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(IN)EFFICIENT COMMUTING AND MIGRATION CHOICES: THEORY AND POLICY IN AN URBAN SEARCH MODEL

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(IN)EFFICIENT COMMUTING AND MIGRATION CHOICES: THEORY AND POLICY IN AN URBAN SEARCH MODEL

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Abstract. We develop a monocentric urban search-and-matching model in which workers can choose to commute or to migrate within the region. The equilibrium endogenously allocates the population into three categories: migrants (relocate from their hometown to the city), commuters (traveling to work in the city) and home stayers (remaining in their hometown). We prove that the market equilibrium is usually not optimal: a composition externality may generate under- or over-migration with respect to the central planner's solution, which in all cases results in under-investment in job vacancies and therefore production. We calibrate the model to the Greater Paris area to reproduce several gradients observed in the data, suggesting over-migration. We show how policy interventions can help to reduce inefficiencies.

JEL Codes: E24, J68, R13, R23.

Keywords: Migration, Commuting, Urban search-and-matching, Efficiency, Policy.

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Résumé non technique

Les zones urbaines sont des pôles d'attraction où la création d'emplois stimule l'activité économique, et où la population urbaine – bien que dense – ne suffit pas à répondre au besoin de main-d'oeuvre des entreprises. La force de travail y est donc alimentée par les habitants des zones urbaines (nous les nommerons «résidents») mais aussi par les travailleurs vivant à l'extérieur de ces zones (nous les nommerons «navetteurs»). Cette combinaison résidents/navetteurs se retrouve dans de grandes villes, comme Paris et Londres, mais aussi dans de petits États souverains, tel que le Grand-Duché de Luxembourg.

Dans cette étude, nous développons un modèle urbain de recherche et d'appariement («search-and-matching») comprenant des éléments d'économie urbaine et d'économie du travail. Dans notre modèle, il existe deux zones, le centre-ville – où se trouvent tous les emplois – et la périphérie. Il comporte également deux types de travailleurs : les «navetteurs», c'est-à-dire les personnes habitant dans la périphérie et payant moins de loyer mais effectuant des longs trajets journaliers de leur domicile à leur travail ; et les «résidents», c'est-à-dire les personnes habitant en ville et payant plus de loyer pour résider près de leur lieu de travail. La principale contribution de notre modèle réside dans la prise en compte du choix opéré par les travailleurs, qui peuvent changer de résidence pour se rapprocher de l'emploi ou accepter des longs trajets «domicile-travail». Jusqu'ici, la littérature s'est focalisée sur une décision (changer de résidence ou ne pas le faire) ou l'autre (faire la navette ou ne pas la faire), tandis que nous considérons les deux simultanément.

L'objectif de notre étude est d'analyser le choix d'habiter en ville (résidents) ou en périphérie (navetteurs) ainsi que les implications économiques de ce choix. Bien que ces décisions puissent générer de nombreuses externalités (comme la congestion routière, la pollution, etc.), nous nous focalisons sur une externalité émanant du marché du travail, que nous appelons «externalité de composition», parce qu'elle est liée à la proportion de navetteurs et de résidents dans la force de travail. En effet, le statut navetteur/résident d'un demandeur d'emploi influence sa négociation avec des employeurs potentiels, ce qui peut aboutir à des différences salariales. Par conséquent, au niveau macroéconomique, la proportion de navetteurs et de résidents dans la force de travail a un impact sur les coûts des entreprises et, donc, sur la création d'emploi. Or, dans un équilibre compétitif

(c'est-à-dire sans intervention de l'État), les individus ne prennent pas en compte cette externalité lorsqu'ils décident de devenir navetteurs ou résidents.

Nous obtenons trois résultats majeurs. Premièrement, en distinguant explicitement entre navetteurs et résidents, notre modèle peut reproduire diverses caractéristiques observées dans les données, comme par exemple, que la densité de population baisse au fur et à mesure que l'on s'éloigne du centre-ville. Deuxièmement, nous montrons analytiquement que l'externalité de composition implique que l'équilibre compétitif n'est pas forcément efficient (allocation sous-optimale des ressources). En effet, trop ou pas assez de résidents (ou de manière équivalente, pas assez ou trop de navetteurs) conduira à un niveau sous-optimal d'emplois et donc d'activité économique. Finalement, après avoir calibré le modèle pour reproduire différentes caractéristiques des données, nos simulations illustrent comment l'intervention de l'État, à travers des politiques du transport et du logement (aides et taxation), peut améliorer l'allocation des ressources.

1. Introduction

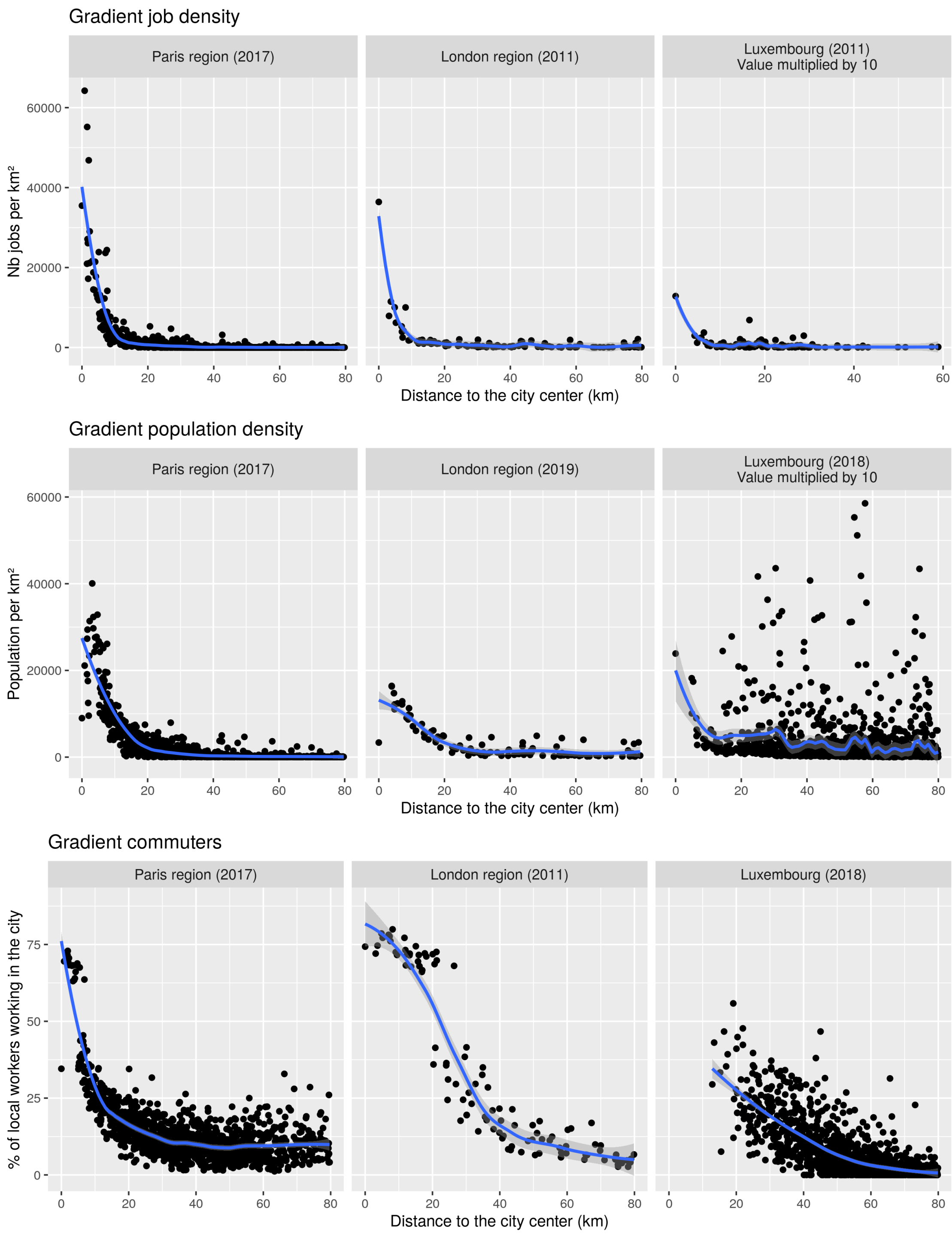
Urban areas are poles of attraction where job creation bolsters economic activity. Employment is fueled by newly arrived citizens but also by commuters from surrounding locations. For instance, in Paris and London, most jobs are located in the inner city, but there are not enough residents in the core to fill all jobs (first two columns in Figure 1). Indeed, although population density is high downtown, firms must also rely on commuters from the greater city area. More precisely, 59% of the jobs located in Paris and 21% of those in London are held by workers living outside the city center. These observations may be generalized to small States, who attract both migrants and cross-border commuters. In the Grand Duchy of Luxembourg, foreign-born resident workers represent 27% of the workforce, while cross-border workers make up 46% (last column in Figure 1).

Though relocation and commuting decisions may induce various types of externalities (e.g. pollution or congestion, see Section 2 for a review of the literature), we here focus on an externality originating from the labor market. Indeed, in a model combining elements from both labor and urban economics (see Zenou, 2009a, for an overview), individual Nash bargaining may imply different wages for migrants and commuters. As a result, the migrant/commuter composition of job seekers has implications for firms' expected profits and therefore also for job creation and economic activity. This creates an externality, as individuals ignore these effects when looking for a job. In this paper, we develop a monocentric urban search-and-matching model with endogenous migration and commuting. We prove that the decentralized equilibrium is generally not optimal and show how policy interventions can help to reduce inefficiency.

Our monocentric urban model comprises a central business district (CBD), also called city, where all production takes place, and the city outskirts (or more simply outskirts). Individuals are distributed along two dimensions: the distance d between their original location (which we may also call hometown) and the city; and the psychological attachment m to their original location (or equivalently their psychological aversion to migrate from their original location to the city).[1] Individuals may choose to (i) relocate to the city (migrants), (ii) stay in their hometown and commute to the city (commuters) or

[1] $d = 0$ means that their original location is the city whereas $d > 0$ means that their original location situates in the city outskirts.

FIGURE 1. Job density, population and commuting flows spatial distribution



Notes. Authors' calculations based on data from INSEE, ONS, STATEC, LISER, IGSS and Eurostat. See Appendix A for full details. Luxembourg differs from Paris and London in terms of population density because other relatively large foreign cities are not distant. However, these cities are comparatively less attractive in terms of the number of job opportunities and the general salary level. Note also that in the case of Luxembourg, distance to the city center extends beyond the country's national borders. In the context of urban economics, the term *gradient* refers to the mean of a variable, conditional on the distance to the city center.

(iii) stay in their hometown without commuting to the city (home stayers). Commuting implies transport costs proportional to the distance d; migration implies one-time relocation costs depending on hometown attachment m and endogenous housing rents; and staying home removes all costs but also removes wage income (Albouy and Lue, 2015).[2] In the model, individuals living in the city and in the city outskirts may have distinct values for leisure, reflecting differences in amenities like restaurants, sport facilities or movie theaters (Brueckner, Thisse, and Zenou, 1999a).

We obtain three main results. First, while most of the related literature focuses on commuting only, we explicitly distinguish between commuters and migrants in an urban search-and-matching model to obtain endogenous population gradients. Second, the decentralized equilibrium is in general not efficient, even when the Hosios condition holds. Indeed, workers have different values for leisure if they live in the city or in the city outskirts and therefore reservation wages also differ. With individual Nash bargaining over wages, this results in wage differences between the city and the outskirts. When deciding between migration or commuting, workers do not realize that their individual choice affects firm profits, and hence the opening of vacancies, i.e. job creation. This *composition externality* may generate under- or over-migration with respect to the central planner solution. In all cases, this externality results in under-investment in vacancies.[3] Third, to investigate the empirical relevance of this composition externality, we calibrate the model to the Greater Paris area. Within our parsimonious setup, we are able to reproduce several gradients observed in data. The Paris calibration produces over-migration (or equivalently under-commuting) due to the presence of the composition externality and the decentralized equilibrium is thus not efficient, with output 2.8% below the central planner solution. We show that a policy acting on commuting and housing rents

[2]Housing rents must be understood in a broad sense that includes the user cost of housing for owner-occupiers.

[3]The term *composition externality* is borrowed from Lehmann, Montero Ledezma, and Van der Linden (2016), who find a similar effect, but arising in a different framework where migration is absent and where commuters choose between two business districts.

helps reducing inefficiencies. More precisely, the optimal policy subsidizes commuting costs through housing taxation.

Section 2 reviews the related literature. Section 3 develops the theoretical model and compares the decentralized with the centralized equilibrium. Section 4 provides the quantitative analysis using Paris data. Section 5 concludes.

2. Related literature

Our paper belongs to the wide research field investigating the interaction of location decisions and labor markets. We briefly present this literature and then discuss our contribution to it.

Early research develops theoretical search models accounting for commuting distance, but assumes that job and residence offers arrive exogenously (e.g. van Ommeren, Rietveld, and Nijkamp, 2000). More recent papers focus on the externalities generated by commuting. One branch of this literature centers on the negative congestion and/or the positive agglomeration externalities arising from moving and travel decisions. Larsen, Pilegaard, and van Ommeren (2008) theoretically analyze welfare-maximizing taxation when commuters generate congestion, while Flemming (2020) addresses this question quantitatively. Denant-Boemont, Gaigné, and Gaté (2018) focus on pollution externalities and Brinkman (2016) considers the opposing effects of congestion and agglomeration externalities arising from commuting behavior. However, this research relies on job search models where workers respond to exogenously arriving job offers.

Our paper is closer to the strand of the literature specifically examining the labor market effects of location decisions. This research combines a spatial urban structure with a search-and-matching framework à la Pissarides (2000), where job creation is endogenous. An influential paper in this literature is Wasmer and Zenou (2002), who examine the implications of distance and urban structure on labor market outcomes. Firms are exogenously situated in a monocentric city, while employed and unemployed workers endogenously choose at which distance to locate from the city center. Individuals relocate at each employment-unemployment transition and at zero moving cost. The authors distinguish a segregated equilibrium (all unemployed individuals reside far away from jobs and the employed reside close to jobs) and an integrated equilibrium (unemployed close and employed far from jobs), and show that both equilibria are efficient under the

Hosios (1990) condition. Many ingredients have been added to this setup, as for instance relocation costs (Wasmer and Zenou, 2006; Zenou, 2009b), firm compensation for moving costs (Van Ommeren and Rietveld, 2007), rural-urban migration (Zenou, 2011), housing development (Xiao, 2014), housing consumption (Boitier, 2018) or a regulated housing market (Chapelle, Wasmer, and Bono, 2020). Most of these extensions generate additional externalities and the Hosios condition is no longer sufficient to guarantee efficiency.

Our paper differs in several aspects from this branch of the literature. First, in the urban search-and-matching literature à la Wasmer and Zenou (2002), firms typically need to compensate workers for their commuting costs, which are higher than those of unemployed individuals. This implies that wages increase with the distance to jobs, while the opposite is usually observed in the data (see Figure 10 in Appendix A). Instead, we follow Rupert and Wasmer (2012) and assume that wages are unrelated to commuting distance. In our model, individuals living in the city and those residing in the outskirts have different values for leisure and therefore different wages.

Second, most urban search-and-matching models only focus on the implications of commuting. Migration only occurs because individuals are forced to relocate at each unemployment-employment transition (zero relocation cost as in Wasmer and Zenou, 2002) or migration does not occur at all (infinite relocation cost as in Zenou, 2009b).[4] In contrast, our relocation cost is strictly positive but not infinite, which allows us to explicitly distinguish between migrants and commuters and model their choices. In our model, individuals face travel costs if they commute and moving costs plus higher rents if they migrate to the city. This commuting versus migration decision affects wage negotiations, as firms compensate migrants and commuters differently, and creates a composition externality. In this respect, our paper is closely related to Lehmann, Montero Ledezma, and Van der Linden (2016) who extend Wasmer and Zenou (2002) to consider two central business districts. In their model, residential location is determined in a first stage. Since commuting costs are shared by the worker and the firm, job creation will depend on the average commuting distance, which is not internalized by job seekers. Although

[4]One exception is the rural-urban model of Zenou (2011), where individuals in the rural area can choose between staying idle or migrating and becoming unemployed in the city.

the setup is different, this kind of composition externality is similar to ours.[5] However, our approach yields more realistic urban structures with both the city and the outskirts hosting unemployed and employed individuals, while in the related literature, employed and unemployed individuals never reside in the same area (Wasmer and Zenou, 2002).[6] Lastly, we calibrate our urban search-and-matching model to reproduce several gradients observed in data (Greater Paris area). Our quantitative results are in line with the size of other externalities found in literature. Flemming (2020) finds that the welfare cost of the negative congestion externality is 3.8% (our externality leads to a 2.8% production cost) and that the optimal tax on commuters passing in congested areas equals 16% (our optimal commuting subsidy amounts to 9.2%).

3. Theoretical analysis

There are two areas: a monocentric city (denoted by the subscript C) and the city outskirts (denoted by the subscript O). Employment is only available in the city. We normalize the total population to 1. Each individual has a hometown located in the outskirts, at a distance $d \in [0,1]$ from the city (in the particular case $d = 0$, the hometown is the city). Each individual has a psychological attachment $m \in [0,1]$ to her hometown. Each individual is therefore characterized by (d, m), which is time-invariant, where $f(.,.)$ is the probability density function such that $\int_0^1 \int_0^1 f(d,m)\,\mathrm{d}d\,\mathrm{d}m = 1$ and $F(.,.)$ is the cumulative distribution function. Each individual (d, m) may choose between three scenarios: moving to the city and searching for a job there (migration scenario); remaining at the same location in the outskirts and searching for a job in the city (commuting scenario); remaining at the same location in the outskirts without searching for work in the city (home scenario).[7] Commuting implies recurrent costs proportional to the distance d, capturing both monetary and time costs. Migration to the city implies one-time

[5] In contrast to Lehmann, Montero Ledezma, and Van der Linden (2016) and other studies (Zenou, 2009b; Boitier, 2018), our approach can also characterize the equilibrium without assuming a risk-free interest rate converging toward zero. This hypothesis typically facilitates the determination of the central planner equilibrium as it implies that the transition dynamics do not matter anymore.

[6] In the literature, either the employed live in the city and the unemployed in the outskirts (segregated equilibrium) or the reverse (integrated equilibrium). In our model, an urban structure with e.g. a rich city and poor outskirts features high-earning individuals (wage for the employed and reservation wage for the unemployed) living in the city and low-earning individuals residing in the outskirts.

[7] In this setup, there is therefore no possibility to migrate from one location $d > 0$ in the outskirts to another location $d' > 0$ still in the outskirts.

costs proportional to hometown psychological attachment m (see for instance Cameron and Muellbauer, 1998, for a justification of such a cost scheme). In this setup, other things being equal, individuals with small d and high m favor commuting, individuals with high d and small m favor migration, and individuals with high d and high m stay in the outskirts without searching for a job (home scenario). Let $\mathcal{S}_m$, $\mathcal{S}_c$ and $\mathcal{S}_h$ the sets of all (m,d) whose decision is migration, commuting and home, respectively. Then the fraction of the population living in the city, residing in the outskirts and commuting, and residing in the outskirts without commuting is $MI = \int_{(d,m)\in\mathcal{S}_m} f(d,m)\ \mathrm{d}d\ \mathrm{d}m$, $CO = \int_{(d,m)\in\mathcal{S}_c} f(d,m)\ \mathrm{d}d\ \mathrm{d}m$ and $HO = \int_{(d,m)\in\mathcal{S}_h} f(d,m)\ \mathrm{d}d\ \mathrm{d}m$, respectively, with $MI + CO + HO = 1$. Finally, housing rents are normalized to 0 in the city outskirts but are endogenous and positive in the city.

3.1. **Model: Decentralized economy.**

Labor markets

We introduce search-and-matching frictions in the labor market as in Mortensen and Pissarides (1999) and Pissarides (2000). The population living in the city is composed of job seekers U_C and workers N_C

$$MI = U_C + N_C \tag{1}$$

Similarly, the population living in the city outskirts and commuting is also composed of job seekers U_O and workers N_O

$$CO = U_O + N_O \tag{2}$$

There is a continuum of firms opening vacancies V. Vacancies and job seekers meet according to a standard differentiable matching function $M(V, U_C + U_O)$. $M(.,.)$ is increasing and concave in both arguments, and exhibits constant returns to scale. The rate at which a vacancy is filled is

$$\frac{M(V, U_C + U_O)}{V} = q(\theta) \tag{3}$$

where $\theta = V/(U_C + U_O)$ is labor market tightness and $q'(\theta) < 0$. Similarly, the rate at which a job seeker finds a vacancy is

$$\frac{M(V, U_C + U_O)}{U_C + U_O} = p(\theta) = \theta q(\theta) \tag{4}$$

where $p'(\theta) > 0$. As usual in the literature, we assume the following Inada conditions

$$\begin{aligned}
\lim_{\theta \to 0} p(\theta) &= \lim_{\theta \to +\infty} q(\theta) = 0 \\
\lim_{\theta \to +\infty} p(\theta) &= \lim_{\theta \to 0} q(\theta) = +\infty
\end{aligned}$$

Every job may be destroyed with an exogenous probability $\delta \geq 0$. At the steady state, the number of job seekers entering employment is equal to the number of workers leaving employment

$$p(\theta)\, U_C = \delta N_C \tag{5}$$

$$p(\theta)\, U_O = \delta N_O \tag{6}$$

Bellman equations

For those living in the city, $I_C^U(d,m)$ and $I_C^E(d,m)$ denote the inter-temporal values of the unemployed and the employed characterized by (d,m). At the steady state, their Bellman equations are

$$\begin{aligned}
rI_C^U(d,m) &= h_C - (1-t_R)R + p(\theta)\, \left(I_C^E(d,m) - I_C^U(d,m)\right) \\
rI_C^E(d,m) &= w_C(d,m) - (1-t_R)R + \delta\, \left(I_C^U(d,m) - I_C^E(d,m)\right)
\end{aligned}$$

$r > 0$ is the exogenous discount rate. In the city, the unemployed find a job at a rate $p(\theta)$ and benefit from a value $h_C \geq 0$ for leisure. In Appendix B, we show this value for leisure may be closely related to the level of amenities in the city (therefore, in the remainder of the paper, we use these two terms interchangeably). An employee living in the city receives a wage $w_C(d,m)$, which depends on individual characteristics, and loses her job at a rate δ. Both unemployed and employed living in the city must pay an endogenous housing rent R. t_R is a policy parameter corresponding to a proportional housing subsidy (with respect to rent).

Similarly, for those residing in the outskirts, $I_O^U(d,m)$ and $I_O^E(d,m)$ denote the inter-temporal values of an unemployed and an employed characterized by (d,m). At the steady state, their Bellman equations are

$$\begin{aligned}
rI_O^U(d,m) &= \max\left[h_O - (1-t_d)\mu d + p(\theta)\, \left(I_O^E(d,m) - I_O^U(d,m)\right),\ h_O\right] \\
rI_O^E(d,m) &= w_O(d,m) - (1-t_d)\mu d + \delta\, \left(I_O^U(d,m) - I_O^E(d,m)\right)
\end{aligned}$$

An unemployed resident in the outskirts may or may not search for a job in the city.[8] In both cases, the unemployed benefits from a value $h_O \geq 0$ for leisure (or amenities in the outskirts, see Appendix B). Unemployed searching for a job need to go to the city to collect information and attend interviews, therefore incurring a commuting cost $\mu > 0$ per unit of distance. They also find a job at rate $p(\theta)$. A resident in the outskirts with a job in the city receives a wage $w_O(d, m)$, pays a commuting cost μd and loses her job at a rate δ. Rents in the outskirts are constant and normalized to 0. t_d is a policy parameter corresponding to a commuting subsidy proportional to the distance.

For simplicity, we assumed above that (i) unemployed and employed workers have the same commuting costs, meaning they commute to city at the same frequency (for a similar assumption see e.g. Coulson, Laing, and Wang, 2001), and (ii) rents in the city outskirts are exogenous and normalized to 0 (see for instance Zenou, 2011, for a similar assumption). The first assumption yields a discrete wage distribution (w_C and w_O) and implies that wages do not depend on the commuting distance. Supposing instead that employed workers pay a higher commuting cost, as e.g. in Zenou (2011), would produce a continuous wage distribution $w(d)$ with wages increasing in the commuting distance. Analogously, the second assumption enables to keep an analytically tractable discrete rent distribution (R and 0) instead of a continuous one $R(d)$. Moreover, because individuals must pay a rent whatever their job status (unemployed or employed), rent does not enter the wage equations.

Let J_V, $J_C(d, m)$ and $J_O(d, m)$ represent the inter-temporal values of a firm with an open vacancy, with a job filled by a worker (d, m) living in the city and with a job filled by a worker (d, m) living in the city outskirts. At the steady state, their Bellman equations

[8]The possibility of *not* searching for a job in the city (the second argument in the max operator) is not essential for our results. It only aims at giving a more realistic picture of the population density once all migration decisions have been made, i.e. it avoids areas without population (see Section 4 for an illustration and Section 5 for a discussion).

are

$$
\begin{aligned}
rJ_V &= -a + q(\theta)\left(\left(\frac{U_C}{U_C+U_O}\frac{\int_{\mathcal{S}_m} J_C(d,m)\,\mathrm{d}F(d,m)}{\int_{\mathcal{S}_m}\mathrm{d}F(d,m)}\right.\right.\\
&\qquad \left.\left.+\frac{U_O}{U_C+U_O}\frac{\int_{\mathcal{S}_c} J_O(d,m)\,\mathrm{d}F(d,m)}{\int_{\mathcal{S}_c}\mathrm{d}F(d,m)}\right) - J_V\right)\\
rJ_C(d,m) &= y - w_C(d,m) + \delta\left(J_V - J_C(d,m)\right)\\
rJ_O(d,m) &= y - w_O(d,m) + \delta\left(J_V - J_O(d,m)\right)
\end{aligned}
$$

The firm pays a vacancy cost $a > 0$ and fills the vacancy at rate $q(\theta)$. In case of contact with a job seeker, the firm has a probability $U_C/(U_C+U_O)$ of meeting a migrant $(m,d) \in \mathcal{S}_m$ and a probability $U_O/(U_C+U_O)$ of meeting a commuter $(m,d) \in \mathcal{S}_c$. y is the product of a match. Free entry of firms implies $J_V = 0$.[9]

Wages

Firms and workers bargain on wages. As is common in the literature, we assume Nash bargaining

$$
\begin{aligned}
\gamma J_C(d,m) &= (1-\gamma)(I_C^E(d,m) - I_C^U(d,m))\\
\gamma J_O(d,m) &= (1-\gamma)(I_O^E(d,m) - I_O^U(d,m))
\end{aligned}
$$

where $0 < \gamma < 1$ is the worker bargaining power. From the Bellman equations, we have $\forall\ (d,m) \in \mathcal{S}_m$

$$
\begin{aligned}
I_C^E(d,m) - I_C^U(d,m) &= \frac{w_C(d,m) - h_C}{r+\delta+p(\theta)}\\
J_C(d,m) &= \frac{y - w_C(d,m)}{r+\delta}
\end{aligned}
$$

Similarly, $\forall\ (d,m) \in \mathcal{S}_c$

$$
\begin{aligned}
I_O^E(d,m) - I_O^U(d,m) &= \frac{w_O(d,m) - h_O}{r+\delta+p(\theta)}\\
J_O(d,m) &= \frac{y - w_O(d,m)}{r+\delta}
\end{aligned}
$$

[9] Although the firm may have preferences between workers (the firm surplus depends on (d,m)), it will never turn down an application as long as the surplus is positive (see for instance Pissarides, 2000, for a discussion).

We therefore compute

$$
\begin{aligned}
w_C(d,m) \equiv w_C &= \frac{\gamma(r+\delta+p(\theta))y+(1-\gamma)(r+\delta)h_C}{r+\delta+\gamma p(\theta)} \\
w_O(d,m) \equiv w_O &= \frac{\gamma(r+\delta+p(\theta))y+(1-\gamma)(r+\delta)h_O}{r+\delta+\gamma p(\theta)}
\end{aligned}
$$

Because rents and commuting costs (and subsidies) appear in both employment and unemployment states, they do not enter the wage equations. In contrast to Wasmer and Zenou (2002) and related papers, only two different wages co-exist in the economy: one for the migrants and one for the commuters, which only differ through the respective values for leisure h_C and h_O, linked to local amenities (see Section 4 for a discussion on the value of these parameters). We may then use the wage expressions to simplify (see Appendix C) and obtain the free entry condition

$$
a = \frac{q(\theta)(1-\gamma)\left(U_C(y-h_C)+U_O(y-h_O)\right)}{(U_C+U_O)(r+\delta+\gamma p(\theta))} \tag{7}
$$

We observe that when $h_C = h_O$, U_C and U_O disappear from the equation and the free entry condition pins down θ as function of parameters only. In other words, the equilibrium θ is independent from the split of the population between migrants and commuters. When $h_C \neq h_O$, a composition effect arises and θ also depends on the endogenous unemployment shares, that is on the endogenous location decisions. We show later how this creates inefficiencies in the market equilibrium.

Location choices

Migration is bi-directional: an individual may move from the outskirts to the city (in-migration) or may move from the city to her original location in the outskirts (out-migration). To simplify the analysis, we assume migration can only happen when the individual is unemployed (for a similar assumption see Zenou, 2011; Larsen, Pilegaard, and van Ommeren, 2008).[10] $I^U_C(d,m)$ and $I^U_O(d,m)$ are the inter-temporal values of

[10]Focusing on relocation when unemployed is consistent with empirical findings. One-worker households are about 2.5 times more likely to relocate after a job change relative to households that keep their jobs (Clark and Davies Withers, 1999), while workers experiencing an unwanted job loss are 30 to 80% more likely to relocate to a new location compared to employed workers (DaVanzo, 1978; Fackler and Rippe, 2017; Huttunen, Møen, and Salvanes, 2018).

unemployed individuals (d,m) living in the city and in the outskirts, respectively. Then, an individual (d,m) is indifferent between living in the city or living in the city outskirts if and only if

$$I_C^U(d,m) - I_O^U(d,m) = \tau m \tag{8}$$

where $\tau > 0$ and $\tau\, m$ represents the migration cost for an individual with hometown psychological attachment m. This migration equilibrium condition means that, for a marginal individual, the difference between inter-temporal values exactly offsets the one-time migration cost.

We are now able to formulate the indifference conditions between the three different scenarios: migration, commuting and home. An individual (d,m) is *indifferent between the migration and the home scenarios* if and only if equation (8) holds and $rI_O^U(d,m) = h_O$. Using expressions from Appendix C, we obtain the first indifference equation

$$r\tau m = h_C - h_O - (1-t_R)R + \frac{p(\theta)\gamma(y-h_C)}{r+\delta+\gamma p(\theta)} \tag{9}$$

An individual (d,m) is *indifferent between the migration and the commuting scenarios* if and only if equation (8) holds and $rI_O^U(d,m) = h_O-(1-t_d)\mu d+p(\theta)\ (I_O^E(d,m)-I_O^U(d,m))$. Using again Appendix C, we get the second indifference equation

$$r\tau m = h_C - h_O - (1-t_R)R + (1-t_d)\mu d - \frac{p(\theta)\gamma(h_C-h_O)}{r+\delta+\gamma p(\theta)} \tag{10}$$

An individual (d,m) is *indifferent between the home and the commuting scenarios* if and only if $h_O = h_O-(1-t_d)\mu d+p(\theta)\ (I_O^E(d,m)-I_O^U(d,m))$. This gives the third indifference equation

$$(1-t_d)\mu d = \frac{p(\theta)\gamma(y-h_O)}{r+\delta+\gamma p(\theta)} \tag{11}$$

Indifference equations (9) to (11) split the population into three distinct areas, which correspond to the three scenarios: commuting $\mathcal{S}_c$, home $\mathcal{S}_h$ and migration $\mathcal{S}_m$.

Definition 1.

$$\begin{aligned} d^\star &= \frac{p(\theta)\ \gamma(y-h_O)}{(1-t_d)\mu(r+\delta+\gamma p(\theta))} \\ m^\star &= \frac{p(\theta)\ \gamma y + (r+\delta)h_C}{r\tau(r+\delta+\gamma p(\theta))} - \frac{h_O+(1-t_R)R}{r\tau} \end{aligned}$$

Under Definition 1, equations (9) to (11) become

$$\begin{aligned} m &= m^\star \\ m &= m^\star + \frac{(1-t_d)\mu}{r\tau}\,(d-d^\star) \\ d &= d^\star \end{aligned}$$

Assumption 1. *Parameters are such that $d^\star \in [0,1]$ and $m^\star \in [0,1]$*

This assumption implies that commuters and migrants make up non-empty sets. Under Definition 1 and Assumption 1, Figure 2 illustrates the population distribution and the split into the migration, commuting and home scenarios. For clarity of exposition, we define

$$d_0 = d^\star - \frac{r\tau m^\star}{(1-t_d)\mu} \tag{12}$$

as the intersection between the $m = 0$ line and the indifference equation between migration and commuting.[11]

Because the one-time migration cost and the recurrent commuting costs are linear in m and d, the population split also follows linear rules. Individuals close to the city (small d) choose commuting whereas individuals with a low psychological aversion to migrate (small m) choose migration. Individuals far from the city (high d) and with important aversion (high m) are uninterested in migration or commuting.

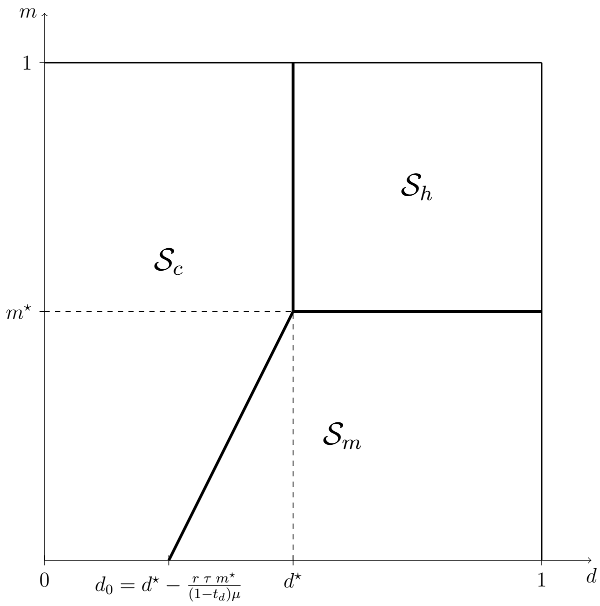
To provide more intuitions from Definition 1, let us assume no policy and $h_C = h_O$. Using also results from Appendix C, the first expression simplifies to $\mu d^\star = p(\theta)\gamma(y - h_C)/(r+\delta+\gamma p(\theta)) = p(\theta)(I_O^E - I_O^U)$. This means that the maximum commuting cost an individual is willing to pay is equal to the probability of finding a job multiplied by the worker surplus from this job. Similarly, the second expression from Definition 1 simplifies to $r\tau m^\star + R = \mu d^\star$, which indicates that the discounted one-time migration cost augmented by the recurrent housing rent is equal to the recurrent commuting cost, i.e. indifference between migration and commuting scenarios. The next assumption simplifies the theoretical analysis.

Assumption 2. *Population distribution $f(d,m)$ is uniform.*[12]

[11] Figure 2 shows the case $d_0 \geq 0$. The other possibility is $d_0 < 0$. In this case, the indifference curve between commuting and migration intersects the vertical axis.

[12] Introducing a two-step function with a higher density in the city center complicates the analysis without affecting the main findings. In any case, assuming a uniformly distributed population is not

FIGURE 2. Population distribution and choices



Notes. Population is distributed along d (distance between the city and the hometown) and m (psychological attachment to live in the hometown). The city is located at $d = 0$. $\mathcal{S}_c$ is the set of all (d, m) choosing the commuting scenario, $\mathcal{S}_m$ is the set of all (d, m) choosing the migration scenario and $\mathcal{S}_h$ is the set of all (d, m) choosing the home scenario. Definition 1 gives $d^\star$ and $m^\star$.

Proposition 1. *Under Definition 1, and Assumptions 1 and 2, the share of the population in the commuting, migration and home scenarios are given respectively by*

$$\begin{aligned} CO &= d^\star - \frac{r\tau}{2(1-t_d)\mu}\, m^{\star 2} \\ MI &= m^\star \left(1 - d^\star + \frac{r\tau}{2(1-t_d)\mu}\, m^\star\right) \\ HO &= (1-m^\star)(1-d^\star) \;=\; 1 - CO - MI \end{aligned}$$

Proof. Straightforward. □

farfetched. In 2007, only 22% of Parisians aged 20 and more were born in Paris (Moreau, Molinier, and Roger, 2017). In 2018, only 25%-32% of Londoners were born in London (Bosetti, 2018) and in 2021, 47% of the Luxembourg population did not have Luxembourg citizenship (STATEC, 2021).

Housing market

As in Zenou (2011), all housing is owned by absentee landlords. The housing supply is equal to the housing demand, i.e. to the share MI of the population living in the city. Landlords receive a rent R per unit of housing supplied but they also incur a maintenance cost $\epsilon\ MI^{1+\alpha}$, with ϵ, $\alpha \geq 0$, which is increasing and convex in the number of housing units supplied. The representative landlord therefore maximizes $R\ MI - \epsilon\ MI^{1+\alpha}$, which gives

$$(1+\alpha)\epsilon\ MI^{\alpha} = R \tag{13}$$

α is the elasticity of the rent with respect to housing units. In the extreme case in which $\alpha = 0$, R is a constant equal to ϵ and landlord profits $\alpha\epsilon MI^{1+\alpha}$ disappear (see for instance Kline and Moretti, 2014, for a similar housing representation).

3.2. **Steady state properties.**

We define the relative unemployment share as $u \equiv U_C/(U_C + U_O)$. The free entry condition (7) can be re-written as $\mathcal{G}(\theta, u) = 0$. Using equations (1), (2), (5) and (6), we also obtain the mobility condition $u = MI/(CO + MI)$. Using Proposition 1 and the rent equilibrium (13), we observe that the mobility condition only includes θ and u, and may therefore be written as $\mathcal{H}(\theta, u) = 0$.

Definition 2. *Under Assumptions 1 and 2, a steady state equilibrium is a pair* $\{\theta, u\} \in]0, +\infty[\ \times [0, 1]$ *satisfying both the free entry and mobility conditions*

$$\begin{aligned} \mathcal{G}(\theta, u) &= 0 \\ \mathcal{H}(\theta, u) &= 0 \end{aligned}$$

Note that when $h_C = h_O$, the free entry condition simplifies to $\mathcal{G}(\theta) = 0$.

Proposition 2. *From the free entry condition* $\mathcal{G}(\theta, u) = 0$*, we have* $h_C < h_O \Leftrightarrow \frac{\mathrm{d}\theta}{\mathrm{d}u} > 0$*. From the mobility condition* $\mathcal{H}(\theta, u) = 0$*, we have* $\left.\frac{\partial u}{\partial t_d}\right|_{\theta,\, R=cst} < 0$ *and* $\left.\frac{\partial u}{\partial t_R}\right|_{\theta,\, R=cst} > 0$*.*

Proof. We apply the Implicit Function Theorem with $\frac{\partial \mathcal{G}(\theta,u)}{\partial \theta}\,\mathrm{d}\theta + \frac{\partial \mathcal{G}(\theta,u)}{\partial u}\,\mathrm{d}u = 0$ to prove the first part of the proposition. The second part is straightforward. □

Results from Proposition 2 are intuitive. When $h_C < h_O$, $w_C < w_O$ and $J_C > J_O$. Therefore, firms with a vacancy prefer to hire migrants. When the share of migrant job seekers increases, firms open more vacancies and labor market tightness θ rises. Mobility subsidy t_d increases $d^\star$ and hence reduces the share of migrant job seekers. In contrast, housing subsidy t_R increases $m^\star$ and hence the share of migrant job seekers. When rents are endogenous, they move positively with migration, which counteracts the effects of subsidies on mobility and housing (see Section 4 for an illustration).

Unlike the related literature, we have characterized the model equilibrium without assuming that the risk-free interest rate converges toward zero.[13] Nevertheless, to explore the $\mathcal{G}(\theta,u)-\mathcal{H}(\theta,u)$ equilibrium, we simplify equations by assuming $r\to 0$. Corollary 1 provides this steady state equilibrium, which we call $(DE_{r\to 0})$.

Corollary 1. *When $r\to 0$ and under Assumptions 1 and 2, the decentralized equilibrium without policy interventions ($t_R=t_d=0$) is*

$$(DE_{r\to 0})\quad\left\{\begin{array}{rcl} \mu\, CO &=& \frac{p(\theta)\gamma(y-h_O)}{\delta+\gamma p(\theta)} \\ (1+\alpha)\epsilon MI^{\alpha} &=& (h_C-h_O)+\frac{p(\theta)\gamma(y-h_C)}{\delta+\gamma p(\theta)} \\ a &=& (1-\gamma)\;q(\theta)\;\frac{(y-h_C)u+(y-h_O)(1-u)}{\delta+\gamma p(\theta)} \end{array}\right.$$

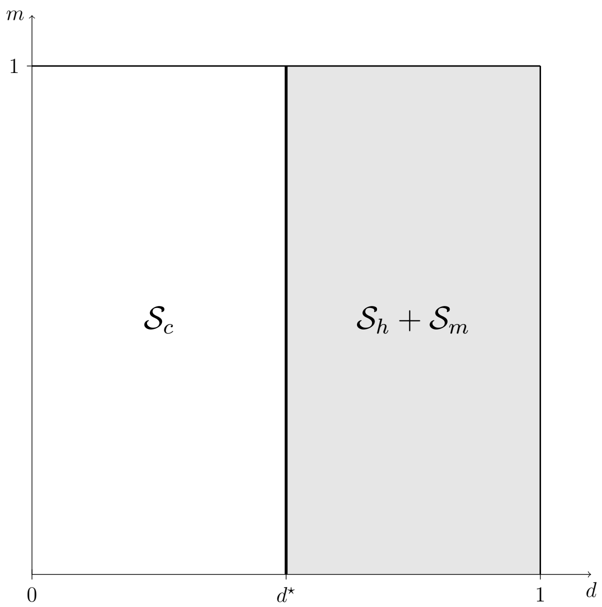
with $u=MI/(MI+CO)$, $p(\theta)=\bar{m}\theta^{1-\eta}$ and $q(\theta)=\bar{m}\theta^{-\eta}$.

Proof. We impose $r\to 0$ in equations (7) and (9) to (11) to obtain the system of equations $(DE_{r\to 0})$. □

When $r\to 0$, the transition no longer matters. The one-time migration cost τ becomes irrelevant as does m. The first two equations in Corollary 1 show that τ does not affect the CO and MI equilibrium. As a result, the split of the population into the migration, commuting and home scenarios simplifies, as shown in Figure 3. When $d\le d^\star=CO$, all individuals commute independently of their m. When $d>d^\star$, individuals either migrate or stay home. Again, the split between these two scenarios does not depend on m but

[13] Under this assumption, workers have no preference for the present, which facilitates the determination of the equilibrium as it implies that unemployed and employed individuals have the same expected utility (Zenou, 2009b; Lehmann, Montero Ledezma, and Van der Linden, 2016; Boitier, 2018).

FIGURE 3. Population distribution and choices when $r \to 0$



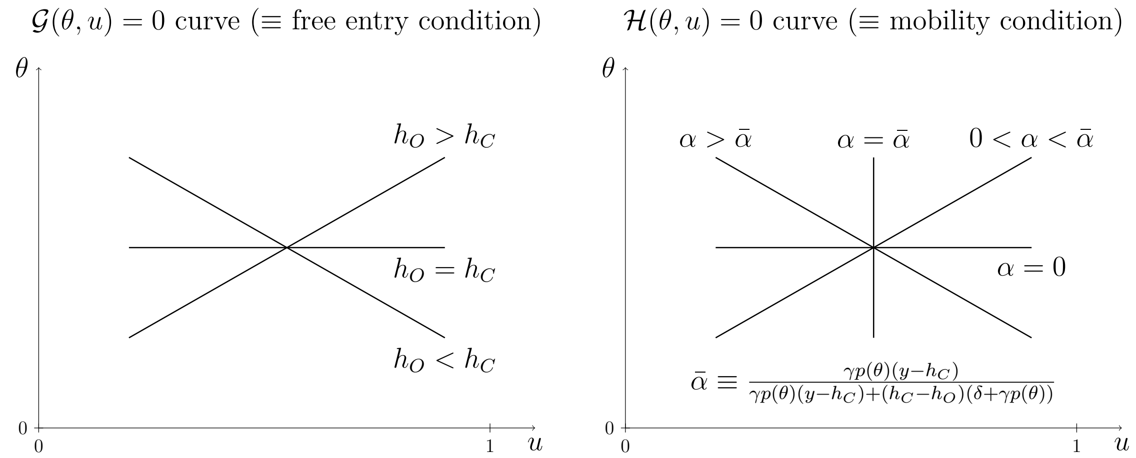
Notes. Population is distributed along d (distance between the city and the hometown) and m (psychological attachment to live in the hometown). The city is located at $d = 0$. $\mathcal{S}_c$ is the set of all (d, m) choosing the commuting scenario, $\mathcal{S}_h + \mathcal{S}_m$ is the set of all (d, m) choosing either the home or the migration scenarios. Corollary 1 gives $d^\star = CO$ and the split between $\mathcal{S}_h$ and $\mathcal{S}_m$.

only on aggregate variables. For instance, more migration increases rents until there is no longer an incentive to migrate.

We now use the simplified equilibrium from Corollary 1 to study existence and uniqueness. The last equation of the System ($DE_{r\to 0}$) is the $\mathcal{G}(\theta, u)$ equation, whereas the combination of the first two equations of the System ($DE_{r\to 0}$) through $u = MI/(MI + CO)$ yields the $\mathcal{H}(\theta, u)$ equation. Figure 4 plots these equilibrium curves.

The left panel shows the free entry condition $\mathcal{G}(\theta, u) = 0$. We know from Proposition 2 that the slope is positive when $h_O > h_C$ and negative when $h_O < h_C$. The right panel shows the mobility condition $\mathcal{H}(\theta, u) = 0$. When α is low (resp. high), that is when the elasticity of the rent to migration is moderate (resp. strong), an increase in θ raises the probability of finding a job and shifts the job seeker composition towards migration (resp. commuting). As a result, the slope of the curve is positive (resp. negative). When α is equal to an intermediate value $\bar{\alpha}$, unemployment composition u does not react to a change in θ. Note that when $h_O = h_C$, $\bar{\alpha} = 1$. Figure 4 implies that the existence and

FIGURE 4. Equilibrium curves when $r \to 0$



Notes. This is a schematic representation, since most $\mathcal{G}(\theta, u)$ and $\mathcal{H}(\theta, u)$ curves are non linear (linear exceptions are the $h_O = h_C$, $\alpha = 0$ and $\alpha = \bar{\alpha}$ curves). These results follow immediately from applying the Implicit Function Theorem to the $\mathcal{G}(\theta, u) = 0$ and $\mathcal{H}(\theta, u) = 0$ curves. The right panel displays the general case when $\bar{\alpha} > 0$. Under extreme conditions ($h_O >> h_C$), $\bar{\alpha}$ become negative. See Appendix C. All cases presented here can also be obtained numerically.

uniqueness of an equilibrium is not always granted. For instance, when $h_O = h_C$ and $\alpha = 0$ (exogenous rent), the two curves are horizontal and there is either no equilibrium or an infinity of equilibrium (u is indeterminate). In the next section, we verify whether the decentralized equilibrium is efficient and, if not, how policy interventions might restore efficiency.

3.3. **Central planner equilibrium.**

As in Wasmer and Zenou (2002), the social welfare function is defined as the discounted sum of aggregate net output every period, with instantaneous net output equal to all production (including the values for leisure) minus all costs (vacancy, housing, commuting and migration). Welfare is therefore

$$W = \int_0^\infty e^{-rt} \Big\{ y(N_C + N_O) + h_C U_C + h_O(1 - N_C - N_O - U_C) \\ -aV - \epsilon(N_C + U_C)^{1+\alpha} - \mathcal{C}_c - \mathcal{C}_m \Big\} \,\mathrm{d}t$$

where $\mathcal{C}_c$ and $\mathcal{C}_m$ are the commuting and migration costs. The central planner must choose the level of labor market tightness θ but also decide who commutes, migrates or stays home.[14] The division between commuting and home does not depend on m (vertical line) and the division between home and migration does not depend on d (horizontal line). We then assume that the curve dividing commuting and migration is linear, i.e. the frontier between $\mathcal{S}_c$ and $\mathcal{S}_m$ is linear

Assumption 3. *The central planner curve dividing commuting and migration is linear.*

As a consequence of Assumptions 2 and 3, the share of the population in the commuting and migration scenarios, as well as the commuting costs, are fully determined by the triplet $\{d_0, d^\star, m^\star\}$: $CO(d_0, d^\star, m^\star)$, $MI(d_0, d^\star, m^\star)$ and $\mathcal{C}_c(d_0, d^\star, m^\star)$. There is a one-time migration cost which only appears during the transition. This takes the form $\mathcal{C}_m(d_0, d^\star, m^\star, \dot{d}_0, \dot{d}^\star, \dot{m}^\star)$ with $\mathcal{C}_m(.,.,.,0,0,0) = 0$. Appendix D provides the expressions for CO, MI, $\mathcal{C}_c$ and $\mathcal{C}_m$, under the Assumptions 2 and 3. We also observe that unemployment is

$$\begin{aligned} U_C &= MI(d_0, d^\star, m^\star) - N_C \\ U_O &= CO(d_0, d^\star, m^\star) - N_O \end{aligned}$$

and that the law of motion for employment is

$$\begin{aligned} \dot{N}_C &= U_C\, \bar{m}\theta^{1-\eta} - \delta N_C \\ \dot{N}_O &= U_O\, \bar{m}\theta^{1-\eta} - \delta N_O \end{aligned}$$

where $\bar{m}$ is the matching efficiency parameter and $0 < \eta < 1$ is the elasticity of matches with respect to unemployment. Finally, we can define

$$\begin{aligned} C_{d_0} &= \dot{d}_0 \\ C_d &= \dot{d}^\star \\ C_m &= \dot{m}^\star \end{aligned}$$

[14]For the sake of convenience, we drop the superscript *cp* from all the variables in this section.

We are now able to write the Hamiltonian function used to solve the central planner problem

$$
\begin{aligned}
H &= e^{-rt}\{y(N_C+N_O)+h_C(MI(d_0,d^\star,m^\star)-N_C)+h_O(1-MI(d_0,d^\star,m^\star)-N_O) \\
&- a\theta(MI(d_0,d^\star,m^\star)+CO(d_0,d^\star,m^\star)-(N_C+N_O))-\epsilon(MI(d_0,d^\star,m^\star))^{1+\alpha} \\
&- \mathcal{C}_c(d_0,d^\star,m^\star)-\mathcal{C}_m(d_0,d^\star,m^\star,C_{d_0},C_d,C_m) \\
&+ \lambda_c((MI(d_0,d^\star,m^\star)-N_C)\,\bar{m}\theta^{1-\eta}-\delta N_C) \\
&+ \lambda_r((CO(d_0,d^\star,m^\star)-N_O)\,\bar{m}\theta^{1-\eta}-\delta N_O) \\
&+ \lambda_{d_0}C_{d_0}+\lambda_d C_d+\lambda_m C_m\}
\end{aligned}
$$

where θ, C_{d_0}, C_d and C_m are control variables; N_C, N_O, d_0, $d^\star$ and $m^\star$ are state variables; and λ_c, λ_r, λ_{d_0}, λ_d and λ_m are the associated co-state variables.

Proposition 3. *Under Assumptions 2 and 3, the central planner solution at the steady state is*

$$
\begin{aligned}
a &= (1-\eta)\,q\,\frac{(y-h_C)u+(y-h_O)(1-u)}{r+\delta+\eta p} \\
\frac{r\tau m^\star}{3}+R &= \frac{\mu(d^\star+2d_0)}{3}+\frac{(r+\delta)(h_C-h_O)}{r+\delta+p} \\
r\tau m^\star\left(1-\frac{d_0}{3}-\frac{2d^\star}{3}\right) &= \frac{a\theta\eta}{1-\eta}(1-d^\star) \\
&+ \frac{p(h_C-h_O)}{(r+\delta+p)}\left(u\frac{d_0-d^\star}{2}-(1-u)\left(1-\frac{d_0+d^\star}{2}\right)\right) \\
&+ (h_C-h_O-R)\left(1-\frac{d_0+d^\star}{2}\right)+\frac{\mu}{6}(d^\star-d_0)(2d^\star+d_0) \\
\frac{r\tau\left(m^\star\right)^2}{3} &= -\frac{a\theta\eta}{1-\eta}(1-m^\star) \\
&+ \frac{p(h_O-h_C)}{(r+\delta+p)}\left(u\left(1-\frac{m^\star}{2}\right)+(1-u)\frac{m^\star}{2}\right) \\
&+ (h_C-h_O-R)\frac{m^\star}{2}-\frac{\mu m^\star}{6}(4d^\star-d_0)+\mu d^\star
\end{aligned}
$$

where, to simplify the exposition, we define $u\equiv u(d_0,d^\star,m^\star)=U_C/(U_C+U_O)=MI(d_0,d^\star,m^\star)/(MI(d_0,d^\star,m^\star)+CO(d_0,d^\star,m^\star))$, $p\equiv p(\theta)=\bar{m}\theta^{1-\eta}$, $q\equiv q(\theta)=\bar{m}\theta^{-\eta}$ *and* $R\equiv R(d_0,d^\star,m^\star)=(1+\alpha)\epsilon MI(d_0,d^\star,m^\star)^\alpha$.

Proof. See Appendix D. □

The central planner solution in Proposition 3 is characterized by a system of four equations with the four unknowns $\{\theta, d_0, d^\star, m^\star\}$. The first equation is the socially optimal job creation condition and corresponds to the free entry condition (7) from the decentralized equilibrium when (i) $\eta = \gamma$ (the so-called Hosios 1990 condition), (ii) there is no policy intervention in the decentralized economy, and (iii) u are similar in both the decentralized and the centralized equilibrium. In general, the Hosios condition is therefore a necessary but not a sufficient condition. The last three equations in Proposition 3 determine $\{d_0, d^\star, m^\star\}$, which in turn fix u. Their counterparts in the decentralized equilibrium are given by Definition 1 and equation (12). We observe that they are not equivalent (except if $\eta = \gamma$ and $h_C = h_O$). In the next section, we compare the decentralized versus the centralized equilibrium.

3.4. **Comparing centralized and decentralized equilibrium.**

Proposition 4. *When the value for leisure is the same everywhere ($h_C = h_O$) and there is no policy intervention in the decentralized economy ($t_d = t_R = 0$), then the Hosios condition $\eta = \gamma$ is a sufficient and necessary condition to guarantee that the decentralized equilibrium described in Definition 2 is efficient, i.e. is equivalent to the central planner equilibrium described in Proposition 3.*

Proof. See Appendix E. □

When $h_C = h_O$, migrants and commuters receive the same wage and the firm is indifferent between hiring workers from different locations. The only externality is therefore the standard search and matching externality, which can be internalized through the Hosios condition. When $h_C \neq h_O$, a second composition externality arises. Wages are different for migrants and commuters, and firms' expected profit (and the number of vacancies) depends on the composition of job seekers, i.e. the share of migrants and commuters. However, in the decentralized equilibrium, individuals consider the probability to find a job as exogenous, failing to realize that firms will respond to the moving decision. Compared to the central planner decision, the decentralized equilibrium may therefore produce over-migration or under-migration (or equivalently under-commuting or over-commuting). For instance, let us assume a higher value for leisure in the city outskirts ($h_O > h_C$). Commuters will obtain a higher wage than migrants, leading individuals to prefer commuting to migration (Appendix C shows that in this case $I_O^E - I_O^U > I_C^E - I_C^U$).

$U_C/(U_C + U_O)$ decreases and this composition effect reduces firms' expected surplus, depressing vacancy openings (see equation 7). The equilibrium is not efficient and over-commuting (or under-migration) could be corrected through policy interventions. The opposite reasoning holds when the value for leisure is higher in the city ($h_O < h_C$).

To render more explicit the difference between the decentralized and the centralized equilibrium, we assume $r \to 0$. As already explained in Section 3.2 and Figure 3, in this zero-discount economy, the transition no longer matters and the one-time migration cost τ becomes irrelevant. This simplifies Proposition 3 to

Corollary 2. *When $r \to 0$ and under the Assumptions 2 and 3, the centralized equilibrium is*

$$(CP_{r\to 0}) \left\{ \begin{array}{rcl} \mu\, CO & = & \frac{p(\theta)\eta(y-h_O)}{\delta+\eta p(\theta)} + \frac{\delta(1-\eta)p(\theta)(h_C-h_O)u}{(\delta+p(\theta))(\delta+\eta p(\theta))} \\ (1+\alpha)\epsilon MI^{\alpha} & = & (h_C-h_O) + \frac{p(\theta)\eta(y-h_C)}{\delta+\eta p(\theta)} - \frac{\delta(1-\eta)p(\theta)(h_C-h_O)(1-u)}{(\delta+p(\theta))(\delta+\eta p(\theta))} \\ a & = & (1-\eta)\, q(\theta)\, \frac{(y-h_C)u+(y-h_O)(1-u)}{\delta+\eta p(\theta)} \end{array} \right.$$

with $u = MI/(MI + CO)$, $p(\theta) = \bar{m}\theta^{1-\eta}$ and $q(\theta) = \bar{m}\theta^{-\eta}$.

Proof. We impose $r \to 0$ in Proposition 3 to obtain the system of equations ($CP_{r\to 0}$). □

As before, we immediately see that when $\eta = \gamma$ (Hosios condition) and $h_C = h_O$ (composition condition), the System ($DE_{r\to 0}$) from Corollary 1 is equivalent to the System ($CP_{r\to 0}$) from Corollary 2, and the decentralized equilibrium is therefore efficient. When only the Hosios condition is met ($\eta = \gamma$ and $h_C \neq h_O$), comparing the first two equations of Systems ($DE_{r\to 0}$) and ($CP_{r\to 0}$) shows that when $h_C > h_O$, $CO^{cp} > CO^{de}$ and $MI^{cp} < MI^{de}$, which gives $u^{cp} < u^{de}$. Then the free entry condition (last equation of Systems $DE_{r\to 0}$ and $CP_{r\to 0}$) implies $\theta^{cp} > \theta^{de}$ (see Proposition 2): the decentralized equilibrium generates a bias towards migration, which results in under-investment in vacancies. Using a similar reasoning, when $h_C < h_O$, the decentralized equilibrium generates a bias towards commuting, which also results in under-investment in vacancies. In conclusion, a violation of the Hosios condition may lead to either over- or under-investment in vacancies, while a violation of the composition condition always leads to under-investment in vacancies.

4. Empirical analysis

In this section, we calibrate the model to the Paris metropolitan area and show that it can reproduce many of the gradients observed in the data. We then compare the decentralized equilibrium to the central planner solution and investigate the role of policy interventions and housing to reduce inefficiencies.

4.1. **Calibration and fit to the data.**

We set the time period to one month. We use a yearly discount rate of 4%, which implies a monthly discount rate $r = 0.33\%$. Cahuc, Postel-Vinay, and Robin (2006) use French administrative data to estimate bargaining power between 0 and 20% for low-skilled workers and between 20% and 40% for high-skilled workers. Hence, we choose a bargaining power of $\gamma = 1/4$, which is within the range of credible values reported by the authors. As usual in the literature (Lehmann, Montero Ledezma, and Van der Linden, 2016), we assume that the Hosios condition holds and impose $\eta = \gamma$. We normalize production to $y = 10$. Cahuc, Carcillo, and Le Barbanchon (2019) use French data to estimate a high value for leisure, equal to 94% of output (see also Hagedorn and Manovskii, 2008, for a similar estimate on US data). Therefore, we set the value for leisure in the city to 90% of production ($h_C = 0.9 \times y$). Hairault, Le Barbanchon, and Sopraseuth (2015) estimate from French labor force surveys that the monthly job finding rate is $p = 7.5\%$. This value implies an average duration of unemployment of approximately 13.3 months, which is close to the value reported by OECD for the period 2010-2017 (14.3 months). The average unemployment rate in the Paris region for the period 2010-2020 was 8.25% according to INSEE. Ignoring movements in and out of the labor force, the expression for the steady state value of unemployment pins down the monthly job separation rate at $\delta = 0.7\%$.[15] Cahuc, Carcillo, and Le Barbanchon (2019) report that the average duration of an unfilled vacancy in France is 2 months, which implies $q = \frac{1}{2}$. The values of p and q determine the vacancy cost parameter a and the matching efficiency parameter $\bar{m}$.

We calibrate the model so that the city center corresponds to the city of Paris and the city outskirts correspond to rest of the Ile-de-France region. Using data from INSEE, we calculate a share of commuters $CO/(CO + MI) = U_O/(U_C + U_O) = N_O/(N_C + N_O) = 0.59$.[16] There are about 12 million inhabitants in the region Ile-de-France, with

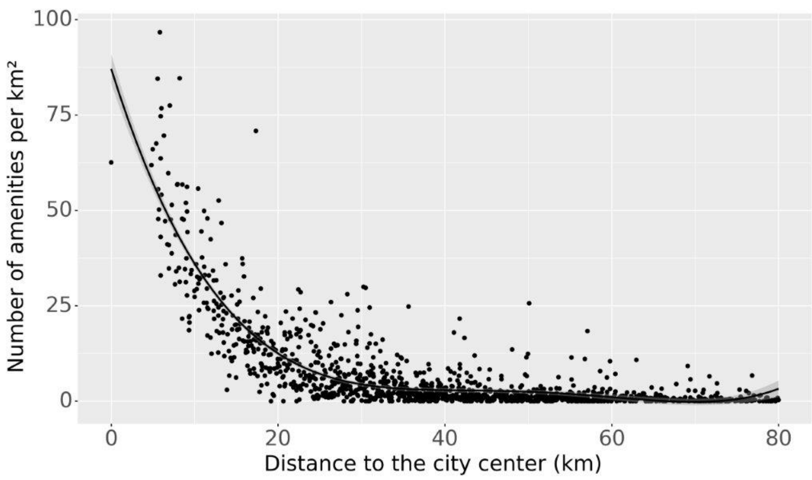
[15]At the steady state, the unemployment rates are the same: $U_C^r \equiv \frac{U_C}{U_C+N_C} = \frac{\delta}{\delta+p} = \frac{U_O}{U_O+N_O} \equiv U_O^r$.

[16]Using the database on professional mobility from the INSEE, we find approximately 1 751 600 jobs are located in the city of Paris in 2017, of which only 717 500 are held by people living in Paris.

approximately 2 million residents in the inner city of Paris, which implies $MI = 1/6$.[17] The values for the share of commuters and MI pin down the values for μ and τ.

In most European cities, amenities (museums, parks, fine architecture, etc.) are concentrated in the city center, which pushes up the value for leisure (see for instance Brueckner, Thisse, and Zenou, 1999b; Koster and Rouwendal, 2017).[18] Figure 5 illustrates amenities distribution in the Paris metropolitan area: amenities are highly concentrated in the Paris city center but decay quickly as the distance from the center increases. We set $\frac{h_C}{h_O} = 4$. Within our framework, amenities raise the value for leisure, which in turn raises the reservation wage. Hence, this calibration will yield a wage premium for those living in the city (see Figure 6 below).

Figure 5. Amenities distribution



Notes. Authors' calculations based on the database "base permanente des équipements" from INSEE, which contains geolocalised observations on amenities (for instance, theaters, parks, swimming pools, libraries).

The last two parameters to calibrate relate to the housing market. The parameter α is the elasticity of rent to population in the city. Combes, Duranton, and Gobillon (2019) find an elasticity for Paris of $\alpha = 0.38$. Since we normalize rent in the city

[17]In the city outskirts, the shares of the population aged 0 to 14 years and 60+ are both larger (about 5-6 percentage points), which is consistent with the model's assumption that people move to the city to work.

[18]Brueckner, Thisse, and Zenou (1999a) also note that the reverse is often observed in several US cities with fancy outskirts and dull centers. The authors show that differences in the amenity patterns between European and American cities can explain the location of rich and poor individuals.

outskirts to zero in the model, the rent R actually represents the difference between the average rent R_C in the city of Paris and the average rent R_O in surrounding areas. Empirically, the rent-to-earnings ratio is approximately equal to 37% in the city of Paris ($R_C/w_C = 0.37$). We also observe that rents are approximately three times higher in the city of Paris ($R_C/R_O = 3$, see Figure 10 in Appendix A). This allows us to derive $R = 2.34$ and calibrate ϵ accordingly. Table 1 summarizes the calibration, with all policy parameters set to zero.

Table 1. Calibrated Parameters

Symbol	r	δ	γ	η	a	$\bar{m}$	y	h_C	h_O	ϵ	α	μ	τ	t_d	t_R
Value	0.003	0.007	0.25	0.25	65	0.31	10	9	2.25	3.52	0.38	19	7680	0	0

Our calibration implies that the maximum migration cost an individual is willing to pay represents 12.5 years of labor income ($\tau m^\star/w_C = 150$), which is close to the relocation costs of 15.6 years of annual income reported by Kennan and Walker (2011). We also check how far this calibration produces empirically relevant population and price gradients. To take the model to the data, we need to express geographical distances in discrete form. Indeed, we assumed that the city is a point at the origin, which implies an infinite population density. We thus split the unit distance into n intervals of equal length $1/n$. Then, we project the distance dimension into a unit disc. Each bin $i \in \{1, ...n\}$ of length $1/n$ therefore becomes a ring of area $\pi(2i-1)/(2n^2)$.[19] Each individual now lives on a ring with a strictly positive area, which allows us to calculate non-degenerate population and job densities and to move from a 1-dimensional to a 2-dimensional spatial representation. Finally, we assume that the city is constituted of $j < n$ bins and we split the population and the jobs from the city equally into the first j bins.

We also need to rescale the distance (between 0 and 1 in the model), jobs and population (unit mass in the model), and output (production $y = 10$ per job in the model). We rescale the distance between 0 and 80 km. A circle with a radius of 80 km centered in Paris contains 99.7% of the population living in the Ile-de-France region. In this discretization exercise, we take $n = 50$ bins and $j = 5$ 'city' bins. The length of each bin therefore corresponds to 1.6 km and the inner city has a radius of 8 km. To convert jobs

[19] We hence get $\sum_{i=1}^{n} \pi(2i-1)/(2n^2) = \pi/2$.

and workers into their empirical counterparts, we multiply the gradient of job density by a scaling factor s_1, where s_1 is chosen so that the maximum value of job density in the model is equal to the maximum value of job density in the data. Then, to obtain the model's counterpart of the empirical population density, we also multiply the model's population density by the same factor s_1.

We next convert rents and wages from the model into euros. To do so, we multiply wages by a scaling factor s_2, chosen so that the maximum wage in the model is equal to the maximum of the average monthly wage of individuals resident in the Paris region. Then, the same scaling factor s_2 can be used to express rents in euros. Because we set the rent in the city R_C to be 37% of the wage in the city w_C (see above), the rent in the city in terms of euros is given by $s_2 \times 0.37 \times w_C$. Using the assumption that the ratio $\frac{R_C}{R_O} = 3$ (see above), we can also infer the rent in the city outskirts R_O in terms of euros. By construction, the scaling factors s_1 and s_2 ensure that the model matches perfectly the maximum job density and the maximum wage observed in the data (first and last panels in Figure 6, respectively). However, other values are not constrained and the degree to which the model can reproduce the different gradients in Figure 6 is a valid test of the model's ability to match the data.

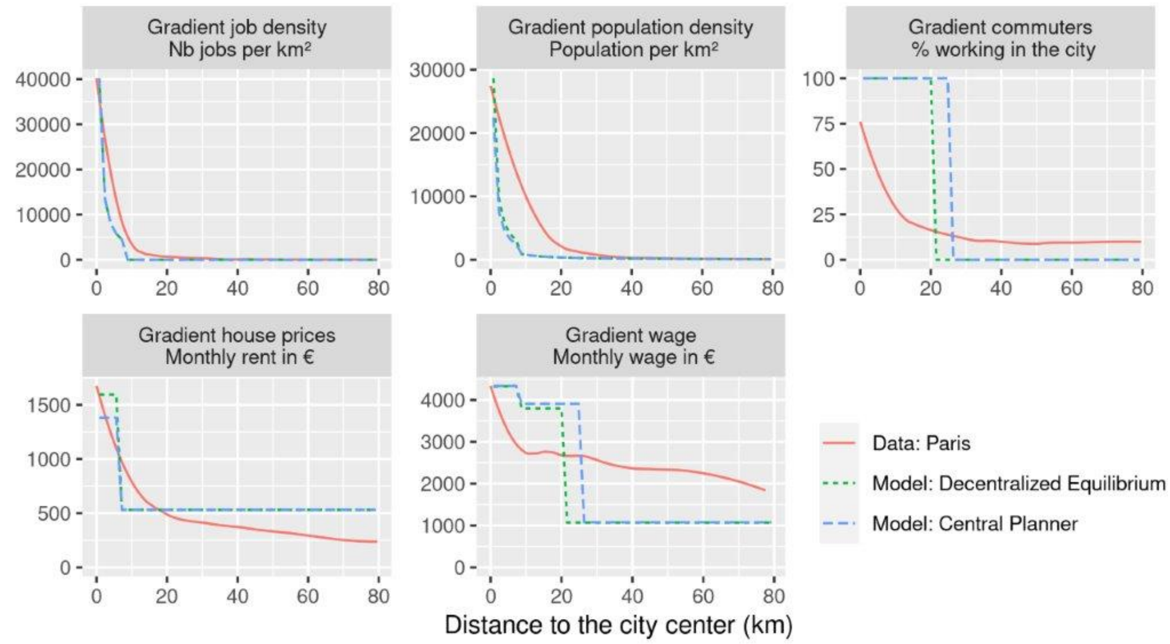
The model performs well matching the gradients for job and population density, even though population density is steeper than in the data. In terms of matching the flow of commuters, the model captures well the qualitative feature that the share of residents working in the city sharply drops when the distance to the city center is more than 20 km. One limitation is that our model predicts a corner solution (100% working in the city below a distance threshold of 22 km; 0% above it).[20]

Our model is also unable to match the continuous nature of the gradient for rents, because we assumed that only two levels of rent were possible. However, it does a good job at reproducing the sharp decrease in rents as the distance from the city center increases. The interpretation of the monthly wage is more challenging. Moving from left to right, the last panel in Figure 6 shows the wage in the city, the wage of commuters, and the income of non-commuters living in the city outskirts. The latter's wage is

[20]A possible extension of our model would be to add idiosyncratic preference shocks over locations, as in Kennan and Walker (2011), to match the continuity of the gradient of commuters.

unobserved and we assume that non-commuters living in city outskirts operate a low-level production technology earning a monthly income equal to the value for leisure. We have no information of the wage difference between commuters and residents. Instead, we observe the average wage as a function of distance, which is a mixture of the wage for commuters working in the city and non-commuters working locally. In our model, all jobs are located in the city. With these caveats in mind, the model predicts approximately a 20% wage premium for workers in the city compared to commuters, whereas the data suggests a $30-60\%$ premium.[21]

Figure 6. Selected gradients from model and data



Notes. Authors' calculations based on data from INSEE, ONS, STATEC, LISER, IGSS, Eurostat and Seloger.com. See more details in the Appendix A.

4.2. **Decentralized equilibrium versus central planner.**

Since $h_C \neq h_O$, we know from Proposition 4 that the decentralized equilibrium (DE) equilibrium is not efficient. Figure 6 illustrates the differences between the DE equilibrium and the central planner solution (CP). Table 2 provides more details. The first

[21]One extension of our work could be to consider two groups of workers (skilled and unskilled), with different preferences over amenities, as in Brueckner, Thisse, and Zenou (1999b). If skilled workers (with higher wages) prefer living in the city to enjoy a higher level of amenities, the model would generate a steeper income gradient.

line DE displays the decentralized equilibrium and the second line CP shows the central planner solution. Because $h_C > h_O$, we see that the central planner chooses a higher $d^\star$ and a lower $m^\star$, i.e. she prefers more commuting and less migration. As a result, the central planner equilibrium displays more vacancies and more production.

Table 2. Equilibrium

	$d^\star$	$m^\star$	d_0	CO	MI	HO	V	Y
Decentralized equilibrium (DE)	0.26	0.19	0.01	0.24	0.17	0.59	0.01	3.7
Central planner (CP)	+3.5	-3.0	+7.5	+4.2	-3.5	-0.8	+6.9	+2.8
Decentralized equilibrium (policy mix)	+3.5	-1.5	+3.2	+3.7	-1.9	-1.8	+7.9	+5.0

Notes. $d^\star$, $m^\star$ and d_0 divides the population as shown in Figure 2; CO, MI and $HO = 1-CO-MI$ are the shares of commuters, migrants and home stayers, respectively; $Y = (N_C + N_O)y$ is gross output. The central planner equilibrium (CP) and the decentralized equilibrium (policy mix) are expressed in percentage point (all population variables) or percentage (V and Y) deviation from the decentralized equilibrium (DE) without policy.

Figure 7 compares the DE and CP equilibrium for different parameter values. The first row varies the value for leisure h_O in the city outskirts. From a central planner's point of view, a higher h_O reduces the firm surplus $S = (y-h_C)u+(y-h_O)(1-u)$, which decreases incentives to open vacancies and hence labor market tightness θ. Higher value for leisure in the city outskirts also leads to a substitution from work (migration or commuting scenarios) to the home scenario, and from migration to commuting (the unemployment value is higher in the city outskirts). As a result, u decreases. For the reasons already explained above, we see that the decentralized equilibrium generates over-migration and under-investment in vacancies when $h_O < h_C = 0.9$. When $h_O = h_C = 0.9$, the market and the planner equilibrium coincide.

The second row of Figure 7 considers variations in worker bargaining power γ. A change in γ does not modify the centralized equilibrium. In the decentralized equilibrium, a higher γ lowers the firm's surplus and therefore θ. Less vacancies reduce the probability of finding a job, which induces more individuals to opt for the home scenario. However, a higher γ also induces an opposite effect through higher wages. The resulting effect on u is ambiguous.

In the third row of Figure 7, we consider changes in the discount rate r. A higher discount rate implies a higher preference for the present, which encourages commuting (costs are paid every period) instead of migration (costs are only in the present), and therefore lowers u. Because we calibrate $h_O < h_C$ (see Table 1), this commuting bias pushes up the surplus S, which favors the opening of vacancies and hence θ. Varying r has similar effects in the centralized and decentralized economies.

Finally, the last row of Figure 7 investigates the role of α. We observe that the housing maintenance cost $\epsilon MI^{1+\alpha}$ decreases in α because $0 \leq MI \leq 1$. A higher α therefore supports migration, which is detrimental to the opening of vacancies because $h_O < h_C$. As above, the effects of changing α are similar in the centralized and decentralized economies.

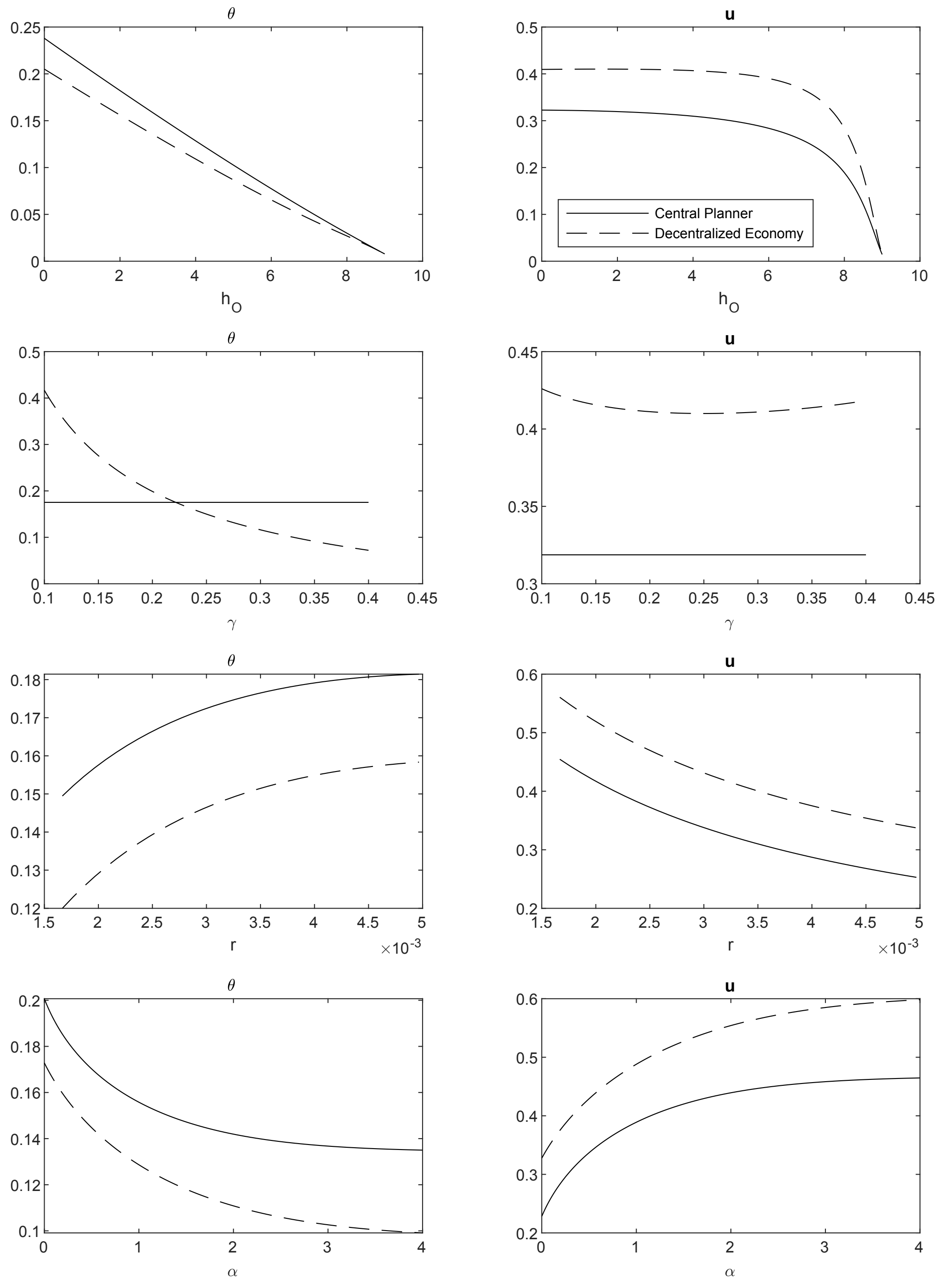
In Figure 8, we conduct the same type of exercise, but focusing on the policy parameters. Of course, these have no effect on the central planner equilibrium. The first line shows that an increase in the commuting subsidy rate t_d promotes commuting. Because $h_O < h_C$, more commuting also entails more vacancies. Therefore, a commuting subsidy closes part of the gap between the decentralized and the centralized equilibrium. A reduction in the housing subsidy t_R leads to similar results, as shown in the second row of the figure.

A key characteristic of most simulations above is that results depend on the relative calibration of h_C and h_O, i.e. the structural difference between migrants and commuters. We differentiate them through the value for leisure (or equivalently the level of amenities) but we would obtain similar conclusions with other criteria, such as productivity or bargaining power.

4.3. **Optimal policies.**

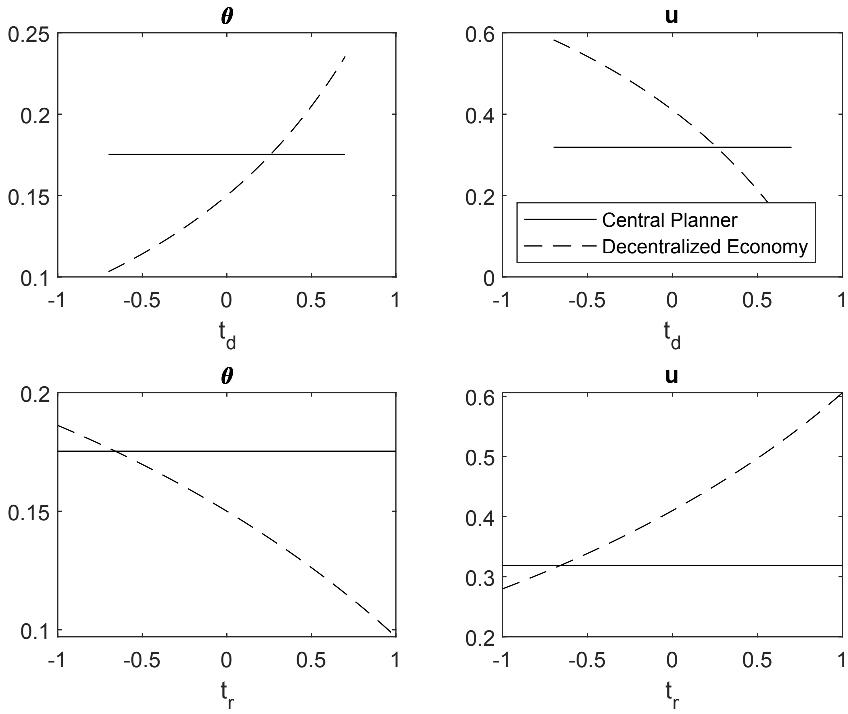
As a last experiment, we compute the policy mix (t_d, t_R), which (i) minimizes the distance between the decentralized equilibrium and the central planner equilibrium and (ii) satisfies the government budget constraint. The central planner equilibrium is determined by the population split among the migration, commuting and home scenarios, characterized by $\{d_0, d^\star, m^\star\}$ (see Figure 2), and the level of vacancies V. We therefore

FIGURE 7. Equilibrium sensitivity to selected structural parameters



Notes. $\theta = V/(U_C + U_O)$ is labor market tightness and $u = U_C/(U_C + U_O) = MI/(MI + CO)$ is the migration share. h_O is the value for leisure in the city outskirts, γ is worker bargaining power, r is the discount rate and α is the rent elasticity to migration.

FIGURE 8. Equilibrium sensitivity to policy parameters



Notes. $\theta = V/(U_C + U_O)$ is labor market tightness and $u = U_C/(U_C + U_O) = MI/(MI + CO)$ is the migration share. t_d is the mobility subsidy rate and t_R is the housing subsidy rate.

define the distance between the decentralized and the centralized equilibrium (denoted by the superscript *cp*) as

$$D = |\, d_0 - d_0^{cp}| + |\, d^\star - (d^\star)^{cp}| + |\, m^\star - (m^\star)^{cp}| + \left|\frac{V - V^{cp}}{V^{cp}}\right|$$

The government budget constraint is

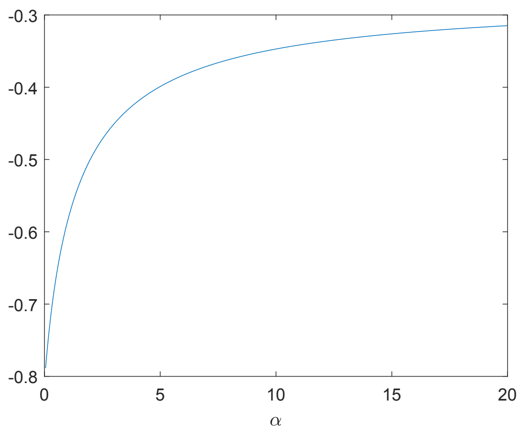
$$0 = t_d\, \mathcal{C}_c + t_R\, R\, MI \tag{14}$$

where $\mathcal{C}_c$ is total commuting costs as defined in Appendix D. The last line in Table 2 shows the minimization results. Since the optimization problem features two instruments, one constraint and four targets, we cannot fully recover the centralized equilibrium. There is therefore no 'divine coincidence' in this exercise. The optimal policy mix is $(t_d, t_R) = (9.2\%, -22.2\%)$. Since we have over-migration in the decentralized equilibrium, the optimal policy mix subsidizes mobility policy through a housing tax. These results are robust to alternative distance definitions and weights. Our results are qualitatively in line with some aspects of the policies already in place in the Paris region.

In 2017, users of the public transport system only paid 22% of the total cost of their transport (Rapoport, Carrez, Savary, Quinet, Pélissier, Crozet, Leurent, and Mirabel, 2018). The rest was financed by firms (41%), tax-payers (34%) and advertising contracts (3%). As of 2021, rent income is subject to a social security levy of 17.2% and an income tax, with marginal tax rates ranging between 11 and 45%.

Finally, we note that the effectiveness of public policy also depends on the response of the housing market. The mobility subsidy suggested above aims to reduce migration. When the elasticity of rents to migration is important (high α), the policy will generate a severe decline in rents. This is counterproductive, as it makes the city attractive as a place to live undermining the initial goal of the policy, which becomes ineffective. Figure 9 illustrates this interaction between policy effectiveness and the housing market. We observe that when α increases, there is a decline in the absolute value of the elasticity of the migration share $u = MI/(MI + CO)$ with respect to a mobility subsidy financed through a tax on housing.

FIGURE 9. Elasticity of u with respect to t_d, for alternative values of α



Notes. $u = U_C/(U_C + U_O) = MI/(MI + CO)$ is the migration share, t_d is the mobility subsidy rate and α is the elasticity of rents with respect to migration. The chart displays $\Delta u/\Delta t_d$, with $\Delta t_d = 0.10$ under the budget constraint (14) (i.e. the housing subsidy/tax t_R adjusts). When changing α, we modify ϵ accordingly to keep an unchanged initial decentralized equilibrium (first line DE in Table 2).

5. Conclusion

In this paper, we develop an urban search-and-matching model which differs from the existing literature because of (i) an elastic housing supply and (ii) a strictly positive but finite one-time relocation cost. We show that the market equilibrium divides the population into migrants, commuters and home stayers, and performs quite well matching population gradients observed in data. However, this market equilibrium is usually inefficient and appropriate policy interventions may help to correct these inefficiencies.

To obtain theoretical results, we rely on several simplifying assumptions. Future work could relax these assumptions at the cost of lower analytical tractability. For instance, housing supply currently only exists in the city. Introducing housing supply also in the city outskirts would create an endogenous rent gradient, which will probably make policy interventions less effective. Similarly, our model only considers residential property. Adding a commercial property market (firms and households compete for space) would also dampen the effects of policies. Finally, so far the treatment of home stayers is quite rudimentary (they just benefit from the same value for leisure as unemployed commuters) and probably deserves a deeper analysis. More generally, moving from a theoretical analysis to a numerical approach would allow a much more detailed representation of the urban structure.

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Appendix A. Data sources and data wrangling

Table 3. Main metropolitan areas in EU and North-America

		Population (*millions*)	Area (km^2)
1.	New-York (Greater)	19.961	23 880
2.	Los Angeles (Greater)	17.914	83 882
3.	Paris	12.915	17 584
4.	London	12.435	6 968
5.	Chicago	9.499	18 935
6.	Washington (Greater)	9.115	23 799
	Luxembourg(*)	11.619	65 406

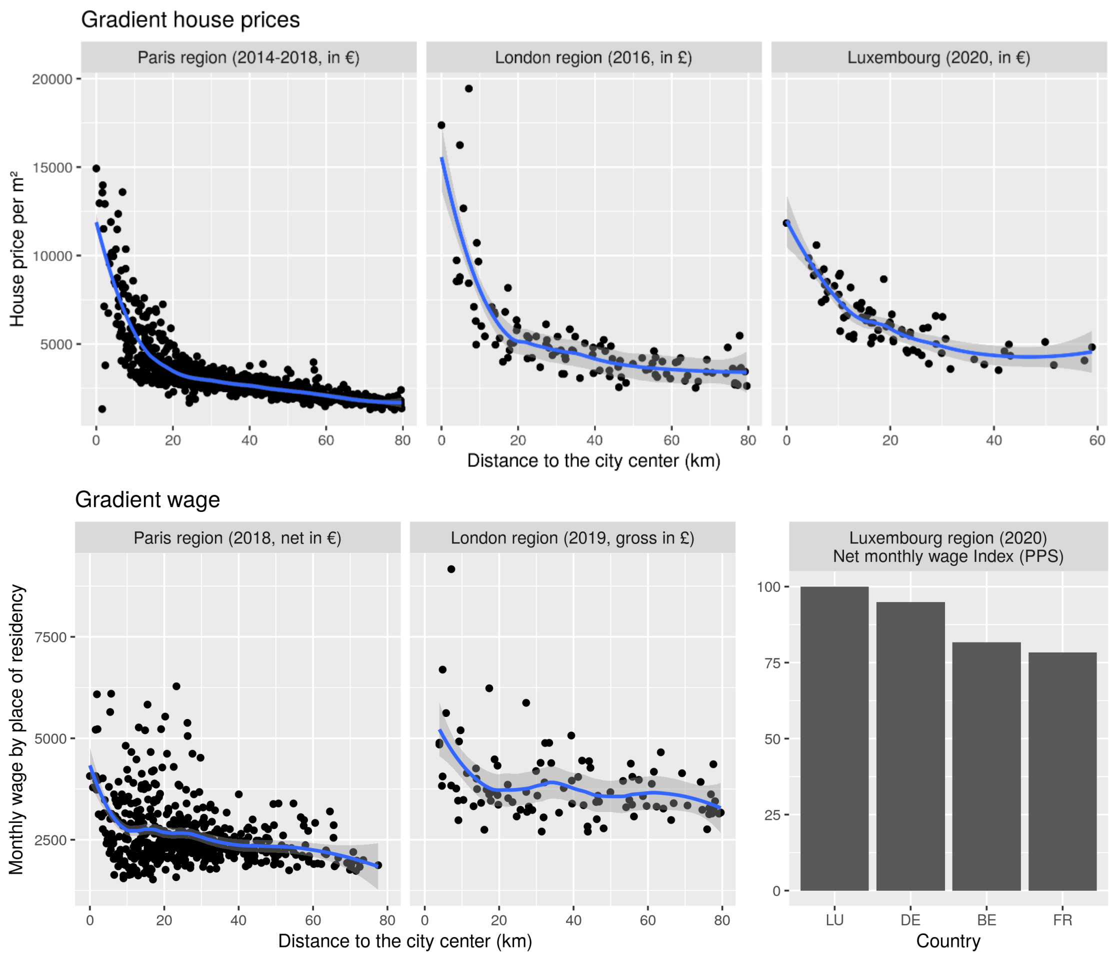
Notes. Data for 2018. Sources: `https://stats.oecd.org` for the ranked cities and `https://www.grande-region.lu` for Luxembourg. (*) Luxembourg includes the Grand Duchy of Luxembourg and its catchment area (Grande Région): Saarland and Rheinland-Pfalz (in Germany), the departments of the former administrative region Lorraine (in France) and Wallonia (in Belgium).

In Figures 1 and 10, the kilometer zero (km 0) is defined as the center of the first Arrondissement for Paris, as the center of the City for London, and as the center of Luxembourg City for Luxembourg. Blue lines represent smoothed conditional means estimated using local polynomial regressions. Regarding the commuting flows in Figure 1 (third line), workers are counted as working in the city if they work in the city of Paris, the London region or the country of Luxembourg. In Figure 10, the monthly wage index (bottom-right plot) is based on series expressed in Purchasing Power Standards (PPS), which controls for differences in the cost of living across countries. The index is normalized to 100 for Luxembourg. The underlying series of net monthly wages is for a full-time single worker without children, earning an average wage. See below for more details on data sources. To produce all the gradients, we combine several public datasets. We describe hereafter the data sources and explain the data preparation steps.

A.1. **Paris.** Observations on the housing market for the Paris regions are from the DVF database[22], which contains information on all housing transactions in France, starting in 2014. We exclude transactions that do not involve apartments and houses. Data

[22] `https://files.data.gouv.fr/geo-dvf/latest/csv/`

FIGURE 10. House prices and wages spatial distribution



Notes. Authors' calculations based on data from INSEE, ONS, STATEC, LISER, IGSS and Eurostat. See Appendix A for more details.

on prices and areas appear to be quite noisy. Hence, we trim the sample by dropping observations that are in the top and lowest 2% percentiles in terms of price per square meter. Then we calculate the median price per square meter at the municipality level for the period 2014-2018. Data on average net hourly wage is only available at the municipality level for municipalities with more than 2000 inhabitants.[23] We calculate an approximate monthly net wage using the average number of hours worked in France.[24] Data on population at the municipality level is from INSEE.[25] We calculate the share

[23] https://www.insee.fr/fr/statistiques/2021266

[24] https://www.insee.fr/fr/statistiques/4501612?sommaire=4504425

[25] https://www.insee.fr/fr/statistiques/4515539?sommaire=4516122

of the employed workers working in the city of Paris (1st-20th Arrondissement) at the municipality level using INSEE database on commuting for the latest year available.[26] We calculate the total number of jobs at the municipality level by summing over the flows of workers commuting to a given destination.

A.2. **London.** Observations for the housing market in London from the Office for National Statistics (ONS). We use the series on the average cost of property sold in 2016 at the local authority level.[27] To geolocalise observations, we merge the series with a shape file of UK local authorities from ONS.[28] Data on labor earnings at the local authority level is from ONS.[29] We use the series on annual gross pay in 2019 for a for full-time employee. The monthly gross labor earnings is obtained by dividing the yearly value by twelve. Data on population at the local authority level is also from ONS.[30] We calculate the share of the employed workers working in the region of London using the Place of Residence by Place of Work dataset from ONS.[31] We calculate the total number of jobs at the local authority level by summing over the flows of workers.

A.3. **Luxembourg.** Observations for the housing market in Luxembourg are from LISER Observatoire de l'Habitat.[32] We use the series on advertised sale prices for houses and apartments for the year 2020.[33] We merge the series on house prices with a shape file delimiting the Luxembourgish municipalities from the Administration du cadastre et de la topographie, which allows us to geolocalise observations.[34] Data on median monthly

[26] `https://www.insee.fr/fr/statistiques/4509353`

[27] `http://webarchive.nationalarchives.gov.uk/20170726163612///visual.ons.gov.uk/wp-content/uploads/2017/10/map.csv`

[28] `https://geoportal.statistics.gov.uk/datasets/local-authority-districts-december-2019-boundaries-uk-bfc`

[29] `https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/earningsandworkinghours/datasets/placeofresidencebylocalauthorityashetable8`

[30] `https://www.ons.gov.uk/peoplepopulationandcommunity/populationandmigration/populationestimates/datasets/populationestimatesforukenglandandwalesscotlandandnorthernireland`

[31] `https://data.london.gov.uk/dataset/place-residence-place-work-local-authority`

[32] `http://observatoire.liser.lu/index.cfm?pageKw=serie3`

[33] `http://observatoire.liser.lu/index.cfm?pageKw=prixcommune`

[34] `https://data.public.lu/en/datasets/limites-administratives-du-grand-duche-de-luxembourg/`

wage per municipality for 2017 is from STATEC.[35] Observations are also geolocalised using the shape file from the Administration du cadastre et de la topographie. Data on population density for Luxembourg and its neighboring countries is obtained in three steps. We use the dataset JRC-GEOSTAT 2018[36], which is a grid map of 1 x 1 km cells with the number of residents for the year 2018 for European countries. This data is merged with a shapefile of European municipalities[37], which allows us to calculate the resident population at the municipality level across several countries in a unified manner. We infer the working age population (15-64) at the municipality level by using the share of the population aged between 15 and 64 for each country.[38] The total number of cross-border workers at the municipality level working in Luxembourg in 2018 is from the Luxembourg IGSS.[39] The percentage of cross-border workers in Luxembourg is obtained using the working age population at the municipality level, as calculated above. Data on the location of jobs within Luxembourg is from STATEC.[40] The location of jobs is available only for the population of residents (not for the cross-border workers).

A.4. **Empirical targets.** The model predicts a rent, while we have empirical observations on the price per square meter to buy or sell a property. Seloger.com[41], a popular website aggregating real estate ads in France, reports that the average monthly rent for an apartment in Paris is 1600 euros (for July 2021). The same website reports an average price per square meter in Paris of 11361 euros. We scale the observed price per square meter by a factor of $\frac{1600}{11361} \approx 0.141$, which produces an approximation of a rent gradient to which we can compare the rent gradient obtained by our model. We use this approximation in the bottom left panel in Figure 6.

[35] `https://statistiques.public.lu/catalogue-publications/bulletin-Statec/2017/PDF-Bulletin2-2017.pdf`

[36] `https://ec.europa.eu/eurostat/web/gisco/geodata/reference-data/population-distribution-demography/geostat`

[37] `https://gisco-services.ec.europa.eu/distribution/v1/communes-2016.html`

[38] `https://data.worldbank.org/indicator/SP.POP.1564.TO.ZS?locations=EU&most_recent_year_desc=true`

[39] `https://data.public.lu/en/datasets/emploi-total-par-commune-de-residence-au-luxembourg-et-dans-les-pays-frontaliers/`

[40] `https://data.public.lu/en/datasets/population-ayant-une-activite-professionnelle-selon-la-commune-de-residence-et-de-travail-situation-au-1er-fevrier-2011-1/`

[41] `https://www.seloger.com/prix-de-l-immo/location/ile-de-france/paris.htm`

Appendix B. Value for leisure and amenities

In our model, unemployed receive a value for leisure h_C or h_O, depending on their place of residence (city or outskirts). In this appendix, we show explicitly that we can relate these values for leisure to the levels of amenities.

We assume that a_C is the level of amenities in the city and both employed and unemployed living in the city may equally benefit from these amenities. Their Bellman equations are therefore

$$\begin{aligned} rI_C^U(d,m) &= a_C - (1-t_R)R + p(\theta)\left(I_C^E(d,m) - I_C^U(d,m)\right) \\ rI_C^E(d,m) &= w_C(d,m) + a_C - (1-t_R)R + \delta\left(I_C^U(d,m) - I_C^E(d,m)\right) \end{aligned}$$

a_O is the level of amenities in the outskirts and unemployed living in the outskirts benefit from them. However, employed commuters (residing in the outskirts but working in the city) enjoy the amenities from the city, since they spend most of their active time in the city. The Bellman equations for those residing in the outskirts are therefore

$$\begin{aligned} rI_O^U(d,m) &= \max\left[a_O - (1-t_d)\mu d + p(\theta)\left(I_O^E(d,m) - I_O^U(d,m)\right),\ a_O\right] \\ rI_O^E(d,m) &= w_O(d,m) + a_C - (1-t_d)\mu d + \delta\left(I_O^U(d,m) - I_O^E(d,m)\right) \end{aligned}$$

After computations, we obtain the following wages

$$\begin{aligned} w_C(d,m) \equiv w_C &= \frac{\gamma(r+\delta+p(\theta))y}{r+\delta+\gamma p(\theta)} \\ w_O(d,m) \equiv w_O &= \frac{\gamma(r+\delta+p(\theta))y + (1-\gamma)(r+\delta)(a_O - a_C)}{r+\delta+\gamma p(\theta)} \end{aligned}$$

We immediately see that when $a_C > a_O$, $w_C > w_O$. Pushing further the computations, we obtain the free entry condition

$$a = \frac{q(\theta)(1-\gamma)\left(U_C\, y + U_O(y - (a_O - a_C))\right)}{(U_C + U_O)(r+\delta+\gamma p(\theta))} \tag{15}$$

When $a_C = a_O$, U_C and U_O disappear from this condition and the composition effect no longer holds. Finally, $d^\star$ and $m^\star$ fully characterize the population split into the migration, commuting and home scenarios, i.e. the mobility condition, with

Definition 3.

$$
\begin{aligned}
d^\star &= \frac{p(\theta)\ \gamma(y-(a_O-a_C))}{(1-t_d)\mu(r+\delta+\gamma p(\theta))} \\
m^\star &= \frac{p(\theta)\ \gamma y}{r\tau(r+\delta+\gamma p(\theta))} - \frac{(a_O-a_C)+(1-t_R)R}{r\tau}
\end{aligned}
$$

To summarize, in the model presented in Section 3, with the values for leisure h_C and h_O, the equilibrium is determined by the free entry condition (7) and by the Definition 1 (which determines the population split and hence the mobility condition). In the alternative with amenities a_C and a_O presented in this appendix, the equilibrium is determined by the free entry condition (15) and by the Definition 3. These equations are strictly equivalent when $h_C = 0$ and $h_O = a_O - a_C$. In other words, the value of leisure in the city is normalized to 0 and the value for leisure in the outskirts corresponds to the difference in amenities between the outskirts and the city. We also have $h_C = h_O$ when $a_C = a_O$, $h_C > h_O$ when $a_C > a_O$ and $h_C < h_O$ when $a_C < a_O$. Accordingly, (i) all the results in the main paper still hold with this alternative approach and (ii) we can indistinguishably refer to value for leisure or level of amenities.

A second alternative to model amenities would be that all individuals benefit from the amenities where they reside and employed have less time ($\lambda \leq 1$) to enjoy amenities than unemployed (time normalized to 1). Doing exactly the same exercise as above, it is easy to see when $h_C = (1-\lambda)a_C$ and $h_O = (1-\lambda)a_O$, this second alternative is also equivalent to the model from Section 3.

Appendix C. Decentralized economy

We use the wage expressions to simplify firm and worker surpluses. $\forall\ (d,m) \in \mathcal{S}_m$, we obtain

$$
\begin{aligned}
I_C^E(d,m) - I_C^U(d,m) &\equiv I_C^E - I_C^U = \frac{\gamma(y-h_C)}{r+\delta+\gamma p(\theta)} \\
J_C(d,m) &\equiv J_C = \frac{(1-\gamma)(y-h_C)}{r+\delta+\gamma p(\theta)}
\end{aligned}
$$

Similarly, $\forall\ (d,m) \in \mathcal{S}_c$, we get

$$
\begin{aligned}
I_O^E(d,m) - I_O^U(d,m) &\equiv I_O^E - I_O^U = \frac{\gamma(y-h_O)}{r+\delta+\gamma p(\theta)} \\
J_O(d,m) &\equiv J_O = \frac{(1-\gamma)(y-h_O)}{r+\delta+\gamma p(\theta)}
\end{aligned}
$$

Finally, the life time return of a job seeker in the city is

$$\begin{aligned} rI_C^U(d,m) \equiv rI_C^U &= h_C - (1-t_R)R + p(\theta)\,(I_C^E - I_C^U) \\ &= h_C - (1-t_R)R + \frac{p(\theta)\gamma(y-h_C)}{r+\delta+\gamma p(\theta)} \end{aligned}$$

All expressions are independent from the individual characteristics (d,m).

From Corollary 1, we get

$$\begin{cases} \mathcal{H}(\theta,u) &= 1 - \frac{1}{u} + \frac{p(\theta)\gamma(y-h_O)}{\mu(\delta+\gamma p(\theta))}\left(\frac{(1+\alpha)\epsilon}{(h_C-h_O)+\frac{p(\theta)\gamma(y-h_C)}{\delta+\gamma p(\theta)}}\right)^{\frac{1}{\alpha}} \\ \mathcal{G}(\theta,u) &= (1-\gamma)\,q(\theta)\left((y-h_C)u + (y-h_O)(1-u)\right) - a\left(\delta+\gamma p(\theta)\right) \end{cases}$$

We compute

$$\begin{cases} \frac{\mathcal{H}(\theta,u)}{\partial u} &= 1/u^2 > 0 \\ \frac{\mathcal{H}(\theta,u)}{\partial \theta} &= \left(\frac{1}{u}-1\right)\left(\frac{p'\delta}{p(\delta+\gamma p)}\right)\left(1-\frac{\bar\alpha}{\alpha}\right) \\ \frac{\mathcal{G}(\theta,u)}{\partial u} &= (1-\gamma)\,q(h_O-h_C) \\ \frac{\mathcal{G}(\theta,u)}{\partial \theta} &= (1-\gamma)\,q'((y-h_C)u+(y-h_O)(1-u)) - a\gamma p' < 0 \end{cases}$$

where $\bar\alpha = \frac{\gamma p(y-h_C)}{\gamma p(y-h_C)+(h_C-h_O)(\delta+\gamma p)}$. As a result, using the Implicit Function Theorem, the sign of $\mathrm{d}\theta/\mathrm{d}u$ for the $\mathcal{H}(\theta,u)=0$ curve is given by the sign of $\left(\frac{\alpha}{\bar\alpha-\alpha}\right)$. In general, $\bar\alpha > 0$ and the slope is positive for $\alpha < \bar\alpha$ and negative for $\alpha > \bar\alpha$. Note that when $h_O >> h_C$, $\bar\alpha$ may become negative and the slope is then always negative. The sign of $\mathrm{d}\theta/\mathrm{d}u$ for the $\mathcal{G}(\theta,u)=0$ curve is given by the sign of (h_O-h_C).

Appendix D. Proof of Proposition 3

First, we compute the different functions $CO(d_0,d^\star,m^\star)$, $MI(d_0,d^\star,m^\star)$, $\mathcal{C}_c(d_0,d^\star,m^\star)$ and $\mathcal{C}_m(d_0,d^\star,m^\star,\dot d_0,\dot d^\star,\dot m^\star)$. Under Assumptions 2 and 3, Figure 2 gives

$$\begin{aligned} CO(d_0,d^\star,m^\star) &= (1-m^\star)d^\star + m^\star(d^\star+d_0)/2 \\ MI(d_0,d^\star,m^\star) &= m^\star(1-d^\star+1-d_0)/2 \end{aligned}$$

Note that $\partial CO/\partial d_0 = m^\star/2$, $\partial CO/\partial d^\star = 1 - m^\star/2$, $\partial CO/\partial m^\star = (d_0-d^\star)/2$, $\partial MI/\partial d_0 = -m^\star/2$, $\partial MI/\partial d^\star = -m^\star/2$ and $\partial MI/\partial m^\star = 1-(d_0+d^\star)/2$. To compute the costs, we make use that the analytical expression of the linear border between $\mathcal{S}_c$ and $\mathcal{S}_c$ is $d = g(m\,;\,d_0,d^\star,m^\star) = d_0 + m(d^\star-d_0)/m^\star$, or equivalently $m = g^{-1}(d\,;\,d_0,d^\star,m^\star) =$

$(d-d_0)m^\star/(d^\star-d_0)$. Hence we have from Figure 2

$$\begin{aligned}\mathcal{C}_c(d_0,d^\star,m^\star) &= \int_0^{d_0}\mu d\,\mathrm{d}d+\int_{d_0}^{d^\star}\mu d\left(1-g^{-1}(d\,;\,d_0,d^\star,m^\star)\right)\mathrm{d}d\\ &= \frac{\mu\left(d^\star\right)^2}{2}-\frac{\mu m^\star}{6}(d^\star-d_0)(2d^\star+d_0)\end{aligned}$$

Note that $\partial\mathcal{C}_c/\partial d_0=\mu m^\star(2d_0+d^\star)/6$, $\partial\mathcal{C}_c/\partial d^\star=\mu d^\star-\mu m^\star(4d^\star-d_0)/6$, $\partial\mathcal{C}_c/\partial m^\star=-\mu(d^\star-d_0)(2d^\star+d_0)/6$. Similarly

$$\begin{aligned}\mathcal{C}_m(d_0,d^\star,m^\star,\dot{d}_0,\dot{d}^\star,\dot{m}^\star) &= \int_0^{m^\star}\tau m\left(g(m\,;\,d_0,d^\star,m^\star)-g(m\,;\,d_0+\dot{d}_0,d^\star+\dot{d}^\star,m^\star+\dot{m}^\star)\right)\mathrm{d}m\\ &+ \int_{m^\star}^{m^\star+\dot{m}^\star}\tau m\left(1-g(m\,;\,d_0+\dot{d}_0,d^\star+\dot{d}^\star,m^\star+\dot{m}^\star+\dot{m}^\star)\right)\mathrm{d}m\end{aligned}$$

We immediately see that $\mathcal{C}_m(d_0,d^\star,m^\star,0,0,0)=0$. To solve the above equation, we first compute

$$\int m\,g(m\,;\,d_0,d^\star,m^\star)\,\mathrm{d}m=m^2\left(\frac{d_0}{2}+\frac{m(d^\star-d_0)}{3m^\star}\right)\equiv G(m\,;\,d_0,d^\star,m^\star)$$

We observe that $G(0\,;\,d_0,d^\star,m^\star)=0$. Likewise, we also define $G(m\,;\,d_0+\dot{d}_0,d^\star+\dot{d}^\star,m^\star+\dot{m}^\star)$. The migration costs therefore simplify into

$$\begin{aligned}\mathcal{C}_m(d_0,d^\star,m^\star,\dot{d}_0,\dot{d}^\star,\dot{m}^\star) &= \frac{\dot{m}^\star(2m^\star+\dot{m}^\star)}{2}+\tau G(m^\star\,;\,d_0,d^\star,m^\star)\\ &- \tau G(m^\star+\dot{m}^\star\,;\,d_0+\dot{d}_0,d^\star+\dot{d}^\star,m^\star+\dot{m}^\star)\end{aligned}$$

with

$$\begin{aligned}G(m^\star\,;\,d_0,d^\star,m^\star) &= \frac{(m^\star)^2}{3}\left(\frac{d_0}{2}+d^\star\right)\\ G(m^\star+\dot{m}^\star\,;\,d_0+\dot{d}_0,d^\star+\dot{d}^\star,m^\star+\dot{m}^\star) &= \frac{(m^\star+\dot{m}^\star)^2}{3}\left(\frac{d_0+\dot{d}_0}{2}+d^\star+\dot{d}^\star\right)\end{aligned}$$

The partial derivatives of $\mathcal{C}_m$ are at the steady state (since we are only interested in the long run equilibrium, we take $\dot{d}_0=\dot{d}^\star=\dot{m}^\star=0$ after the derivation): $\partial\mathcal{C}_m/\partial d_0=$

$\partial\mathcal{C}_m/\partial d^\star = \partial\mathcal{C}_m/\partial m^\star = 0$, $\partial\mathcal{C}_m/\partial\dot{d}_0 = -\tau(m^\star)^2/6$, $\partial\mathcal{C}_m/\partial\dot{d}^\star = -\tau(m^\star)^2/3$ and $\partial\mathcal{C}_m/\partial\dot{m}^\star = \tau m^\star(1 - d_0/3 - 2d^\star/3)$.

Second, we solve the Hamiltonian. The first order conditions are at the steady state (we take $\dot{\lambda}_c = \dot{\lambda}_r = \dot{\lambda}_{d_0} = \dot{\lambda}_d = \dot{\lambda}_m = 0$ after the derivation)

$$\begin{array}{lllll} \dfrac{\partial H}{\partial \theta} = 0 & \dfrac{\partial H}{\partial C_{d_0}} = 0 & \dfrac{\partial H}{\partial C_d} = 0 & \dfrac{\partial H}{\partial C_m} = 0 & \\ \dfrac{\partial H}{\partial N_C} = r\lambda_c & \dfrac{\partial H}{\partial N_O} = r\lambda_r & \dfrac{\partial H}{\partial d_0} = r\lambda_{d_0} & \dfrac{\partial H}{\partial d} = r\lambda_d & \dfrac{\partial H}{\partial m} = r\lambda_m \end{array}$$

The first line corresponds to the first order conditions related to the four control variables whereas the last line corresponds to the first order conditions related to the five state variables.

Third, make use of all above functions and partial derivatives to develop the solution of the Hamiltonian problem. Remember that to simplify the notation, we denoted $C_{d_0} = \dot{d}_0$, $C_d = \dot{d}^\star$ and $C_m = \dot{m}^\star$. After computations and simplifications, we obtain the system of equations in Proposition 3.

Appendix E. Proof of Proposition 4

When $\gamma = \eta$, $t_d = t_R = 0$ and $h_C = h_O = h$, the decentralized equilibrium given by equation (7), Definition 1 and equation (12) simplifies into

$$\begin{aligned} a &= \frac{q(1-\eta)\,(y-h)}{r+\delta+\eta p} \\ \mu d^\star &= \frac{p\,\eta(y-h)}{r+\delta+\eta p} \\ r\tau m^\star &= \mu d^\star - R \\ d_0 &= \frac{R}{\mu} \end{aligned}$$

When $h_C = h_O = h$, the central planner equilibrium given by Proposition 3 simplifies into

$$\begin{aligned}
a &= (1-\eta)\, q\, \frac{y-h}{r+\delta+\eta p} \\
\frac{r\tau m^\star}{3} + R &= \frac{\mu(d^\star + 2d_0)}{3} \\
r\tau m^\star \left(1 - \frac{d_0}{3} - \frac{2d^\star}{3}\right) &= \frac{\eta p(y-h)(1-d^\star)}{r+\delta+\eta p} - R\left(1 - \frac{d_0 + d^\star}{2}\right) + \frac{\mu}{6}(d^\star - d_0)(2d^\star + d_0) \\
-\frac{r\tau\,(m^\star)^2}{3} &= \frac{\eta p(y-h)(1-m^\star)}{r+\delta+\eta p} + R\frac{m^\star}{2} + \frac{\mu m^\star}{6}(4d^\star - d_0) - \mu d^\star
\end{aligned}$$

We see that the first equation in each system is the same. To prove that the last three equations are also the same (and hence that the decentralized equilibrium is equivalent to the central planner solution), we inject the decentralized equilibrium into the central planner solution and show the equations are satisfied. More precisely, in each central planner equation, we replace $m^\star$, d_0 and $\eta p(y-h)/(r+\delta+\eta p)$ by their corresponding values in the decentralized equilibrium, which are $(\mu d^\star - R)/(r\tau)$, R/μ and $\mu d^\star$, respectively. After simplifications, we see that all equations are satisfied.

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