent work by Wu and Zhang (2019) argues that the impact of such unconventional measures on the economy is identical to that of a negative "shadow" interest rate, so that the reported IRFs also yield valuable information regarding the potential effects of unconventional measures.

[24] Domestic technology shocks include the transitory and permanent neutral productivity shocks (ϵ and g_z) and the investment-specific technology shock (ϵ^I). Domestic cost-push shocks correspond to the markup shocks for domestic intermediate goods (θ_H), exports (θ_{EX}), and wages (θ_W). Domestic demand shocks include the consumption preference shock (ϵ^{PREF}), the labor demand shock (ϵ^N), the import demand shock (ϵ^{IM}), and the government consumption shock (g). Finally, foreign shocks correspond to the markup shock for imports (θ_{IM}), the shock to foreign demand ($\mu^\star$), the aggregate demand and supply shocks in the euro area ($\epsilon^{ad^{EA}}$ and $\epsilon^{as^{EA}}$), the euro-area monetary policy shock ($\epsilon^{mp^{EA}}$), the rest-of-world output shock (y^{RW}), the exchange rate shock (s^{RW}), and the oil price (p_O).

TABLE 6. Variance decomposition: Real GDP

Horizon (quarters)	Technology (supply) shocks	Cost-push (markup) shocks	Domestic demand shocks	Foreign shocks
1	13	3	20	64
2	14	7	15	64
4	14	15	14	57
8	13	16	19	52
16	12	16	18	53
32	12	17	19	52
∞	12	16	19	52

Notes. Entries are posterior mean estimates for the forecast error variance decomposition of real GDP growth at the horizons of 1, 2, 4, 8, 16, and 32 quarters, as well as for the unconditional variance decomposition. Computations consider only the structural shocks, with the contributions of measurement errors being set to zero. See Footnote 24 for the shock classification. Because of rounding errors, the contributions may not sum exactly to 100%.

The most obvious result from Table 6 is that in Luxembourg real GDP growth is primarily driven by foreign shocks, which account for 64% of output movements in the short run and 52% in the long run. While this conclusion may not be surprising in light of Luxembourg's economic openness, it is remarkable because DSGE models typically have difficulties accounting for international spillovers of realistic magnitude (Justiniano and Preston, 2010). Among these external disturbances, the most important ones drive foreign demand addressed to Luxembourg: these are the shock to demand for Luxembourg exports, the aggregate demand shock in the euro area, and the output shock in the rest of the world, which respectively account for about 19, 15, and 14 percent of output movements in the medium run. Turning to the domestic shocks, demand forces explain about 20% of GDP fluctuations at all horizons, while the contributions of both technology and cost-push shocks are more limited.

As regards consumption deflator inflation, Table 7 suggests that domestic demand shocks and foreign shocks together explain about 80% of the variability in inflation at all horizons. Interestingly, among domestic demand shocks the most important contributor to movements in inflation is the disturbance to import demand, which reinforces the relevance of LED's international dimension. Among foreign shocks, the main drivers of inflation are the aggregate demand shock in the euro area and the import cost shock. On the other hand, the oil price shock has a very limited role and accounts for less than 1% of inflation at all forecast horizons, reflecting the small share of oil in Luxembourg's imports.[25] Finally, domestic cost-push shocks explain bless than 20% of inflation variability. The smaller role of these

[25]The contribution of the oil price shock to aggregate price movements is certainly underestimated since LED ignores feedback effects from oil prices to domestic production costs and to foreign prices.

TABLE 7. Variance decomposition: Consumption deflator inflation

Horizon (quarters)	Technology (supply) shocks	Cost-push (markup) shocks	Domestic demand shocks	Foreign shocks
1	3	13	62	22
2	4	17	49	29
4	4	16	42	37
8	5	18	39	38
16	5	19	39	37
32	5	18	38	38
∞	5	18	38	39

Notes. Entries are posterior mean estimates for the forecast error variance decomposition of consumption deflator inflation at the horizons of 1, 2, 4, 8, 16, and 32 quarters, as well as for the unconditional variance decomposition. See the notes to Table 6.

shocks, which usually explain the bulk of inflation in DSGE models,[26] reflects the relatively high value estimated for the slope of the price Phillips curve in Luxembourg, which ensures a strong propagation of demand shocks to prices.

Variance decompositions also help to identify the frictions that are most useful to reproduce important patterns in the data. In Luxembourg, the most relevant feature of the economy is its marked dependence on foreign shocks, a property correctly identified by LED. To identify the friction(s) responsible for the domestic propagation of external disturbances, Table 8 presents the contributions of domestic and foreign shocks to the variance of real GDP growth at the 1-year horizon in different versions of the model, in which various frictions (limited substitution between domestic and foreign inputs, consumption habits, investment costs, sticky prices, sticky wages) are practically removed one at a time while keeping other parameters constant.[27]

Starting with real frictions, increasing the elasticity of substitution between domestic and imported inputs in the production functions for the four final goods leads to a marked decrease in the ability of foreign shocks to explain GDP fluctuations in Luxembourg: at the 1-year horizon, their contribution drops from to 57% to 45%. It follows that limited input substitution is an important mechanism in LED since it helps propagate external disturbances to domestic variables. The logic is straightforward: in a high-substitution economy, domestic production and imports would comove negatively as final producers would switch

[26]For instance, domestic price and export markup shocks account for more than 50% of the variance of inflation in the euro area according to the estimated NAWM (Coenen, Christoffel, and Warne, 2008).

[27]A better way to assess the role of specific frictions would be to reestimate the model shutting off specific channels one at a time to compare the restricted specifications with the baseline model using Bayes factors. This procedure would allow other parameters to adjust to compensate as much as possible for the excluded feature, at the cost of much higher computing time. Such an analysis is left for future work.

TABLE 8. Real GDP variance decomposition — Empirical role of real and nominal frictions

Model version	Technology (supply) shocks	Cost-push (markup) shocks	Domestic demand shocks	Foreign shocks
Baseline	14	15	14	57
High substitution production	12	22	21	45
Low consumption habits	11	13	28	48
Low investment costs	18	13	13	56
Flexible domestic prices	19	31	12	38
Flexible wages	10	50	15	25

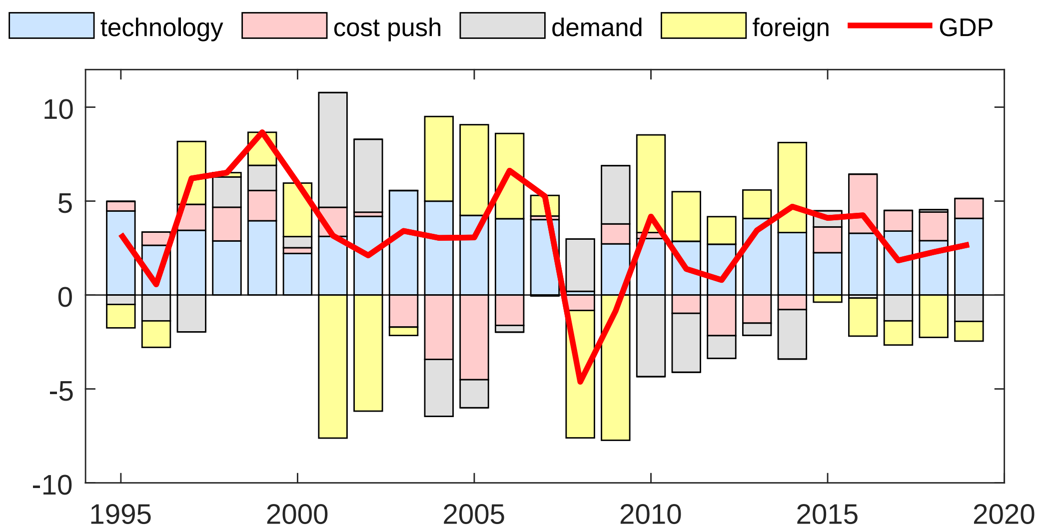
Notes. Entries are estimates for the forecast error variance decomposition of real GDP growth at the horizon of 4 quarters in different versions of the model. The version with high substitution in production sets the CES elasticities at $\mu_C = \mu_I = \mu_G = \mu_X = 5.5$; the version with low consumption habits sets $\kappa = 0.01$; the version with low investment adjustment costs sets $\gamma_I = 0.01$; and the versions with flexible prices or wages set $\xi_H = 0.01$ or $\xi_W = 0.01$. See the notes to Table 6.

to the cheapest input, which would limit the transmission of shocks affecting import prices. Habit formation in consumption is also relatively important, as reducing it decreases the contribution of foreign shocks to GDP movements. Since stronger habits tend to isolate consumption from shocks, the explanation for this result is not obvious. One likely interpretation is that reducing the strength of habits makes consumption, and thus GDP, more reactive to domestic shocks, which mechanically limits the relative influence of external forces. Finally, Table 8 shows that removing investment adjustment costs has no noticeable impact on the variance decomposition of GDP, so that this friction appears to be of minor importance.

Turning to nominal frictions, it is clear that increasing the flexibility of domestic prices and especially wages entails drastic changes in the variance decomposition of real GDP. This is not surprising, as nominal frictions are very important mechanisms in New Keynesian models. With flexible prices, technology shocks and cost-push shocks generate similar dynamics and both contribute more to output fluctuations, at the expense of foreign disturbances. With flexible wages, markup shocks become pure labor supply shocks, which on their own can explain 50% of the volatility of GDP at the 1-year horizon, while the contribution of foreign shocks drops to a low 25%. Clearly, these results highlight the major role played by both price and wage stickiness in shaping the model's properties.

5.3. **Historical decompositions.** Estimated DSGE models can decompose the observed variables used in estimation into separate contributions originating from the various structural shocks. Such decompositions help to understand the sources of macroeconomic fluctuations in an economy and can also identify what shock(s) explains a specific historical event such as a recession. To illustrate these possibilities, this section uses LED to decompose annual real GDP growth in Luxembourg between 1995 and 2019. To facilitate the

FIGURE 8. Sources of real GDP growth in Luxembourg.



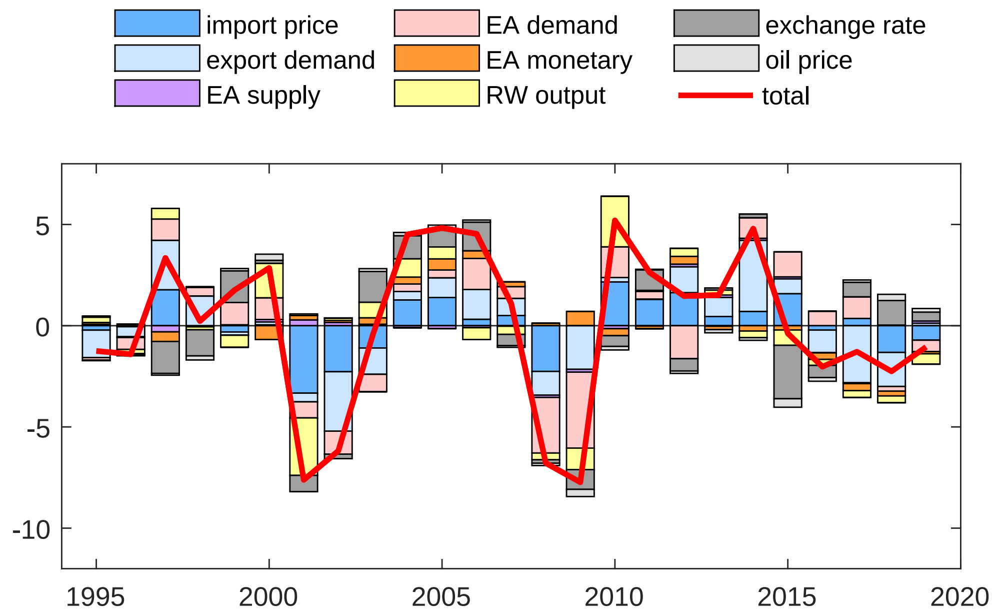
Notes. The figure reports the decomposition of annual real GDP growth over the estimation sample into contributions originating from the four groups of shocks defined in Footnote 24. Computations are performed at the posterior mode and consider only the structural shocks, with the contributions of measurement errors being set to zero.

presentation, the shocks are grouped in the same four categories introduced in Footnote 24: technology shocks, markup shocks, demand shocks, and foreign shocks. Figure 8 reports the main decomposition, whereas Figure 9 focuses on individual foreign disturbances.

Starting with the broad picture, it is clear that, according to the model, the main contributors to sustained growth in Luxembourg over the last 20 years have been technology shocks. This is an expected finding, as neoclassical models like LED prevent non-technology shocks from generating sustained economic growth in the long run. In fact, the regular contribution of technology shocks to GDP growth apparent from Figure 8 directly reflects the calibration strategy setting average technical progress to match the average growth rate of real GDP in the data. It is also important to emphasize that LED abstracts from population growth, so that the steady contribution of technology shocks in the model should be decomposed in two parts: an increase in labor productivity (of about 1.6% each year), which corresponds to 'true' technological progress, and an additional increase in population (of about 1.7% each year), implicitly attributed to technological progress in LED. Together, these trends account for the 3.2% steady-state annual real growth rate of GDP used in Section 4.3.

Turning to cyclical developments, the decomposition suggests only a minor role for technology shocks, since their contribution to GDP growth remained quite stable over time. On the other hand, the group of foreign disturbances seems especially important, as it is associated with the most volatile contributions. In particular, the figure highlights that periods

FIGURE 9. Decomposition of the foreign shock group.



Notes. The figure decomposes the contribution from foreign shocks to annual real GDP growth into contributions originating from individual shocks. See the notes to Figure 8.

of weaker economic growth in Luxembourg tend to coincide with negative foreign shocks reflecting a decline in worldwide activity, as was the case during the early 2000s recession or the 2008-2009 Great Recession. The depth of the latter is evident: 2008 stands out as the only year in the sample in which Luxembourg experienced a significant economic contraction. It is also interesting to note that foreign disturbances have been slowing domestic GDP growth since 2015, consistent with the weak recovery in exports. On the other hand, they largely contributed to economic growth between 2003 and 2006, during the boom that preceded the Great Financial Crisis, as well as in the 2010-2014 period, when import prices evolved favourably and export demand recovered gradually.

The historical decomposition also allows us to focus on specific episodes. For instance, compare the early 2000s recession and the 2008-2009 Great Recession. As shown in Figure 8, LED estimates that foreign disturbances weighed on Luxembourg GDP growth by about 7 percentage points during both episodes. However, actual GDP growth behaved very differently, remaining positive in 2001-2002 while reaching a deep trough of -5% in 2008. LED interprets these different paths as signaling an important role for expansionary domestic shocks in the early 2000s, but not during the Great Recession. This is confirmed by the graphical decomposition: domestic supply and demand shocks contributed more than 10 percentage points to real GDP growth in 2001, but were much less relevant for the Great

Recession. Of course, LED is not able to identify precisely the factors that supported domestic activity in 2001-2002, but the model-based decomposition provides a good starting point for further analyses. For instance, one may argue that the strong development of the Luxembourg financial sector following the launch of the euro area explained the positive technology contribution in the early 2000s, while a succession of fiscal reforms in 2001 and 2002 stimulated demand by lowering effective tax rates on households and businesses.

The last groups of shocks, related to domestic demand and markups, also played an important role in cyclical developments over the estimation sample. One interesting finding is that domestic forces tend to be negatively correlated with foreign disturbances. In particular, it is striking that since 2001 the contributions from domestic demand and cost-push shocks on the one hand, and the contributions from foreign shocks on the other hand, have consistently worked in opposite directions.

Finally, decomposing the contribution of the foreign shock group, as in Figure 9, shows that the most important external disturbances originate from euro-area aggregate demand and demand for Luxembourg exports. These two shocks drive the dynamics of exports in the model and the decomposition highlights their key contribution during the 2008-2009 slowdown. Other important disturbances include shocks to the price of imports, which is not surprising given the large import content of production in Luxembourg, and shocks to the exchange rate, which also affect foreign demand addressed to Luxembourg firms. Finally, the euro-area supply and monetary policy shocks and the oil price shock only made minor contributions throughout the historical sample.

6. Conclusion

This paper presented LED, an estimated DSGE model of Luxembourg as a small open economy within a monetary union. The model structure is sufficiently rich to constitute an attractive framework for quantitative analyses and policy simulations. The estimates are reasonable and informative about important dimensions of the Luxembourg economy, for example regarding the imperfect substitution between domestic and imported goods or the magnitude of nominal frictions. They will also help refine the calibration of the LU-EAGLE model already in use at the BCL, which shares a number of parameters with LED. The estimated model has a reasonable fit and tells coherent stories about the determinants of economic growth and cyclical fluctuations in Luxembourg.

Further work needs to be done to refine the model. For instance, the current version of LED maintains several simplifying assumptions that may be worth relaxing in the future. Interesting extensions could include:

(1) A richer specification of fiscal policy, with shocks to taxes and transfers, a distinction between public consumption and public investment, public capital in the production

function, and public employment (Coenen, Straub, and Trabandt, 2012, 2013, developed such extensions of the NAWM to study fiscal policy in the euro area). These additional mechanisms would enhance the model's ability to evaluate the effects of fiscal policy in Luxembourg.

(2) Taking into account population growth. Over recent decades, Luxembourg witnessed a steady increase in the number of residents, especially via positive immigration flows. This dimension, currently omitted in LED, could be worth exploring.

(3) The incorporation of a financial sector, not so much as a source of shocks and frictions as in most of the DSGE literature, but rather as a productive sector and a source of economic growth (Marchiori and Pierrard, 2015, provide an interesting first step in that direction). As the importance of the financial sector in Luxembourg keeps growing over time, an explicit description of its functioning should improve the quantitative performance of the model.

(4) The inclusion of the housing market, with a relative price of houses and a distinction between home owners and tenants. Such an extension would be interesting to study the macroeconomic effects of the observed upward trend in house prices in Luxembourg.

(5) Further economic applications, for instance related to forecasting or to the estimation of potential output and the output gap in Luxembourg.

References

Adolfson, M., S. Laseen, L. Christiano, M. Trabandt, and K. Walentin (2013): "RAMSES II - Model Description," Occasional Paper Series 12, Sveriges Riksbank (Central Bank of Sweden).

Adolfson, M., S. Laseen, J. Linde, and M. Villani (2007): "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," *Journal of International Economics*, 72(2), 481–511.

Bayoumi, T., H. Faruqee, D. Laxton, P. D. Karam, A. Rebucci, J. Lee, B. L. Hunt, and I. Tchakarov (2004): "GEM: A New International Macroeconomic Model," IMF Occasional Papers 239, International Monetary Fund.

Burgess, S., E. Fernandez-Corugedo, C. Groth, R. Harrison, F. Monti, K. Theodoridis, and M. Waldron (2013): "The Bank of England's Forecasting Platform: COMPASS, MAPS, EASE and the Suite of Models," Bank of England working papers 471, Bank of England.

Burriel, P., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2010): "MEDEA: a DSGE model for the Spanish economy," *SERIEs: Journal of the Spanish Economic Association*, 1(1), 175–243.

Calvo, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Christiano, L. J., M. Trabandt, and K. Walentin (2011): "Introducing Financial Frictions and Unemployment into a Small Open Economy Model," *Journal of Economic Dynamics and Control*, 35(12), 1999–2041.

Clarida, R., J. Gali, and M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37(4), 1661–1707.

Coenen, G., K. Christoffel, and A. Warne (2008): "The New Area-Wide Model of the Euro Area: A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis," Working Paper Series 944, European Central Bank.

Coenen, G., R. Straub, and M. Trabandt (2012): "Fiscal Policy and the Great Recession in the Euro Area," *American Economic Review*, 102(3), 71–76.

——— (2013): "Gauging the Effects of Fiscal Stimulus Packages in the Euro Area," *Journal of Economic Dynamics and Control*, 37(2), 367–386.

Erceg, C. J., L. Guerrieri, and C. Gust (2006): "SIGMA: A New Open Economy Model for Policy Analysis," *International Journal of Central Banking*, 2(1).

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46(2), 281–313.

Fernandez-Villaverde, J., J. F. Rubio-Ramirez, and F. Schorfheide (2016): *Solution and Estimation Methods for DSGE Models*vol. 2 of *Handbook of Macroeconomics*, chap. 9, pp. 527–724. Elsevier.

Garcia Sanchez, P., and A. Moura (2019): "The LU-EAGLE Model with Disaggregated Public Expenditure," BCL working papers 135, Central Bank of Luxembourg.

Justiniano, A., and B. Preston (2010): "Can Structural Small Open-Economy Models Account for the Influence of Foreign Disturbances?," *Journal of International Economics*, 81(1), 61–74.

Justiniano, A., G. Primiceri, and A. Tambalotti (2011): "Investment Shocks and the Relative Price of Investment," *Review of Economic Dynamics*, 14(1), 101–121.

Kilponen, J., S. Orjasniemi, A. Ripatti, and F. Verona (2016): "The Aino 2.0 Model," Research Discussion Papers 16/2016, Bank of Finland.

Krebs, B. (2019): "Revisions to Quanterly National Accounts Data in Luxembourg," BCL working papers 136, Central Bank of Luxembourg.

Lunnemann, P., and T. Matha (2005): "Consumer Price Behaviour in Luxembourg: Evidence from Micro CPI Data," BCL working papers 17, Central Bank of Luxembourg.

Lunnemann, P., and L. Wintr (2009): "Wages are Flexible, aren't they? Evidence from Monthly Micro Wage Data," BCL working papers 39, Central Bank of Luxembourg.

Marchiori, L., and O. Pierrard (2012): "LOLA 2.0: Luxembourg OverLapping generation model for policy Analysis," BCL working papers 76, Central Bank of Luxembourg.

——— (2015): "LOLA 3.0: Luxembourg OverLapping generation model for policy Analysis," BCL working papers 100, Central Bank of Luxembourg.

Moura, A. (2018): "Investment Shocks, Sticky Prices, and the Endogenous Relative Price of Investment," *Review of Economic Dynamics*, 27, 48–63.

Moura, A., and K. Lambrias (2018): "LU-EAGLE: A DSGE Model for Luxembourg within the Euro Area and Global Economy," BCL working papers 122, Central Bank of Luxembourg.

Pagan, A. (2017): "Some Consequences of Using Measurement Error Shocks when Estimating Time Series Models," CAMA Working Papers 2017-12, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University.

Pierrard, O., and H. R. Sneessens (2009): "LOLA 1.0: Luxembourg Overlapping Generation Model for Policy Analysis," BCL working papers 36, Central Bank of Luxembourg.

Ratto, M., W. Roeger, and J. in't Veld (2008): "QUEST III: An Estimated DSGE Model of the Euro Area with Fiscal and Monetary Policy," European Economy - Economic Papers 2008 - 2015 335, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Sangaré, I. (2019): "Housing Sector and Optimal Macroprudential Policy in an Estimated DSGE Model for Luxembourg," BCL working papers 129, Central Bank of Luxembourg.

Schmitt-Grohe, S., and M. Uribe (2003): "Closing Small Open Economy Models," *Journal of International Economics*, 61(1), 163–185.

Schmitz, M., M. D. Clercq, M. Fidora, B. Lauro, and C. Pinheiro (2012): "Revisiting the Effective Exchange Rates of the Euro," Occasional Paper Series 134, European Central Bank.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1(5), 1123–1175.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Wu, J. C., and J. Zhang (2019): "A Shadow Rate New Keynesian Model," *Journal of Economic Dynamics and Control*, 107(C).

Appendix A. Comparing Estimation Results across Data Vintages

In common with all DSGE models, LED is estimated from a set of macroeconomic variables extracted from national accounts, which are subject to data revisions. In particular, the most recent observations used to estimate the model are not final, in the sense that they may not incorporate all relevant information and could be adjusted in the future. It follows that observables are subject to statistical uncertainty, which raises the question of the robustness of the results. This issue seems especially relevant for Luxembourg, where quarterly national accounts are surrounded by even more uncertainty than in other countries (Krebs, 2019).

To deal with this difficulty, LED incorporates measurement errors for all variables extracted from Luxembourg national accounts (see p. 27). This appendix evaluates the robustness of the results by comparing the benchmark estimation reported in the text with an alternative one, based on an earlier vintage of the data, running from 1995Q1 to 2018Q3 and released early in 2019. Compared to this earlier vintage, the dataset exploited in the text features two differences: (i) it incorporates five new data points corresponding to 2018Q4, 2019Q1, 2019Q2, 2019Q3, and 2019Q4, and (ii) observations prior to 2018Q3 were revised by the STATEC, as more information became available.

Table 9 reports the estimation results for the older data vintage. The presentation is the same as in the main text and, to save on space, the discussion focuses on structural parameters from the Luxembourg block of the model. Compared to results reported in Table 3, we see that most parameter take values close to their benchmark estimates. For instance, the estimated degree of consumption habits is the same in both datasets, while the estimated costs of adjusting investment and utilization are very similar. This is also the case of the estimated elasticities of substitution and adjustment costs in the final production sector. In particular, it is striking that the ranking of the estimated substitution parameters is the same across datasets, with the consumption sector featuring the highest substitution between imported and domestic goods, and the investment and government sector featuring the lowest. Likewise, parameters related to the labor market and to price and wage frictions take similar values in the two datasets.

Overall, estimated structural parameters appear fairly robust across data vintages. This finding suggests that the measurement error strategy does a good job at disentangling relevant information from statistical noise in the data. This is not surprising given that the major source of information for estimated DSGE models lies in the comovements between different observables, or between an observable and its lagged values: these moments should not be affected by uncorrelated measurement error, so that identification remains valid.

As a second check, Figure 10 shows the historical decomposition of real GDP growth in Luxembourg using the earlier data vintage. Given that estimated parameters take similar values in both datasets, there is no reason to expect large differences in the decompositions. Indeed, a straightforward comparison with Figure 8 reveals that the model yields equivalent

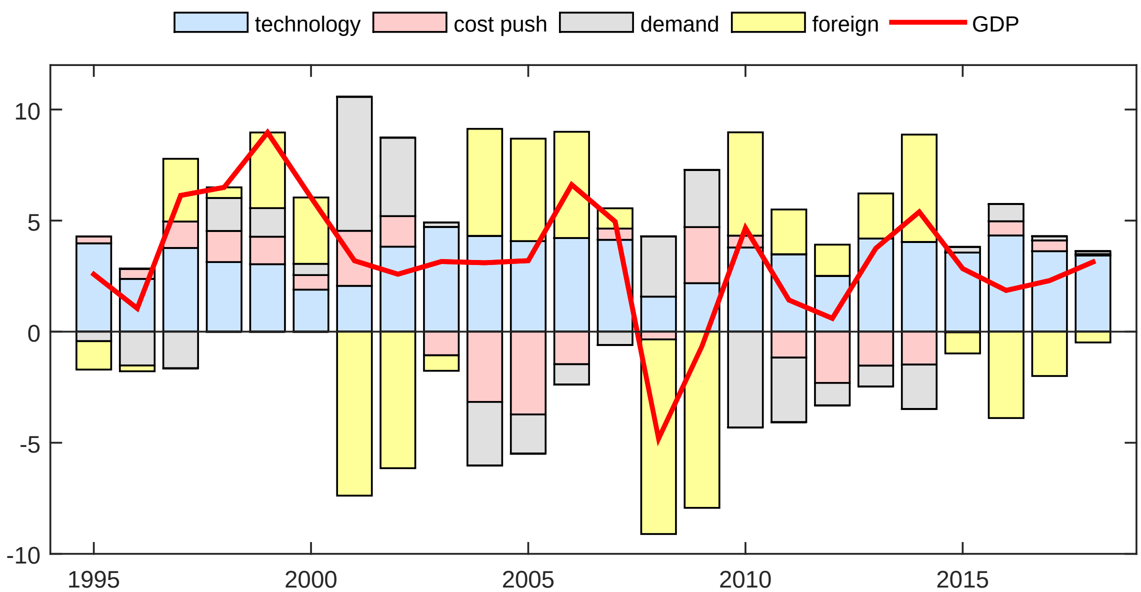
TABLE 9. Estimation results — Robustness Analysis

Parameter	Description	Prior distribution			Posterior distribution	
		Distribution	Mean	SD	Mode	[5%, 95%]
Standard real frictions						
κ	Consumption habits	Beta	0.65	0.05	0.83	[0.80, 0.86]
γ_I	Adj. cost: investment	Gamma	4.00	2.00	1.27	[0.46, 2.63]
γ_{u2}	Utilization cost	Gamma	0.20	0.10	0.54	[0.37, 0.84]
Production functions: Elasticities of substitution						
$\mu_C/(1+\mu_C)$	C sector	Beta	0.35	0.10	0.83	[0.75, 0.88]
$\mu_I/(1+\mu_I)$	I sector	Beta	0.35	0.10	0.35	[0.19, 0.53]
$\mu_G/(1+\mu_G)$	G sector	Beta	0.35	0.10	0.35	[0.22, 0.57]
$\mu_X/(1+\mu_X)$	EX sector	Beta	0.35	0.10	0.49	[0.41, 0.57]
Production functions: Adjustment costs						
γ_{HC}	Domestic content: C sector	Gamma	1.00	0.50	0.13	[0.11, 0.17]
γ_{HI}	Domestic content: I sector	Gamma	1.00	0.50	0.73	[0.29, 1.70]
γ_{HG}	Domestic content: G sector	Gamma	1.00	0.50	0.76	[0.27, 1.85]
γ_{HX}	Domestic content: EX sector	Gamma	1.00	0.50	1.87	[1.28, 2.67]
γ_{IMC}	Import content: C sector	Gamma	1.00	0.50	1.43	[1.02, 2.31]
γ_{IMI}	Import content: I sector	Gamma	1.00	0.50	1.31	[0.68, 2.13]
γ_{IMG}	Import content: G sector	Gamma	1.00	0.50	0.92	[0.32, 1.97]
γ_{IMX}	Import content: EX sector	Gamma	1.00	0.50	0.25	[0.14, 0.53]
Labor market						
$\mu_N/(1+\mu_N)$	Subst. elasticity: labor	Beta	0.35	0.10	0.85	[0.82, 0.88]
γ_R	Adj. cost: resident labor	Gamma	1.00	0.50	1.61	[1.09, 2.31]
γ_F	Adj. cost: foreign labor	Gamma	1.00	0.50	0.12	[0.08, 0.17]
ξ_E	Calvo: employment	Beta	0.75	0.10	0.52	[0.47, 0.60]
Nominal frictions						
ξ_H	Calvo: domestic prices	Beta	0.75	0.10	0.65	[0.56, 0.78]
χ_H	Indexation: domestic prices	Beta	0.50	0.20	0.71	[0.48, 0.91]
ξ_{IM}	Calvo: import prices	Beta	0.75	0.10	0.80	[0.73, 0.84]
χ_{IM}	Indexation: import prices	Beta	0.50	0.20	0.13	[0.05, 0.32]
ξ_{EX}	Calvo: export prices	Beta	0.25	0.10	0.40	[0.33, 0.54]
χ_{EX}	Indexation: export prices	Beta	0.50	0.20	0.22	[0.07, 0.51]
ξ_W	Calvo: wages	Beta	0.75	0.10	0.76	[0.71, 0.81]
χ_W	Indexation: wages	Beta	0.50	0.20	0.12	[0.03, 0.30]
Foreign demand						
$\mu^\star/(1+\mu^\star)$	Price elasticity: foreign demand	Beta	0.35	0.10	0.43	[0.35, 0.53]
γ_{EX}	Adj. cost: foreign demand	Gamma	1.00	0.50	2.32	[1.75, 3.36]

Notes. The estimation sample is 1995Q1-2018Q3. See the notes to Table 1.

interpretations of aggregate dynamics in Luxembourg across the two data vintages. Therefore, here too the results from LED appear robust to revisions in quarterly national accounts and the release of new observations.

Figure 10. Sources of real GDP growth in Luxembourg — Robustness Analysis.



Notes. The estimation sample is 1995Q1-2018Q3. See the notes to Figure 8.

BANQUE CENTRALE DU LUXEMBOURG
EUROSYSTÈME

2, boulevard Royal
L-2983 Luxembourg

Tél.: +352 4774-1
Fax: +352 4774 4910

www.bcl.lu • info@bcl.lu