2. OPTIMAL LEVELS OF BORROWER-BASED MEASURES IN THE PRESENCE OF MORTGAGE DEFAULT

Ibrahima Sangaré[107]

ABSTRACT

This study investigates the optimal calibration for borrower-based measures in Luxembourg in the framework of a DSGE model with mortgage default and two borrowing constraints (LTV and DSTI). Using a welfare-based approach, we find that the optimal values for the LTV and DSTI ratios in the context of the COVID-19 pandemic are 85 % and 32 %, respectively. We also find that the optimal macroprudential policy welfare-dominates the non-optimal policy. Moreover, the optimal policy stabilizes mortgage lending and output more effectively than the policy based on the current average data. Finally, our findings suggest that an LTV limit calibrated above its optimal level increases mortgage default risk while a relatively high DSTI limit has no noticeable effects on the mortgage default risk under COVID-19-related shocks.

1. INTRODUCTION

In recent years, the Luxembourg residential real estate market has been strongly dynamic, characterized by a rapid growth in both house prices and mortgage loans leading to high and increasing household indebtedness. The residential property prices were continuing to rise at the beginning of 2021. The real and nominal residential property prices in Luxembourg respectively rose by 16.08 % and 16.71 % in annual terms in the last quarter of 2020. This ongoing increase in RRE prices is driven by both excess of demand for housing and supply limitations. The persistent low interest rate environment, in combination with high dwelling prices, has fuelled the increase in household indebtedness levels.

Households' indebtedness in Luxembourg is at a high level, even compared to other European countries, and continues to increase. The country features ratios of household debt-to-disposable income and mortgage debt-to-disposable income at above 100 % and continue to have a strong growth in mortgage loans that has often been driven by loosening lending standards. In particular, mortgage debt-to-disposable income amounted to 132 % in 2020Q4 while household debt-to-disposable income reached 167 % in the same quarter. The latter largely exceeds the average European countries household debt-to-income ratio of 104.46 % in 2020Q4.

These developments, forming the main vulnerabilities in the residential real estate market in Luxembourg, taken in combination with adverse economic or financial conditions could pose risks to financial stability risks both from the perspective of households' debt sustainability as well as housing affordability. In the absence of demand-side policy actions accompanying the supply-side policies, these vulnerabilities could have adverse effects for the real economy.

Therefore, in June 2019, the European Systemic Risk Board (ESRB) issued a recommendation for remedial actions on medium-term residential real estate vulnerabilities to Luxembourg, among five other EU countries[108]. More specifically, the ESRB has recommended to Luxembourg to establish a legal framework for borrower-based measures (such as LTV, DSTI, DTI and maturity limits) and to activate them as well as to curb the structural factors that have driven the vulnerabilities identified in Luxembourg.

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[107] Financial Stability and Macroprudential Surveillance Department, Banque centrale du Luxembourg.

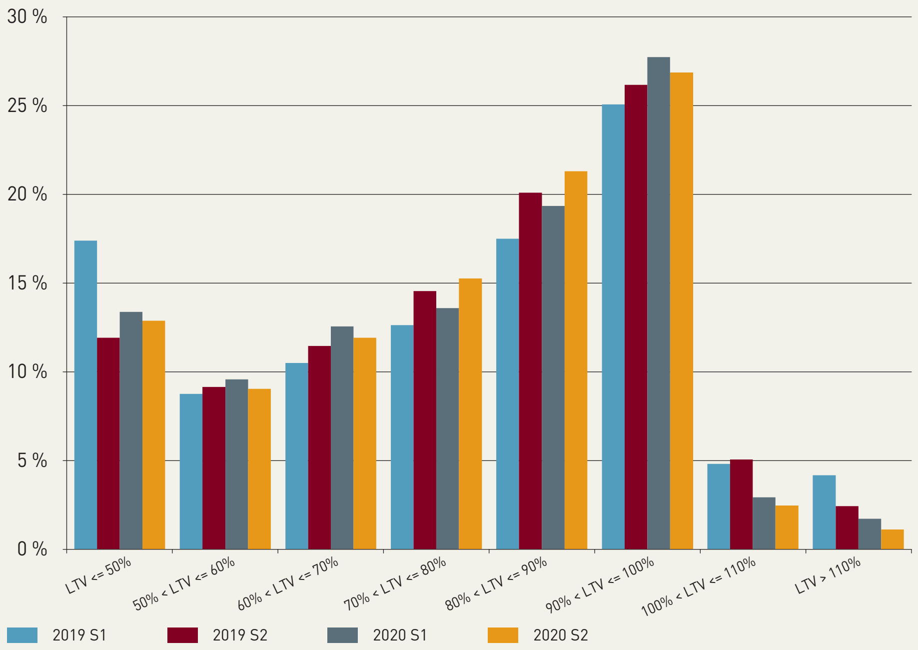
[108] Recommendation/ESRB/2019/6.

Consequently, the legal framework for borrower-based measures in Luxembourg has been adopted by the Luxembourg parliament in November 2019, rendering these demand-side instruments legally available in the national macroprudential policy toolkit. It has followed a recommendation issued by the *Comité du Risque Systémique* (the Luxembourg macroprudential authority) in November 2020 toward the *Commission de Surveillance du Secteur Financier* (CSSF) for activating the LTV limits in Luxembourg[109]. Accordingly, among borrower-based measures, only legally-binding LTV limits have been activated with differentiated limits according to borrower categories and entered into force on 1 January 2021. Especially, a maximum LTV limit is set to 100 % for first-time buyers acquiring their primary residence. For borrowers other than first-time buyers acquiring a primary residence, the LTV limit is 90 %. To enable flexibility, lenders may issue 15 % of the portfolio of new mortgages granted to these borrowers with an LTV above 90 % but below the maximum of 100 %. For all other borrowers, including for the buy-to-let segment, the LTV cap is set to 80 %.

However, before the legal activation of the LTV limits, Luxembourg banks applied various LTV and DSTI limits depending on their own assessment of household creditworthiness as illustrated by the distribution of new loans granted by LTV and DSTI in Figure 1 and Figure 2. Figure 1 and Figure 2 show that the amount of new loans with a

Figure 1

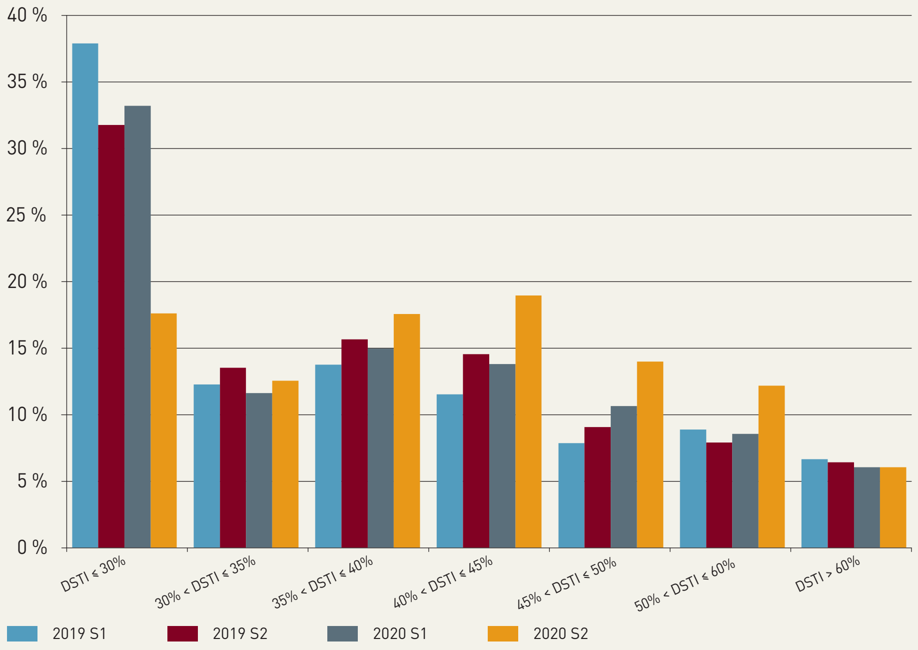
Distribution of new loans granted by LTV



Source: CSSF

Figure 2

Distribution of new loans granted by DSTI



Source: CSSF

[109] For more details, see Recommendation/CRS/2020/005 and CSSF Regulation N° 20-08.

LTV higher than 80 % has decreased from 53.3 % in the second semester of 2019 to 51.3 % in 2020S2. However, in the same period, the amount of new loans with the debt service cost (DSTI) higher than 40 % has increased from 38.4 % to 51.7 %.

As only the LTV cap is legally binding, lenders would continue to extend new loans with varieties of DSTI caps depending on their own assessment of borrowers.

Therefore, there is a need to assess the effectiveness of the combined legally-binding LTV and DSTI in addressing vulnerabilities in the residential real estate market, especially in the context of the pandemic crisis.

This study addresses the question of what is the optimal calibration of borrower-based measures in the context of the coronavirus pandemic. To this end, we use a DSGE model to determine the optimal levels of LTV and DSTI, in the presence of loan default. The macroeconomic effects of such a combination of macroprudential measures is also assessed.

There are two specific objectives of this work. On the one hand, we search for optimal levels of borrower-based measures taken in combination in order to help supporting potential policy actions and to assist in their optimal calibration if it becomes necessary to activate them in combination. Current empirical and theoretical evidence suggests that combinations of macroprudential instruments are more effective in targeting potential risks than the implementation of a single instrument[110]. In addition to providing guidance on the possible calibration of borrower-based measures, this work also provides some insights into the relationship between borrower-based measures and mortgage risk from households.

More specifically, we build a DSGE model with mortgage default and two macroprudential borrower-based instruments namely LTV and DSTI limits. The model is designed to assess the optimal limits of these instruments based on a welfare analysis and is calibrated using Luxembourg data.

Our main findings can be summarized as follows. Using a welfare metric, we first find that, in a context the COVID-19 pandemic characterized by simultaneous adverse demand and supply shocks, the optimal values of LTV and DSTI ratios are 85 % and 32 %, respectively. Second, we find that the optimal macroprudential policy welfare-dominates the non-optimal policy. Moreover, the optimal policy better stabilizes mortgage loans and output than the policy based on the average values of the observed policy instruments. Finally, our findings suggest that a higher levels of both LTV and DSTI limits implies a higher mortgage default risk compared to the optimal calibration of these instruments. This reflects the fact that the main driver of household default risk in the presence of a COVID-19 related shock is the LTV limit.

The rest of the analysis is organised as follows. Section 2 reviews the related literature. Section 3 describes the model and Section 4 presents the model calibration. Section 5 presents the optimal macroprudential policy stance and provides the optimal values of LTV and DSTI limits for Luxembourg as well as the dynamics of the main macro-financial variables in the COVID-19 context. Section 7 concludes.

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[110] See Crowe *et al.* (2013), Cassidy and Hallissey (2016) and Grodecka (2017) for more details.

2. LITERATURE REVIEW

This work is related to four strands of literature. First, the existing studies using the dynamic stochastic general equilibrium (DSGE) modelling approach for analysing the real estate dynamics in Luxembourg are rather limited. Sangaré (2019) studies the optimal macroprudential policy for Luxembourg using a DSGE model with a housing sector and a borrowing constraint. Therefore, the novelty of the current work compared to the previous one is to analyse the optimal macroprdential policy for a combination of borrower-based measures within a DSGE framework that incorporates mortgage default and two borrowing constraints.

This work is also related to numerous papers that model the housing sector with a borrowing constraint in a dynamic stochastic general equilibrium framework (e.g. Iacoviello (2005), Iacoviello and Neri (2010), Gerali *et al.* (2010), Mendicino and Punzi (2014), Rubio and Carrasco-Gallego (2014), Brzoza-Brzezina *et al.* (2017), Guerrieri and Iacoviello (2017)). However, few studies among the mentioned papers explicitly model the banking sector and they do not include mortgage default or study the effectiveness of macroprudential policy. We address this gap by considering a DSGE framework in which banks are explicitly modelled in a monopolistic competitive market and we also include a mortgage default mechanism.

This study is also related to the growing body of literature on the effectiveness of macroprudential policies. Several papers have explored the effectiveness of macroprudential policies using stochastic general equilibrium models, including Lubello and Rouabah (2017) and Fève and Pierrard (2017). However, their models do not account for the housing sector and only consider individual macroprudential instruments without mortgage default modelling.

Few studies with a housing sector have been interested in exploring the optimality of macroprudential policies (Rubio and Carrasco-Gallego (2014), Mendicino and Punzi (2014), Punzi and Rabitsch (2018)). Although they assess optimality, these studies do not focus on the interaction between macroprudential instruments and they do not include either mortgage default or several borrowing constraints. Moreover, most of these papers analyze optimal interactions between the monetary policy and macroprudential policy rather than assessing the optimal combinations of macroprudential instruments.

Some studies (Lambertini *et al.* (2017), Pataracchia *et al.* (2013), Forlati and Lambertini (2011), Clerc *et al.* (2015), Mendicino *et al.* (2018)) do explicitly model mortgage default but they do not include an optimality framework or the combination of macroprudential instruments. Other works, such as those of Rubio and Carrasco-Gallego (2014), Mendicino and Punzi (2014), Punzi and Rabitsch (2018), Mendicino *et al.* (2018) investigate the optimality of macroprudential instruments but the instruments are taken in isolation and not in combination. These authors do not explore the impact of mortgage default.

Finally, our study fits into the literature on combinations of macroprudential instruments. This strand of literature mainly addresses the combination of borrower-based instruments using empirical techniques adopted by Kelly *et al.* (2018) and Albacete *et al.* (2018). Some exceptions include Chen and Columba (2016), Grodecka (2017) and Greenwald (2018) who analysed the combination of borrower-based instruments using a DSGE modelling approach but without default. Benes *et al.* (2016) use a DSGE model for studying the combination of a capital buffer and a borrower-based measure (LTV ratio) but without any optimality analysis.

The current study considers the optimal combination of borrower-based macroprudential instruments (LTV and DSTI) in a DSGE model with mortgage default. To the best of our knowledge, the only work existing in the literature on macroprudential policies that fits our methodology is the one from Aguilar *et al.* (2019). However, the latter paper focuses on combination of capital-based macroprudential measures rather than borrower-based measures.

3. MODEL[111]

We develop a DSGE model with a housing sector, two borrowing constraints (LTV and DSTI ratios) and a mechanism for mortgage defaults. The only source of mortgage default in the model is an idiosyncratic shock that affects the house value. We assume that income-related risks (i.e., household unemployment) do not trigger mortgage default[112].

Two groups of households populate the economy: patient households and impatient households and each group has unit mass. Patient households are savers and have higher discount factors than impatient households who are borrowers ($\beta_P > \beta_I$).

This heterogeneity in agents' discount factors generates positive fund flows in equilibrium: patient households make positive deposits and do not borrow, while impatient households borrow a positive amount of loans. Patient households consume, work and accumulate capital and housing. Impatient households consume, work and accumulate housing. As impatient households are considered to be borrowers, they are constrained by having to collateralize the value of their house which introduce some financial frictions in the economy, to allocate a constant fraction of their income to debt services and by the occurrence of default.

We introduce a monopolistically competitive banking sector *à la* Gerali *et al.* (2010). Banks intermediate the funds that flow from patient households to impatient households. Banks issue loans to impatient households and firms by collecting deposits from patient households and accumulating their own capital out of reinvested profits. Banks face the risk of defaults from their borrowers. Another financial friction is introduced in the model by assuming that banks are subject to a risk weighted capital requirement constraint that translates into an exogenous target for the leverage ratio, the deviation from which implies a quadratic cost.

On the production side, monopolistically competitive intermediate-goods-producing firms produce heterogeneous intermediate goods using physical capital, bought from capital goods producers, and labour supplied by households against flexible wages. The prices of intermediate goods are set in a staggered fashion *à la* Rotemberg (1984). Final goods- producing firms, who bundle intermediate goods into final goods, capital and housing producers operate in perfectly competitive markets.

Finally, a government covers its expenditures by levying lump-sum taxes on households and by collecting the share of defaulting households' wealth that is seized and paid to the government's insolvency agency. The monetary authority follows a standard Taylor-type interest rate rule.

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[111] We only present here a brief summary of the model. The more detailed presentation of the model is in a technical appendix available upon request.

[112] This assumption is made in order to simplify the model.

3.1. HOUSEHOLDS

The economy is composed of two types of agents: patient and impatient households. The only difference between these agents is that the discount factor for impatient households (β_I) is less than the discount factor for patient households (β_P). Both types of households derive utility from consumption, $c_{z,t}$, housing services, $h_{z,t}$ and the number of hours worked, $n_{z,t}$. Households have identical expected discounted utility functions that corresponds, in real terms, to:

$$E_0 \sum_{t=0}^{\infty} \beta_z^t U(c_{z,t}; h_{z,t}; n_{z,t}) = E_0 \sum_{t=0}^{\infty} \beta_z^t \left[A_{c,t}(1-a)\ln\left(c_{z,t} - a.C_{z,t-1}\right) + A_{h,t}\chi_h \ln\left(h_{z,t}\right) - \frac{\chi_n n_{z,t}^{1+\gamma}}{1+\gamma} \right] \tag{1}$$

where z = $\{I,P\}$ with I and P respectively standing for impatient (borrowers) and patient (savers) households. The current individual consumption depends on the lagged smoothed aggregated consumption, $a.C_{z,t-1}$, where the parameter a, denotes the degree of habit formation in consumption for non-durable goods. The parameter χ_h is the weight on housing services, χ_n denotes the weight on hours worked and γ is the elasticity of labour substitution. $A_{c,t}$ and $A_{h,t}$ are two preference shocks to consumption and housing demands, respectively, and both follow an AR(1) process.

A) Patient households

The representative patient household maximises their expected utility (1) and is subject to the following real budget constraint[113]:

$$\begin{aligned} c_{P,t} + q_{h,t}\left[h_{P,t} - h_{P,t-1}\right] + d_t + q_{k,t}[k_t - (1-\delta_k)k_{t-1}] \\ = w_{P,t}n_{P,t} + \frac{R_{t-1}}{\Pi_t}d_{t-1} - T_{P,t} + \Lambda_t + Div_t + \frac{r_{k,t-1}k_{t-1}}{\Pi_t} \end{aligned} \tag{2}$$

where $q_{h,t}$ and $q_{k,t}$ are the respective prices for housing stock, $h_{P,t}$, and physical capital, k_t which depreciates at the rate, δ_k. Patient households receive the wage rate, $w_{P,t}$, for supplying hours of work and earn R_{t-1} on the last period risk-free deposit, d_{t-1} and $r_{k,t-1}$, the rental rate on the physical capital that they own, which depends on gross inflation, $\Pi_t = \frac{P_t}{P_{t-1}}$. Patient households receive a profit Λ_t from both intermediate consumption and capital goods producers and a dividend Div_t from monopolistically competitive banks. Finally, they pay a lump-sum tax, $T_{P,t}$, to the government.

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[113] The first order conditions derived from the maximization problem of patient households are in a technical appendix available upon request.

B) Impatient households

The representative impatient household faces two borrowing constraints.

(i) LTV constraint

In each period, t, households' borrowing is subject to the regulatory LTV constraint defined in real terms as:

$$R^L_{I,t} l_{I,t} \leq LTV q_{h,t} h_{I,t} \tag{3}$$

where LTV denotes the loan-to-value ratio fixed by the macroprudential authority and $R^L_{I,t}$ is the mortgage lending rate.

(ii) DSTI constraint

In addition, the borrowing in period, t, is limited by a regulatory DSTI constraint expressed in real terms as:

$$R^L_{I,t} l_{I,t} \leq DSTI w_{I,t} n_{I,t} \tag{4}$$

For simplification purposes, we assume that only the value of housing is subject to an idiosyncratic shock triggering mortgage default. In other words, the risk of mortgage default is only related to the value of house and not to the borrowers' income. This assumption implies that there is only one source of mortgage default in the model (i.e., house value).

We assume that in $t + 1$, each impatient household faces an idiosyncratic shock to its house value ω_{t+1}, which follows a uniform distribution with the lower and upper bounds, $(\underline{\omega}, \overline{\omega})$[114]. The shock ω_t is i.i.d. and it has positive support with cumulative distribution, $F(x) \equiv prob(\omega_t \leq x)$, with mean $\mu_{\omega,t}$, variance σ^2_ω and density function $f(\omega_t)$.

The borrower is solvent if and only if $\omega_{t+1} \geq \widetilde{\omega}_{t+1}$ where $\widetilde{\omega}_{t+1}$ is the threshold or cutoff point such that, in real terms:

$$R^L_{I,t} l_{I,t} = \widetilde{\omega}_{t+1} E_t q_{h,t+1} h_{I,t} \Pi_{t+1} \tag{5}$$

Default occurs when the expected real value of the impatient household's house at $t + 1$ falls below the amount that needs to be repaid, that is when $E_t(\omega_{t+1} q_{h,t+1} h_{I,t} \Pi_{t+1}) < R^L_{I,t} l_{I,t}$.

From (5) and (3), the cutoff point is determined endogenously as:

$$\widetilde{\omega}_{t+1} = LTV \frac{q_h}{E_t q_{h,t+1} \Pi_{t+1}} \tag{6}$$

The default threshold is therefore driven by the LTV ratio and the deviation of the nominal house price from expectations.

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[114] Impatient households face an identical uniform distribution for the shock.

When default occurs, households cannot repay the loan and the bank can seize, in real terms, $\omega_{t+1} q_{h,t+1} h_{I,t} \Pi_{t+1}$, where $q_{h,t+1}$ denotes the house real price in period, $t + 1$. The bank then pays the fraction $1 - \mu$ of what is seized to the government's insolvency agency.

The bank's participation constraint can be written in real terms as:

$$R^L_{I,t} l_{I,t} = \Phi(\widetilde{\omega}_{t+1}) q_{h,t+1} h_{I,t} \Pi_{t+1} \tag{7}$$

where $\Phi(\widetilde{\omega}_{t+1}) \equiv (1-\Theta)\mathrm{G}_{t+1}(\widetilde{\omega}_{t+1}) + \widetilde{\omega}_{t+1} \int_{\widetilde{\omega}_{t+1}}^{\overline{\omega}} f(\omega_{t+1}) \mathrm{d}\omega_{t+1}$ with $1-\mu = \Theta \in [0,1]$ and $\mathrm{G}_{t+1}(\widetilde{\omega}_{t+1}) \equiv \int_{\underline{\omega}}^{\widetilde{\omega}_{t+1}} \omega_{t+1} f(\omega_{t+1}) \mathrm{d}\omega_{t+1}$ is defined as the expected house value accrued to the bank when default occurs.

The budget constraint of the representative impatient household is given, in real terms, by:

$$c_{I,t} + q_{h,t} h_{I,t} + \frac{R^L_{I,t-1}}{\Pi_{\mathrm{t}}} l_{I,t-1} = w_{I,t} n_{I,t} + l_t - T_{I,t} + [1 - \Theta \mathrm{G}_{\mathrm{t}}(\widetilde{\omega}_t)] q_{h,t} h_{I,t-1} \tag{8}$$

The representative impatient household maximises (1) subject to the budget constraint (8), the regulatory DSTI constraint (4) and the bank participation constraint (7)[115].

3.2. BANKS

A monopolistically competitive banking sector extends loans to impatient households and collect deposits from patient households. Banks are subject to an adjustment cost. As in Gerali *et al.*(2010), we assume that the representative bank has a target τ for their capital-to-risk-weighted-assets ratio and pays a quadratic cost whenever it deviates from that target. The target can be interpreted as an exogenous regulatory capital requirement constraint that imposes the amount of own resources to hold. The existence of a cost for deviating from τ implies that bank leverage affects credit conditions in the economy.

The representative bank's real expected profit is:

$$E_t\left[\Lambda^r_{B,t+1}\right] = E_t[RE^r_{t+1}] - R_t\left(l_{I,t} - k_{B,t}\right) - \frac{\zeta_B}{2}\left(\frac{k_{B,t}}{rwr.\,l_{I,t}} - \tau\right)^2 k_{B,t} \tag{9}$$

where *rwr* denotes the regulatory risk weight on mortgage lending and $E_t[RE^r_{t+1}]$ is the expected real return from lending to impatient households which can be written as:

$$E_t[RE^r_{t+1}] = R^L_{I,t} l_{I,t} - l_{I,t} E_t\left(\frac{q_{h,t+1}\Pi_{\mathrm{t+1}} h_{I,t}}{l_{I,t}}\right) \int_{\underline{\omega}}^{\widetilde{\omega}_{t+1}} (\widetilde{\omega}_{t+1} - \mu\omega_{t+1}) f(\omega_{t+1}) \mathrm{d}\omega_{t+1}.$$

The representative bank chooses the optimal loan supply in order to maximise its real expected profit (9). Solving the maximisation programme leads to the following first order condition:

$$R^L_{I,t} = R_t - \zeta_B \left(\frac{k_{B,t}}{rwr.\,l_{I,t}} - \tau\right)\left(\frac{k_{B,t}}{rwr.\,l_{I,t}}\right)^2 rwr + \rho^L_t \tag{10}$$

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[115] Note that the LTV constraint (3) is included in the household maximization problem through the bank participation constraint (7) as the default threshold, $\widetilde{\omega}_t$, already incorporates the LTV constraint. Furthermore, the first order conditions derived from the maximization problem of impatient households are in a technical appendix available upon request.

where $l_{I,t} = \frac{L_{I,t}}{P_t}$ denotes the real loan and ρ_t^L is the mortgage finance premium, defined as:

$$\rho_t^L = E_t \left[\frac{q_{h,t+1} \Pi_{t+1} h_{I,t}}{l_{I,t}} \right] \int_{\underline{\omega}}^{\widetilde{\omega}_{t+1}} (\widetilde{\omega}_{t+1} - \mu \omega_{t+1}) f(\omega_{t+1}) \mathrm{d}\omega_{t+1} \tag{11}$$

The mortgage finance premium that compensates loan losses is determined by the expected ratio of the real value of houses to the real value of total loans as well as the degree of cross-sectional uncertainty in the economy.

Bank capital is accumulated out of reinvested profits.

3.3. FIRMS

Final goods producers operate under perfect competition, buy differentiated intermediate goods produced by intermediate goods producers. The latter operate under monopolistic competition and are indexed by $j \in [0,1]$. The intermediate goods firm *j* relies on the following technology:

$$y_t(j) = A_{F,t}(k_{t-1}(j))^{\alpha} \left[(n_{I,t}(j))^{\eta} (n_{P,t}(j))^{1-\eta} \right]^{1-\alpha} \tag{12}$$

where α is the share of capital in total production, n is the share of impatient households' labour in the total labour input and $n_{I,t}(j)$ and $n_{P,t}(j)$ stand for labour supplied by impatient and patient households respectively. $A_{F,t+1}$ is an aggregate productivity shock.

Each intermediate producer *j* solves its cost minimization problem subject to (12), which provides the real cost of production factors. Price rigidities are introduced in the model following the New Keynesian literature. Firms are subject to Rotemberg price-setting and the optimal price is found by solving their dynamic problem of profit maximization[116].

Finally, in each period, perfectly competitive capital investment-goods producers purchase last-period undepreciated capital at price $q_{k,t}$ from patient households and capital investment goods from final-goods firms at a relative price of one, and produce the new capital goods. This increases the effective installed capital, which is then sold back to patient households at $q_{k,t}$. This transformation process is subject to adjustment costs in the change in investment. Lastly, $q_{k,t}$ is derived from the capital goods producers' maximization of their expected profits.

3.4. MONETARY POLICY AND GOVERNMENT SPENDING

The central bank sets monetary policy according to a Taylor-type rule. It is assumed that government spending is exogenous and represents a constant fraction of the steady state output.

3.5. MARKET CLEARING CONDITIONS

The model's equilibrium is defined as a set of prices and allocations such that households maximize their discounted present value of utility, banks maximize their real expected profit, and all firms maximize the discounted present value of profits subject to their constraints, and all markets clear.

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[116] As in Rotemberg (1984), it is assumed that price changes are costly with quadratic adjustment costs.

4. CALIBRATION OF THE MODEL

In order to simulate the model, we have selected the values for the model parameters based on both Luxembourg data and literature. Table 1 presents the calibrated values of the various parameters.

We set the discount factor of patient households, β_P, to 0.995 in order to match the average annual real risk free interest rate of 2 %. The discount factor of impatient households, β_I, is assumed to be 0.90 so that the two borrowing constraints are binding.

The degree of habit formation in consumption, a, is set to the estimated value of 0.5 in Sangaré (2019). The capital share in output, α, is equal to 0.3, corresponding to the share of labour income to GDP of 0.7 as per Luxembourg data. The share of impatient households' income of total labour income, η, is set to 0.6 based on the results in Alpanda and Zubairy (2017) and the fact that the BCL's Household Finance and Consumption Survey for Luxembourg (HFCS, 2014) reports a small share of income of wealthier households (top deciles) over the total income declared.

We set the non-residential capital depreciation rate, δ_K, to 0.01 also based on Luxembourg data. The loan-to-value (LTV) ratio, *LTV*, is 0.90 and the debt service-to-income (*DSTI*) ratio is 0.40, which are in line with the CSSF survey. The goods substitution elasticity, ϵ, is set 6, implying the steady-state markup of 20 % as in Chen and Columba (2016) and Hristov and Hülsewig (2017). The inverse Frisch elasticity is γ=1.15 in following with the estimates in Sangaré (2019).

We fix the steady-state ratio of capital-to-risk weighted assets to 12 %, which is inferred from a normalization using the Basel III regulatory rule and data. The regulatory risk weight on mortgage loans, *rwr*, taken from Luxembourg data, is 0.19. The dividend policy parameter, $v = 0.9$, is endogenously determined at the steady state. The banking leverage adjustment cost parameter, ζ_B, is set to 0.66 corresponding to the estimate in Sangaré (2019). The parameters of adjustment costs related to goods prices (ζ_P) and business capital (ζ_K) are respectively set to 10 and 2. These values are broadly consistent with the literature (Hristov and Hülsewig (2017) for ζ_P, Clerc *et al.* (2015) for ζ_K). Bank capital depreciates at the rate of δ_B=0.1 as in Gerali *et al.* (2010).

The weights for housing preference (χ_h) and labour disutility (χ_n) in the utility function are respectively 0.5 and 1, following Clerc *et al.* (2015).

The fraction of the actual house value seized by the bank in case of default, μ, is set to 95 % implying insolvency proceeding costs of 5 %, which is the approximate average value in the literature.

The steady state values of the lower and upper bounds of the idiosyncratic housing value shock are respectively $\underline{\omega} = 0.6$ and $\overline{\omega} = 2.4$, such that the two borrowing constraints in the model are binding and the model is well determined for reasonable values of LTV. Therefore, the steady state value of the probability of mortgage default is in the range from 1 % to 5 %.

The ratio of public spending over GDP is 0.2 based on Luxembourg data. The monetary policy rule has a smoothing parameter of 0.8, a response to inflation about 2, and a response to the output gap of 0.4 following Gerali *et al.* (2010).

Finally, we use 0.8 for the coefficients of the autoregressive parts of the shock processes.

Table 1:

Calibration of the model parameters

β_P	Discount factor of Patient households	0.995
β_I	Discount factor of Impatient households	0.9
a	Degree of habit formation in consumption	0.5
α	Capital share in output	0.3
η	Share of Impatient households' income in labour income	0.6
δ_K	Non-residential capital depreciation rate	0.01
LTV	LTV ratio	0.90
$DSTI$	Debt service-to-income ratio	0.40
γ	Inverse of Frisch elasticity	1.15
τ	Ratio of Capital-to-Risk weighted assets	0.12
ζ_B	Banking leverage adjustment cost	0.66
δ_B	Banking capital used in banking activity	0.1
rwr	Regulatory Risk weight on mortgage loans	0.19
v	Banks' dividend policy parameter	0.9
ζ_P	Parameter of goods price adjustment cost	10
ζ_K	Parameter of business capital-investment adjustment cost	2
ϵ	Goods substitution elasticity	6
μ	Fraction of the house value that seized by banks in case of default	0.95
θ	Fraction of the house value seized to cover insolvency proceeding cost	0.05
χ_h	Weight of housing in the utility	0.5
χ_n	Weight of labour in the utility	1
g	Government spending to GDP ratio	0.2
ϕ_R	Taylor rule smoothing coefficient	0.8
ϕ_Π	Taylor rule coefficient on inflation	2
ϕ_Y	Taylor rule coefficient on output	0.4
$\underline{\omega}$	Lower bound of the idiosyncratic housing shock	0.6
$\overline{\omega}$	Upper bound of the idiosyncratic housing shock	2.4
ρ_c	AR consumption preference shock	0.8
ρ_h	AR housing preference shock	0.8
ρ_b	AR banking capital shock	0.8
ρ_f	AR productivity shock	0.8
ρ_r	AR monetary policy shock	0.8
ρ_k	AR capital-investment shock	0.8
ρ_g	AR government spending shock	0.8
ρ_p	AR risk premium shock	0.8

Source: calculs BCL.

5. OPTIMAL CALIBRATION OF LTV AND DSTI MEASURES

5.1. OPTIMAL POLICY FRAMEWORK

An optimal policy analysis aims at identifying optimal calibration values for the policy instruments that maximize the objective function of the macroprudential authority. Therefore, determining the optimal levels of policy instruments requires defining the objective of the macroprudential policy authority and then defining the optimality criteria.

It is challenging to model the objective of macroprudential policy within a DSGE model context since vulnerabilities in the financial system can arise in various forms and from various sources. Furthermore, there is no specific proxy or widely accepted definition of such policy objectives in macro models.

Given the commonly accepted definition of the objective of the macroprudential authority, which is to safeguard financial stability, some authors such as Rubio and Carrasco-Galego (2014) and Angelini *et al.* (2012) assume that there exists a loss function for the macroprudential authority. This loss function is assumed to depend on a set of weighted variable volatilities and the policy authority minimizes this function subject to the equilibrium conditions of the model. This approach is similar to the monetary economics approach in which the monetary policy authority minimizes its loss function.

However, using loss functions in a DSGE context is generally an approximation of the social welfare analysis. The reason is that the loss function is derived from a second order approximation to the expected utility function of the representative household in the basic New Keynesian (NK) model in the absence of real and financial frictions (only taking price stickiness into account)[117]. The authority's loss function therefore represents an average welfare loss and depends on the variability of some endogenous variables[118]. Moreover, the economic rationale behind the use of the welfare loss function as a policy objective function, which depends on the volatilities of variables, is that the volatility has an impact on welfare. For example, from a financial stability perspective, lower volatility of credit growth can smooth borrowers' consumption, thereby improving their welfare.

For these reasons, we follow a welfare-based approach by assuming that the maximization of social welfare is a proxy for the

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117 See for instance, Gali (2008), Gali and Monacelli (2005, 2008).

118 The monetary policy authority's loss function depends for instance on the variability of both the output gap and the rate of inflation (See Gali (2008) for more details).

objective of the macroprudential authority. We therefore define the optimal macroprudential policy as that which maximises the social welfare of the economy.

We perform a grid search for values of LTV and DSTI that maximise social welfare. This provides an assessment of the benefits of implementing different macroprudential policies. We follow Schmitt-Grohe and Uribe (2007) by computing the conditional welfare of agents using the second order approximation of the model[119].

The welfare loss/gain is computed for each type of household (savers and borrowers) under each policy regime using optimal policy ratios.

To make the welfare results more intuitive, we define a welfare metric in terms of consumption equivalents. This consumption equivalent welfare measure is the constant fraction of steady-state consumption that households are willing to give away in order to obtain the benefits of the macroprudential policy[120].

5.2. OPTIMAL VALUES OF LTV AND DSTI RATIOS WITH COVID-19 RELATED SHOCKS

We search for the values of the LTV and DSTI ratios that provide the highest conditional mean of social welfare under a second order approximation of the model. The optimal LTV and DSTI values are found by searching over a grid defined on [0; 1.2] and [0; 1] respectively[121]. We determine separately the optimal values of the LTV and DSTI caps in such a way that conditional social welfare is maximized. The optimization setup consists of searching for the optimal value of each ratio while taking the other ratio as given and calibrated to its actual data value.

Table 2 presents the optimal and current data values of LTV and DSTI as well as the volatilities and the welfare gains/losses generated by the respective values in a context of simultaneous negative shocks to both demand and supply (i.e., a COVID-19-related shock).

Table 2:

Optimal LTV and DSTI ratios under a COVID-19-related environment

	DATA (AVERAGE)	OPTIMAL LEVELS
LTV	90	85
DSTI	40	32
σ_l	2.9450	2.4816
σ_y	5.6297	5.5779
Social welfare (cost/gain)	-0.1060	-0.1044
Impatients (Borrowers)	0.0820	0.0821
Patients (Savers)	-0.2936	-0.2905

Note: The volatilities and values of macroprudential instruments are expressed in %. The welfare metric used is the conditional welfare, computed conditionally on the initial state being the deterministic steady state of the model. The welfare losses/gains are expressed in terms of their percentage of consumption equivalents. This is the same across scenarios. A second order approximation is used for solving the model and providing the quantitative results.

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[119] Second order approximation methods have a particular advantage of accounting for effects of volatility of variables on the mean levels. See among others Schmitt-Grohe and Uribe (2004).

[120] An analytical expression of the welfare measure is available upon request.

[121] These intervals are chosen in order to ensure the determinacy of the model steady state and to use economically reasonable values.

Under recessionary shocks, the optimal LTV limit is found to be 85 % while the optimal DSTI cap is about 32 %. The optimal values of the policy instruments imply welfare gains for borrowers (i.e., impatient households) while savers (i.e. patient households) face welfare losses.

Overall social welfare is negative as a consequence of stronger welfare losses for savers. The welfare metric displays a concave curve as a function of DSTI and LTV. For a given DSTI, increasing the LTV ratio loosens the collateral constraint, implying more mortgage lending to borrowers who increase their asset (house) holdings, which improves their welfare. However, this implies stronger adverse effects resulting from the recessionary shocks on their consumption, thereby reflecting higher debt service charges. The overall impact of increasing LTV on borrowers' welfare is detrimental. For savers, the increase in LTV leads them to save more at the expense of consumption while their house values improve as a result of the higher asset valuation and their increasing return from saving. Therefore, the net effect of increasing LTV is beneficial for savers as their welfare improves. Overall, the social welfare of the whole economy follows a concave path as a function of the LTV values[122].

Comparing the optimal policy scenario to the outcomes provided by the current average data suggests that optimal levels of policy instruments welfare-dominate their non-optimal levels. Furthermore, in terms of stabilization properties, the optimal policy better stabilizes mortgage credit growth and output than the non-optimal policy in the presence of the considered recessionary shocks.

5.3. EFFECTS OF COVID-19-RELATED SHOCKS UNDER THE OPTIMAL POLICY CALIBRATION

We assume that simultaneous negative demand and supply shocks, triggered by the COVID-19 pandemic, hit the economy. The demand shock is a consumption preference shock, while the supply shock is defined by a productivity shock. Figure 3 shows the effects of a simultaneous negative 1 % demand and supply shock on the main macro-financial variables of the economy subject to the optimal limits of LTV and DSTI. These shocks directly reduce consumption of households (borrowers and savers) and output. As a consequence, savers increase their saving and borrowers' preference for house holding increases. Banks, facing a balance sheet (equilibrium) constraint as deposits have increased, respond to credit demand from borrowers by increasing mortgage loans with higher interest rates. The shocks therefore lead to a rise in mortgage loans and lending rates. House prices increase, reflecting the upward trend in both borrowers' preference for housing and mortgage loans. LTV and DSTI ratios, having been set to their tighter optimal limits, have an adverse impact on mortgage loans and subsequently house prices increase less as it would be the case in the absence of these levels of policy instruments.

Facing these recessionary COVID-19-related shocks, the mortgage default risk declines following the impact of shocks before increasing in the medium and long term. This, combined with the higher expected house values, increases the mortgage finance premium, which in turn raises mortgage lending rates. The negative prospects for banking profits and lending activities deplete bank capital. Finally, these recessionary shocks bring the monetary policy rates down, leading to a decrease in real interest rates.

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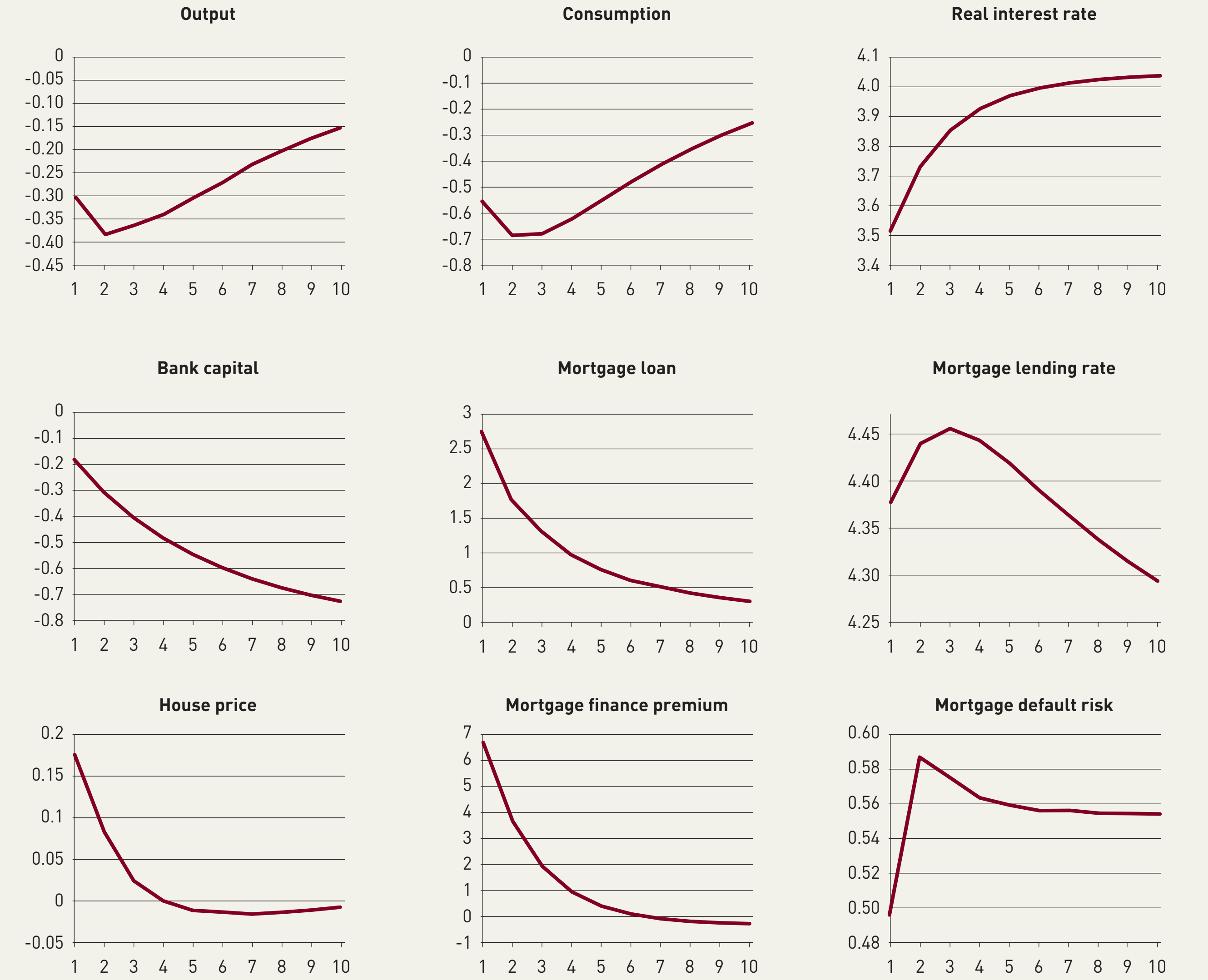
[122] A similar analysis applied to changing DSTI values, when LTV limit remains given, explains the concave path of the overall economy's welfare in function of DSTI.

Figure 3
Effects of COVID-19-related shocks on the main variables of the economy



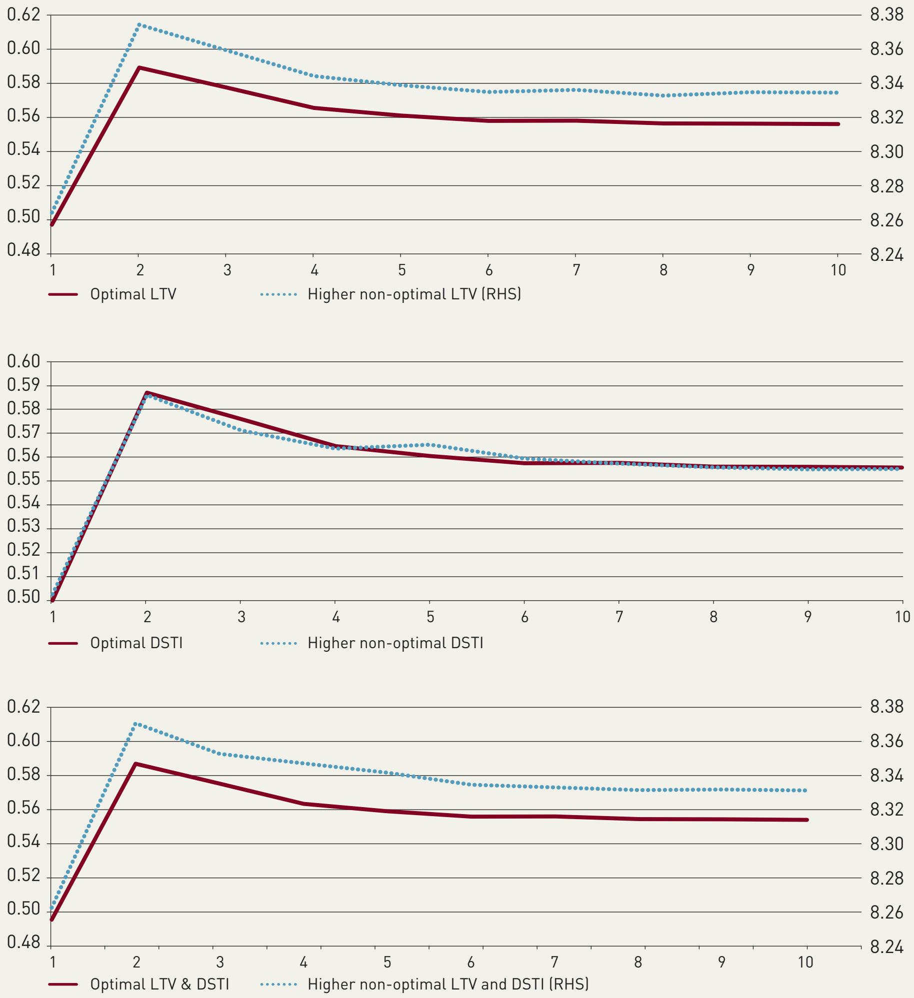
Note: Time, measured in quarters, is on the horizontal axis. All variables are measured in % deviations from steady state, except the mortgage default risk expressed in % levels and the real interest rate and the mortgage lending rate measured in % annualized levels.

5.4. INVESTIGATING THE INTERPLAY BETWEEN LTV AND DSTI AND MORTGAGE DEFAULT RISK

We assume that the macroprudential authority exogenously sets the values of both LTV and DSTI caps to their optimal levels. We perform a counterfactual (i.e., a sensitivity) analysis by assessing the impacts of choosing alternative non-optimal values of DSTI and LTV on the mortgage default risk. We compare the optimal calibration of instruments (LTV=85 % and DSTI=32 %) against three policy scenarios. First, the scenario in which only the LTV limit is modified and set to a higher non-optimal level (LTV=95 %) compared to its optimal value (higher non-optimal LTV scenario). Second, the scenario with higher non-optimal DSTI in which only DSTI has increased (DSTI=50 %) compared to its optimal value and the third scenario assumes that both LTV and DSTI are set to higher values (LTV=95 % and DSTI=50 %) relative to the optimal calibration. We consider a COVID-19-related shock which comprises both contractionary demand and supply shocks.

Figure 4

Effects of the COVID-19-related shocks on mortgage default risk under different policy (LTV and DSTI) scenarios



Notes: Time, measured in quarters, is on the horizontal axis. Mortgage default risk is expressed in % levels.

Figure 4 displays the impacts of simultaneous 1 % negative shocks to households' preference for consumption and total factor productivity under the three policy scenarios. It is clear that LTV and DSTI limits that are higher than their respective optimal values amplify the effects of the shocks on mortgage default risk. In particular, a higher LTV limit increases the risk of mortgage default compared to the optimal LTV cap. The reason is straightforward. Increasing the LTV cap increases the default threshold, which is directly driven by the LTV ratio in the modelling framework. When LTV increases, mortgage loans also increases, thereby leading to the increase in mortgage default risk.

However, Figure 4 shows that, contrary to the LTV cap which affects the probability of mortgage default, the increase in the DSTI limit has a negligible impact on default risk. Consequently, an increase in both LTV and DSTI limits results in greater mortgage default risk compared to the optimal calibration. This reflects the fact that the main driver of default risk in our modelling framework when the COVID-19 related shock occurs is the LTV limit[123].

6. CONCLUSIONS

The objective of this work is to quantitatively determine the optimal calibration values of two borrower based measures for Luxembourg within the framework of a DSGE model with mortgage default. The first contribution of this study is to build a DSGE model that contains a housing sector with mortgage default, two borrowing constraints (LTV and DSTI) and a monopolistically competitive banking sector. The second contribution consists of determining the non-joint optimal values of the LTV and DSTI limits for Luxembourg.

Based on a welfare analysis, we find that the (non-joint) optimal values of LTV and DSTI ratios in the presence of a Covid-19 related adverse shock are 85 % and 32 %, respectively. We also find that the optimal macroprudential policy welfare-dominates the non-optimal policy. Moreover, the optimal policy calibration better stabilizes mortgage lending and output compared to the policy based on the actual data. Finally, our findings suggest that a simultaneous increase in both the LTV and DSTI limits implies a higher mortgage default risk compared to the optimal calibration of these instruments. This reflects the fact that the main driver of default risk in the presence of a COVID-19 related shock is the LTV limit.

As possible extensions of this work, we plan to assess the optimal interactions between LTV, DSTI and the bank capital requirement ratio. It is worth noting that another potential research topic would be to expand the DSGE model with default by introducing an explicit differentiation between the mortgage debt stock and flow, which can facilitate the analysis of amortization requirements and the subsequent macro-financial implications.

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[123] This result reflects the assumption made in the modelling framework that the only source of mortgage default is a shock that affects the house value and that income-related risks do not trigger mortgage default.

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