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LOLA 1.0: LUXEMBOURG OVERLAPPING GENERATION MODEL FOR POLICY ANALYSIS

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Abstract

We build on the DSGE literature to propose an overlapping generation model for Luxembourg. By way of illustration, the model is then used to study the consequences of the ageing of the population and the potential effects of alternative macroeconomic policies.

Keywords: Overlapping Generations, Search Unemployment, Small open economy, Labor Force Participation, Ageing, Labor Market Policy and Institutions

JEL-Code: E24, H55, J26, J64

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Résumé non-technique

L'approche méthodologique actuellement privilégiée pour l'analyse de problèmes de politique macroéconomique est celle des modèles d'équilibre général dynamique. Ces modèles sont à interpréter comme des représentations stylisées (maquettes) du fonctionnement de l'économie. Ils sont construits à partir de représentations cohérentes et rigoureuses des mécanismes de marché et du comportement des agents économiques, fondées sur la théorie microéconomique. Hormis quelques cas particuliers hyper simplifiés, les propriétés et implications de ces modélisations de la réalité économique peuvent rarement être étudiées en termes analytiques généraux. Typiquement, les maquettes sont "calibrées" et leurs propriétés étudiées par simulations numériques, en veillant à spécifier et calibrer le modèle initial (scénario de base) de façon à reproduire des caractéristiques bien établies de l'économie considérée. Les effets de politiques économiques ou autres modifications de l'environnement économique sont simulés en élaborant des variantes du scénario de base.

Le modèle LOLA se conforme à cette approche d'équilibre général dynamique. Il vise principalement à étudier tant les effets de chocs structurels comme les chocs de démographie, que les effets de politiques structurelles telles que les recommandations de l'agenda de Lisbonne. Deux éléments principaux caractérisent ce modèle. Premièrement, le modèle LOLA se base sur les *modèles à générations imbriquées* (OLG models) dont le but est de distinguer différentes générations d'individus (travailleurs, préretraités, retraités) et de modéliser le comportement d'épargne. Cependant, la plupart des modèles à générations imbriquées supposent un marché du travail parfaitement compétitif avec absence de chômage involontaire, ce qui est gênant lorsque qu'il s'agit d'appréhender une variable telle que le taux d'emploi. Lorsque le chômage est pris en compte, la présentation est généralement simplifiée, soit en ignorant la dimension frictionnelle du chômage (liée aux flux d'entrées et sorties), soit en simplifiant la représentation des classes d'âge en adoptant l'hypothèse de "jeunesse perpétuelle" *à la* Blanchard.

Deuxièmement, notre recherche se base sur les *modèles à la Mortensen-Pissarides* qui représentent explicitement les comportements de demande et d'offre de travail, les processus de formation des prix et des salaires, et leurs impacts sur les probabilités d'embauche. Cependant, ces modèles font généralement l'impasse sur la dimension intergénérationnelle. Notre objectif est donc de proposer une modélisation de l'économie luxembourgeoise qui permette de traiter simultanément la dimension frictionnelle inhérente au marché du travail (taux d'emploi, taux de chômage et taux d'activité) et la dimension intergénérationnelle (vieillissement, épargne, pension,...).

Plus précisément, dans LOLA, la vie d'un individu (de 25 ans à 100 ans) est divisée en 15 périodes. Une période représente donc 5 ans. La taille de la population (c'est-à-dire les probabilités de passage d'une génération à une autre) est fixée de manière à avoir une pyramide des âges réaliste. Selon son âge, un individu peut être en emploi, au chômage, en préretraite ou en retraite. Le modèle fait également la distinction entre travailleurs résidents et travailleurs frontaliers. En outre, une seule fonction d'appariement est utilisée, ce qui signifie par exemple que les travailleurs juniors et seniors sont en concurrence pour le même type d'emplois, tout comme sont en concurrence travailleurs résidents et travailleurs frontaliers. Les salaires sont déterminés par une négociation entre l'employeur et l'employé et peuvent donc différer entre générations, de même qu'entre résidents et frontaliers. Le modèle comporte un système de pension par répartition ("pay as you go") mais une pension complémentaire peut être financée par l'épargne individuelle. Outre les pensions, le gouvernement doit également financer les prépensions, les allocations de chômage et les autres dépenses publiques. Toutes ces dépenses sont financées par une taxe sur les salaires, une partie étant à la charge de l'employeur et l'autre partie à la charge de l'employé, et par une taxe sur la consommation. Ce modèle est calibré sur les données luxembourgeoises.

A titre d'illustration, nous utilisons le modèle LOLA afin d'analyser et de comprendre les risques à moyen et long terme que les évolutions démographiques (vieillissement de la population et importance des frontaliers) font peser sur le financement des pensions au Luxembourg. Ce type d'analyse revêt une importance particulière dans un pays tel que le Luxembourg. La petite taille et la forte ouverture du pays compliquent certes tout exercice de projection, surtout sur un horizon de long terme. Cependant, ces mêmes caractéristiques exacerbent la fragilité financière du régime de pension. Les modèles d'équilibre général permettent de mieux appréhender les sources de vulnérabilité et, partant, de choisir en connaissance de cause les mesures susceptibles de pallier ces menaces.

Le modèle suggère que le vieillissement de la population, tant résidente que frontalière, induira un alourdissement progressif et non négligeable des dépenses de pensions et en conséquence une détérioration sensible des finances publiques. Même en supposant une hausse continue de la productivité et du nombre de frontaliers et d'immigrants, le coût pour les finances publiques serait par exemple de l'ordre de 10% du PIB aux alentours de 2040. Le modèle suggère dans le même temps qu'une baisse de 10 points de pourcentage du ratio de remplacement réel des pensions (par exemple en suspendant provisoirement l'indexation des pensions aux salaires réels) couplée à une baisse 10 points de pourcentage du différentiel entre salaire brut et salaire net pour les 55-65 ans (ce qui entraîne une hausse du taux d'activité de ces travailleurs) peut résoudre le futur problème de financement des pensions tout en préservant le bien-être de la population. En conclusion, le financement futur des pensions pourrait être assuré par une réforme initiée dès aujourd'hui, pourvu que cette réforme soit à la fois ciblée et efficace. Un tel effort de préfinancement des charges de pension futures permettrait d'éviter demain des réformes "révolutionnaires" socialement très douloureuses. Il est également important de noter que malgré des approches complètement différentes (modèle d'équilibre général *vs.* modèle comptable), nos résultats sont assez proches de ceux de travaux antérieurs réalisés à la BCL.[1]

Il convient évidemment d'interpréter ces résultats avec prudence. Premièrement, le Luxembourg est une petite économie ouverte qui est de ce fait fortement exposée aux chocs extérieurs. Il n'est donc pas aisé d'élaborer des hypothèses représentatives du futur. Deuxièmement, nos résultats sont basés sur un modèle et donc, par définition, sur une simplification de la réalité. C'est pourquoi ce modèle sera encore développé et raffiné dans le futur (LOLA 2.0). Ces extensions et améliorations concerneront en particulier les parties "finances publiques" et "demande étrangère".

[1] Voir Bouchet (2003) et Bouchet (2006).

1 Introduction

The dynamic stochastic general equilibrium (DSGE) approach is now widespread and often preferred for studying macroeconomic questions. These models are stylized representation of the economy (system of equations) based on rigorous microeconomic foundations explaining agents' behaviour and market mechanisms. They were initially built to answer the Lucas critique and to study the real business cycle fluctuations, see for instance Kydland and Prescott (1982). Since then, these models have been extensively developed (market imperfections, public sector, open economy, nominal dimensions, ...) and used to study many other questions. Except with some specific and very simplified cases, it is difficult to derive general analytical properties for these models. Instead, they are calibrated (each parameter is given a numerical value) in order to reproduce well-established characteristics of the studied economy. Then, the effects of shocks (exogenous shocks, change in economic policy, ...) are studied by modifying the value(s) of selected parameter(s) and simulating the model numerically.

The model we build for Luxembourg is based on the DSGE methodology and encompasses two specific features. First, we make an explicit distinction among generations (worker, early retiree, retiree) to correctly represent consumption and saving behaviour. However, most models with different generations (OLG models: overlapping generation models, see de la Croix and Michel (2002) for an extensive overview or de la Croix and Docquier (2007) for an application) either assume a perfectly competitive labour market (no involuntary unemployment) which is unrealistic and unsatisfactory if we want to look for instance at the activity rate, or alternatively simplify the representation of generations by following Blanchard (1985) and adopting the "perpetual youth" hypothesis. Second, as in for instance Pissarides (2000), we have an extensive representation of the labour market supply and demand, of the wage formation mechanism and of their impact on hiring probabilities. However, these "search and matching" models usually do not account for the inter-generational dimension. We believe that both a fair representation of the generations *and* of the labour market are important to correctly assess the effects of shocks and structural policies.

This research is an extension of past papers. Sneessens et al. (2003), Pierrard (2005) or Pierrard and Sneessens (2008) propose DSGE models with "search and matching" unemployment and workers with different skills. Pierrard (2008) adopts a similar approach but with a distinction between resident and cross-border workers. de la Croix et al. (2008) add the overlapping generation dimension but in a closed economy, both for the capital and the labour market. We build on this last paper (OLG with imperfect labour market) but adopt a small open economy (SOE) approach for the capital market (the interest rate is fixed by the rest of the world) and allow for the possibility of cross-border commuting. These last two hypothesis are particularly relevant for Luxembourg, where the interest rate is fixed by the ECB and half of the jobs are occupied by cross-border commuters.

More precisely, each individual in our model may live from 25 to 100 years and her life is divided into 15 periods. A period therefore represents 5 years. At each period, the probability of death is chosen to a obtain realistic population pyramid. Depending on her age, an individual may work, be unemployed, be in a early retirement scheme or be retired. We also make the distinction between a resident worker and a cross-border worker and use a single matching function (this means that workers of all ages compete for the same jobs, as well as residents and cross-border commuters). The wage is Nash bargained between the employer and the employee and may therefore differ across generations as well as across residents and cross-border commuters. Legal pensions (first pillar) follow a pay-as-you-go scheme but individuals may choose to contribute to a complementary pension (third pillar).[2] The government must pay unemployment benefits and other public consumption as well as the legal pension. These expenses are partly financed by taxation on wages (in charge of employees and employers). For simplicity, we replace all remaining taxes by a single tax on consumption. The model is then calibrated on Luxembourg data and simulated.

In Section 2, we review the already existing models for Luxembourg. We detail our model in Section 3. We extensively explain the calibration in Section 4 and simulate the effects of different shocks in Section 5. By way of illustration, we finally use the model in Section 6 to show the medium- and long-term risks that the ageing of the population and the importance of cross-border employment may raise for the sustainability of the pension system in Luxembourg. We also show how specific structural economic policies (for instance a reduction in employee taxation on senior workers, coupled with a slight decrease in pensions) could enhance long-term prospects of the pension system.

Of course we must remain cautious in interpreting our simulation results. First, the small open economy features of Luxembourg leave the country strongly exposed to idiosyncratic shocks and therefore complicate any attempt to make exogenous assumptions about the future. Second, results are based on a model and therefore on a simplification of the reality. In Section 7, we present the improvements/developments/extensions that we would like to introduce in future versions of LOLA.

2 Existing models for Luxembourg

The STATEC has developed a macroeconometric model (see Adam (2004) for an overview), estimated from annual data (from 1970 onwards) and used for forecasting and scenario analysis. Similarly the BCL, see Guarda (2005), has developed the Luxembourg block of the euro-area

[2] So far we do not look at the possibility of a second pillar (pension funds at the firms level).

multi-country model. The model is estimated with annual data from 1985 onwards and may also be used for projections and policy simulations. These are large-scale and detailed models, especially Adam (2004), that may prove useful for short-term forecasts. However, these two models do not belong to the DSGE literature and are therefore subject to the Lucas critique. The Statec also uses an "hybrid" model (see STATEC (2006) for a technical presentation) partly based on microeconomic foundations. However, the sophistication of the model (disaggregation at the sectoral level) makes it impossible to derive everything from theory and some decisions and behaviour are imposed exogenously.

At present, the only DSGE model already existing for Luxembourg was developed very recently by Fontagné et al. (2008) for the Ministry of Economics. This is an OLG model along the lines of Blanchard (1985). The labour market is represented by a "right-to-manage" setup and wages are bargained between firms and unions. Jobs may be occupied by residents or cross-border commuters, and the interest rate is fixed exogenously (small open economy). They put a strong focus on the goods market with imperfectly competitive intermediate producers, price and/or wage rigidities, and a distinction between tradable and non tradable goods. This is missing from our model and our net exports are simply residuals (as a result, we cannot study a demand shock from the rest of the world or an increase in competition in the goods market). On the other hand, our "pure" OLG representation allows us to study demographic questions (activity rate of seniors, pensions,...) which is impossible with the Blanchard (1985) approach.

3 The Model

In this section, we present the demographics of our model as well as our four different agents: resident households, cross-border households, firms and government.

3.1 Demographics

We consider an overlapping generation model with a home country (denoted by the superscript h hereafter) and a foreign country (denoted by the superscript f hereafter). All production is located in the home country. Employment and capital may be supplied by both the home and the foreign countries. In each country, each member of a generation can live for up to fifteen periods of 5 years each (from age 25 till 100). Let $Z^x_{a,t}$ denote the size of the generation reaching age a at period t in country $x \in \{h, f\}$. The size of new generations changes over time at an exogenous rate x^x_t:

$$Z^x_{0,t} = (1 + x^x_t)\, Z^x_{0,t-1}\,. \tag{1}$$

where x^x_t includes both fertility and migration effects. Abstracting from migration, the size of a given generation t declines deterministically through time. This size is determined by a cumulative survival probability $\beta^x_{a,t+a}$ so that:

$$Z^x_{a,t+a} = \beta^x_{a,t+a}\, Z^x_{0,t} + X^x_{a,t+a}\,, \tag{2}$$

where $0 \leq \beta^x_{a,t+a} \leq 1$ is decreasing in a, with $\beta^x_{0,t} = 1$. Migration flows are taken into account through $X^x_{a,t+a}$. Total (adult) population at time t is equal to $Z^x_t = \sum_{a=0}^{14} Z^x_{a,t}$. The demographic growth and survival probability vector can vary over time.

All individuals above age 65 ($8 \leq a \leq 14$) are inactive, so that 65 is the legal and compulsory retirement age. Our objective is not to explain participation rates of individuals of working age (in particular female participation rates) across time or over the life cycle.[3] However, we want to analyze early retirement decisions, especially the impact of changes in the generosity of early retirement schemes, given the state of the labor market. We thus assume exogenous participation rates except for the component related to early retirement decisions (that take place between 55 and 65). We denote $q^x_{a,t+a}$ the exogenous component of the participation rate, so that $P^x_{a,t+a} = q^x_{a,t+a}\, Z^x_{a,t+a}$ is equal to the active population, broadly defined to include workers on an early retirement scheme. At time t, members of the generation of age a that are participating in the labour market are either employed, unemployed, or on an early retirement scheme:

$$\begin{aligned} P^x_{a,t} &= N^x_{a,t} + U^x_{a,t} + E^x_{a,t}\,, \\ &= \left[n^x_{a,t} + u^x_{a,t} + e^x_{a,t}\right] P^x_{a,t}\,, \qquad 0 \leq a \leq 7\,. \end{aligned}$$

Lower-case letters denote the proportion of individuals in each group. We assume that the decision to go on early retirement does not depend on the initial employment status. Let $\lambda^x_{6,t}$ denote the fraction of people who choose to retire and leave the labor market between 55 and 60, so that the number of early retired workers of that age group is $E^x_{6,t} = \lambda^x_{6,t}\, P^x_{6,t}$. Similarly, let $\lambda^x_{7,t}$ denote the fraction of active workers of age 60-65 who decide to leave the labor market. The total number of workers on an early retirement scheme at time t is then equal to:

$$\begin{aligned} E^x_{6,t} + E^x_{7,t} &= e^x_{6,t}\, P^x_{6,t} + e^x_{7,t}\, P^x_{7,t}\,, \\ \text{with:} \quad & e^x_{6,t} = \lambda^x_{6,t}\,, \\ & e^x_{7,t} = \lambda^x_{6,t-1} + \lambda^x_{7,t}\,(1 - \lambda^x_{6,t-1})\,. \end{aligned} \tag{3}$$

3.2 Labour Market Flows

We use a Mortensen-Pissarides representation of search frictions on the labour market. We assume an exogenous job destruction rate χ and a constant-returns-to-scale matching function. The pool of job seekers in country $x \in \{h, f\}$ at a time t is equal to the new population of junior

[3] See de la Croix and Docquier (2007) for further motivation of this choice.

workers $P^x_{0,t}$, plus the total number of unemployed workers in all older generations. Let us denote $\Omega^x_{a,t}$ the number of job seekers of age a at time t. Given a compulsory retirement age of 65, the total number of job seekers at time t, denoted Ω^x_t, is then equal to:

$$\begin{aligned}\Omega^x_t &= \sum_{a=0}^{7} \Omega^x_{a,t} \,, \\ &= P^x_{0,t} + \sum_{a=1}^{5} \left[1-(1-\chi)\, n^x_{a-1,t-1}\right] P^x_{a,t} \\ &\qquad + (1-\lambda^x_{6,t})\left[1-(1-\chi)\, n^x_{5,t-1}\right] P^x_{6,t} \\ &\qquad + (1-\lambda^x_{7,t})\, \left[(1-\lambda^x_{6,t-1}) - (1-\chi)\, n^x_{6,t-1}\right]\, P^x_{7,t}\,.\end{aligned} \tag{4}$$

The total number of job seekers is therefore $\Omega_t = \Omega^h_t + \Omega^f_t$. Given a matching function:

$$M_t = M(V_t, \Omega_t)\,,$$

the probabilities of finding a job and of filling a vacancy will be given respectively by:

$$p_t = \frac{M_t}{\Omega_t} \qquad \text{and} \qquad q_t = \frac{M_t}{V_t}\,.$$

with V_t the total amount of vacancies. In each country, the number of employed workers in age group a is determined by the sum of non-destroyed jobs (when $a > 0$) and of new hires:

$$\begin{aligned} n^x_{a,t} &= p_t\, \frac{\Omega^x_{a,t}}{P^x_{a,t}}\,, && \text{for } a = 0\,; \\ &= (1-\chi)\, n^x_{a-1,t-1} + p_t\, \frac{\Omega^x_{a,t}}{P^x_{a,t}}\,, && \text{for } 1 \le a \le 5\,; \\ &= (1-\lambda^x_{a,t})\,(1-\chi)\, n^x_{a-1,t-1} + p_t\, \frac{\Omega^x_{a,t}}{P^x_{a,t}}\,, && \text{for } 6 \le a \le 7\,. \end{aligned}$$

After substituting for $\Omega^x_{a,t}$, this equation becomes:

$$\begin{aligned} n^x_{a,t} &= p_t\,, && \text{for } a = 0\,; \\ &= (1-p_t)(1-\chi)\, n^x_{a-1,t-1} + p_t\,, && \text{for } 1 \le a \le 5\,; \\ &= (1-p_t)(1-\lambda^x_{a,t})\,(1-\chi)\, n^x_{a-1,t-1} + p_t(1-\lambda^x_{a,t})\,, && \text{for } a = 6\,; \\ &= (1-p_t)(1-\lambda^x_{a,t})\,(1-\chi)\, n^x_{a-1,t-1} + p_t(1-\lambda^x_{a,t})(1-\lambda^x_{a-1,t-1})\,, && \text{for } a = 7\,. \end{aligned} \tag{5}$$

The same equation can be written in terms of the probability of filling a vacancy q_t by using $p_t = q_t\, V_t/\Omega_t$. Total employment is equal to:

$$N_t = N^h_t + N^f_t = \sum_{a=0}^{7} \left(n^h_{a,t}\, P^h_{a,t} + n^f_{a,t}\, P^f_{a,t} \right)\,.$$

3.3 Households in the home country

For simplicity, in this section we drop the superscript h from all variables. We assume an economy with state-contingent markets, so that each individual can fully insure against idiosyncratic risk at the beginning of his lifetime. Given a sequence of contingent wages and prices, an individual born at time t will determine his optimal contingent consumption plan by maximizing his expected utility, subject to his intertemporal budget constraint. In this setting, the individual optimization problem is identical to the optimization program of a hypothetical large household including all members of a given generation. Provided the instantaneous utility function is separable in consumption and leisure, all members of a given generation will have the same consumption level, whatever their employment or participation status.

Let $c_{a,t+a}$ represent the consumption level of an individual consumer of generation t and age a, while $n_{a,t+a}.q_{a,t+a}$ and $e_{a,t+a}.q_{a,t+a}$ represent respectively the proportion of employed and early retired workers in the total population of age a born at time t. The objective function of the household (effectively of one cohort) is written as follows:

$$W_t^H = \max_{c_{a,t+a},\lambda_{6,t+6},\lambda_{7,t+7}} \sum_{a=0}^{14} \beta^a \, \beta_{a,t+a} \left\{ u(c_{a,t+a}) - d^n \, n_{a,t+a} \, q_{a,t+a} + d_a^e \, \frac{(e_{a,t+a})^{1-\phi}}{1-\phi} \, q_{a,t+a} \right\} Z_{0,t} \,, \quad (6)$$

where β is a subjective discount factor and $0 < \phi < 1$. Instantaneous utility is assumed to be separable in c, n and e. The utility of per capita consumption is represented by a standard concave function (we shall use a logarithmic function). Marginal labour disutility is assumed to be constant, equal to d^n. The extra utility derived from early retirement is represented by a concave function of the early retirement rate [4]. The decision variables are c, λ_6 and λ_7. The last two variables refer to the fraction of agents in the corresponding age groups who decide to go on early retirement and leave the labour market, respectively at age 55 and 60. Inactivity and employment rates are given by (3) and (5).

The household's flow budget constraint at time $t+a$ takes the form:

$$\left[(1-\tau^w_{a,t+a}) \, w_{a,t+a} \, . \, n_{a,t+a} + b^u_{a,t+a} \, . \, u_{a,t+a} + b^e_{a,t+a} \, . \, e_{a,t+a} + b^i_{a,t+a} \, . \, i_{a,t+a} \right] . \, q_{a,t+a}$$
$$+ \frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}} \, R_{t+a} \, s_{a-1,t+a-1} = (1+\tau^c_{t+a}) \, c_{a,t+a} + s_{a,t+a}$$

Wage and consumption tax rates are given by τ^w and τ^c respectively.[5] τ^w may vary across ages to allow for targeted tax cuts. $b^u_{a,t+a}, b^e_{a,t+a}, b^i_{a,t+a}$ are the replacement benefits received respectively by the unemployed, early retired or statutory retirement age worker on a legal pension

[4]This formulation implies -without loss of generality- that the disutility associated with the search activities of the unemployed is normalized to zero.

[5]τ^c must be regarded as more general than a pure consumption tax. For instance, when all firm profits are distributed to households/shareholders, this is also equivalent to a tax on firm profit.

scheme ($i_{a,t+a}$ is a dummy variable equal to zero when $a < 65$ and equal to 1 afterwards); $s_{a,t+a}$ is the financial wealth accumulated at time $t+a$, in per capita terms. This financial wealth is held either in the form of shares or as physical capital rented out to firms. Because there is perfect insurance against individual life uncertainty (as if there were a perfect annuity market), the total return to savings is equal to the gross risk-free interest rate R_{t+a} divided by the survival probability $\beta_{a,t}/\beta_{a-1,t-1}$.

The optimal consumption plan must satisfy the usual Euler equation:

$$\frac{u'_{c_{a,t+a}}}{1+\tau^c_{t+a}} = \beta\, R_{t+a+1}\, \frac{u'_{c_{a+1,t+a+1}}}{1+\tau^c_{t+a+1}} \, .$$

After substitution and rearrangements, the condition determining the optimal proportion of early retired workers aged 60-65 can be shown to be:

$$\frac{b^e_{7,t+7}}{(1+\tau^c_{t+7})\, c_{7,t+7}} + d^e_7\, (e_{7,t+7})^{-\phi} = \pi_{7,t+7} \left[\frac{(1-\tau^w_{7,t+7})\, w_{7,t+7}}{(1+\tau^c_{t+7})\, c_{7,t+7}} - d^n\right] + (1-\pi_{7,t+7}) \left[\frac{b^u_{7,t+7}}{(1+\tau^c_{t+7})\, c_{7,t+7}}\right] ,$$

where π is the unconditional probability that an active worker will be employed. A similar condition holds for early retirement at age 55-60. Details are given in the appendix.

For later use, we also note that the value of an additional job for a household of age a is given by:

$$\begin{aligned}
\frac{1}{u'_{c_{a,t}}}\, \frac{\partial W^H_t}{\partial N_{a,t}} &= \frac{1}{u'_{c_{a,t}}}\, \frac{1}{q_{a,t}\, Z_{a,t}}\, \frac{\partial W^H_t}{\partial n_{a,t}} \\
&= \sum_{j=0}^{7-a} \frac{\beta_{a+j,t+j}}{\beta_{a,t}}\, \beta^j\, \frac{u'_{c_{a+j,t+j}}}{u'_{c_{a,t}}} \left\{ \frac{(1-\tau^w_{a+j,t+j})\, w_{a+j,t+j} - b^u_{a+j,t+j}}{(1+\tau^c_{t+j})} - \frac{d^n}{u'_{c_{a+j,t+j}}} \right\} \frac{\partial n_{a+j,t+j}}{\partial n_{a,t}}
\end{aligned} \tag{7}$$

where $\partial n_{a+j,t+j}/\partial n_{a,t}$ can be obtained from (5).

3.4 Households in the foreign country

Cross-border workers are employed and pay taxes (on wages) in the home country but consume in the foreign country. Unemployment benefits are paid by the foreign country but early-retirement and retirement benefits are paid by the home country. Because we are only interested in the home country, we consider foreign country household decisions exogenous. More precisely, we take as given inactivity choices $\lambda^f_{a,t}$ as well as wages $w^f_{a,t}$. In the simulations, we will simply assume that $\lambda^f_{a,t} = \lambda^h_{a,t}$ and $w^f_{a,t} = w^h_{a,t}$. [6] An extension of this model would be to endogenize the cross-border commuters' behaviour, along the lines of Pierrard (2008).

[6] Brosius (2005) shows that home wages (residents) are on average slightly above wages for cross-border commuters, but this is mainly due to a well-paid public sector that mostly employs residents. At this stage, we do not go into these details and therefore assume similar wages.

3.5 Firms

There are two productive factors, labor and capital. Labour is measured in efficiency units. Efficiency varies across age (because of experience and abilities), across generations (because of education) and may also vary across country of residence. We define total labour input as follows:

$$H_t = \sum_{a=0}^{7} \left(h^h_{a,t}.N^h_{a,t} + h^f_{a,t}.N^f_{a,t} \right) .$$

We assume a constant-return-to-scale production function in labor and capital:

$$Y_t = A_t \, F(K_t, H_t) \, ,$$

where A_t stands for total factor productivity. Firms rent capital from households at cost $v_t = R_t + \delta - 1$ and pay a gross wage $w^x_{a,t}$ to workers of age a from country $x \in \{h, f\}$. We allow the employer wage tax ζ to vary across age groups (to allow for social security tax cuts targeted on specific age groups). The representative firm maximizes the discounted value of all the dividends (profits) that will be distributed to shareholders. Profits at time t are given by:

$$\Pi_t = F(K_t, H_t) - v_t \, K_t - \sum_{a=0}^{7} (1 + \zeta_{a,t}) \, \left(w^h_{a,t} \, N^h_{a,t} + w^f_{a,t} \, N^f_{a,t} \right) - a \, V_t \tag{8}$$

where a stands for the cost of posting a vacancy. The value of the firm can thus be written as follows[7]:

$$W^F_t = \max_{K_t, V_t} \left\{ F(K_t, H_t) - v_t \, K_t - \sum_{a=0}^{7} (1 + \zeta_{a,t}) \, \left(w^h_{a,t} \, N^h_{a,t} + w^x_{a,t} \, N^f_{a,t} \right) - a \, V_t \right\} + R^{-1}_{t+1} \, W^F_{t+1} \tag{9}$$

subject to (5) and $p_t = q_t \, V_t / \Omega_t$. The first-order optimality conditions are:

$$v_t = F_{K_t} \, , \tag{10}$$

$$a = q_t \sum_{a=0}^{7} \left(\frac{\Omega^h_{a,t}}{\Omega_t} \frac{\partial W^F_t}{\partial N^h_{a,t}} + \frac{\Omega^f_{a,t}}{\Omega_t} \frac{\partial W^F_t}{\partial N^f_{a,t}} \right) , \tag{11}$$

where $\frac{\partial W^F_t}{\partial N^x_{a,t}}$ is the value at time t of an additional worker of age a from country $x \in \{h, f\}$. With a job destruction rate χ, this value is equal to:

$$\begin{aligned} \frac{\partial W^F_t}{\partial N^x_{a,t}} = \sum_{j=0}^{7-a} \frac{\beta^x_{a+j,t+j}}{\beta^x_{a,t}} \, R^{-1}_{t,t+j} (1 - \lambda^x_{a+j-1,t+j-1}) \, (1 - \lambda^x_{a+j,t+j}) \, (1 - \chi)^j \\ . \left\{ h^x_{a+j,t+j} \, F_{H_{t+j}} - (1 + \zeta_{a+j,t+j}) \, w^x_{a+j,t+j} \right\} , \end{aligned} \tag{12}$$

where $\lambda^x_{a+i,t+i} \equiv 0$ for $a + i < 6$.

[7] Shareholders from home country may belong to different age groups and have different consumption levels. However, they all have the same discount factor given by $\tilde{\beta}_{t+1} = \beta \, \frac{u'_{c^h_{a+1,t+1}}}{u'_{c^h_{a,t}}} = R^{-1}_{t+1} \, , \; \forall \, a \in \{0, 14\}$. We also assume the same discount factor (that is, implicitly, the same consumption pattern) if shareholders are non-residents.

3.6 Government

We assume that unemployment and (early or legal) retirement benefits are determined by an exogenous fraction of the relevant gross wage, so that

$$\begin{aligned} b^{u}_{a,t} &= \rho^{u}_{t}\, w^{h}_{a,t} && \text{for } 0 \le a \le 7\,; \\ b^{e,x}_{a,t} &= \rho^{e}_{t}\, w^{x}_{a,t} && \text{for } 6 \le a \le 7\,; \\ b^{i,x}_{a,t} &= \rho^{i}_{t}\, \bar{w}^{x}_{t} && \text{for } 8 \le a \le 14\,. \end{aligned}$$

The legal retirement benefit is calculated on the basis of a lifetime average wage $\bar{w}$. Total transfer expenditures are then equal to:

$$T_t = \left[\sum_{a=0}^{7} b^{u}_{a,t}\, u^{h}_{a,t}\, q^{h}_{a,t}\, Z^{h}_{a,t}\right] + \left[\sum_{x\in\{h,f\}} \sum_{a=6}^{7} b^{e,x}_{a,t}\, e^{x}_{a,t}\, q^{x}_{a,t}\, Z^{x}_{a,t}\right] + \left[\sum_{x\in\{h,f\}} \sum_{a=8}^{14} b^{i,x}_{a,t}\, q^{x}_{a,t}\, Z^{x}_{a,t}\right]. \tag{13}$$

Public consumption is assumed to be a fraction of output, i.e. $G_t = \bar{g}_t\, Y_t$. We further assume that the "government" balances its budget in every (five-year) period by adjusting public consumption expenditures (ie, $\bar{g}_t$ is the adjusting variable):

$$\tau^{c}_{t}\, C^{h}_{t} + \sum_{x}\sum_{a} \left(\tau^{w}_{a,t} + \zeta_{a,t}\right) w^{x}_{a,t}\, n^{x}_{a,t}\, P^{x}_{a,t} = G_t + T_t\,, \tag{14}$$

where aggregate consumption $C^{h}_{t} = \sum_a c^{h}_{a,t}\, Z^{h}_{a,t}$.

3.7 Wages

Wages are renegotiated in every period. They are determined by a standard Nash bargaining rule:

$$\max_{w^{h}_{a,t}} \left(\frac{\partial W^{F}_{t}}{\partial N^{h}_{a,t}}\right)^{1-\eta_a} \left(\frac{1}{u'_{c^{h}_{a,t}}}\, \frac{\partial W^{H}_{t}}{\partial N^{h}_{a,t}}\right)^{\eta_a}.$$

The first-order optimality condition can then be written:

$$(1-\eta_a)\, \frac{1}{u'_{c^{h}_{a,t}}}\, \frac{\partial W^{H}_{t}}{\partial N^{h}_{a,t}} = \eta_a\, \frac{1-\tau^{w}_{a,t}}{(1+\zeta_{a,t})(1+\tau^{c}_{t})}\, \frac{\partial W^{F}_{t}}{\partial N^{h}_{a,t}}\,. \tag{15}$$

3.8 Equilibrium

Let Q_t denote the total financial value of firms at time t. In our deterministic setup, the return on equities must be equal to the market interest rate. In other words, the value of equities must be such that:

$$\frac{Q_{t+1} + \Pi_{t+1}}{Q_t} = R_{t+1}\,. \tag{16}$$

With an open (European) capital market, the equilibrium equation is replaced by an interest rate rule. We here simply assume an exogenous interest rate:

$$R_t = \bar{R}. \tag{17}$$

GDP_t may be computed from the net production perspective:

$$GDP_t = Y_t - aV_t, \tag{18}$$

and net exports NX_t are deduced from the demand perspective:

$$NX_t = GDP_t - C_t^h - G_t - K_t + (1-\delta)K_{t-1}. \tag{19}$$

4 Calibration

The model is calibrated on Luxembourg data and the reference year (initial period $t = 0$ in the model) is an average of 2004-2008 (when available).

4.1 Demographics

Each agent is born at the age of 25 and lives a maximum of 15 periods of 5 years. She may work or be unemployed during periods $a \in \{0, 1, ..., 7\}$, she may work, be unemployed or be inactive (early retirement) during periods $a \in \{8, 9\}$ and and she is inactive (retirement) during periods $a \in \{10, 11, ..., 14\}$. In other words, regarding inactivity, we are only interested in early retirement decisions and assume that the exogenous participation rate component is $q^x_{a,0+a} = 1$, with $x \in \{h, f\}$ and $a \in \{0, 1, ..., 7\}$. After 65, all the population becomes inactive and $q^x_{a,0+a} = 0$ with $a \in \{8, 9, ..., 14\}$. Home population $Z^h_{a,0+a}$ at the initial steady state is calibrated to match Luxembourg data, from which we removed inactivity not due to early retirement, as displayed in Figure 1. Remember that in our model we do not take into account population below age 25. As we see in Figure 2, the implied active population $P^h_{a,0+a}$ is quite close to what is observed in Luxembourg. Population in the foreign country $Z^f_{a,0+a}$ at the initial steady state is calibrated to match the current share (across ages) of cross-border commuters in employment, as displayed in Figure 3. We derive the cumulative survival probability $\beta^x_{a,0+a}$ (we assume they are identical in both home and foreign countries) from death probability data for Belgium, see Figure 4. Since we are at the steady state, we have no population growth x^x_0 in equation (1) and the equality in equation (2) is controlled through the migration vector $X^x_{a,0+a}$.

4.2 Labour market flows

Inactivity rates for older workers in Luxembourg (respectively 50.2% for 55-60 and 87.3% for 60-65) are among the highest in OECD. Again, we remove inactivity not due to early retirement

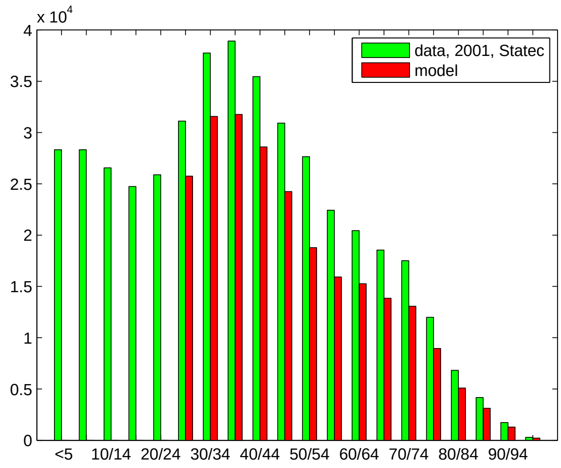

Figure 1: Population across ages: data *vs.* model

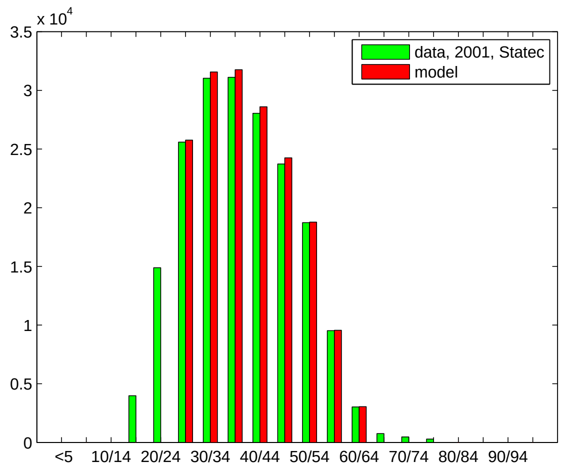

Figure 2: Active population across ages: data *vs.* model

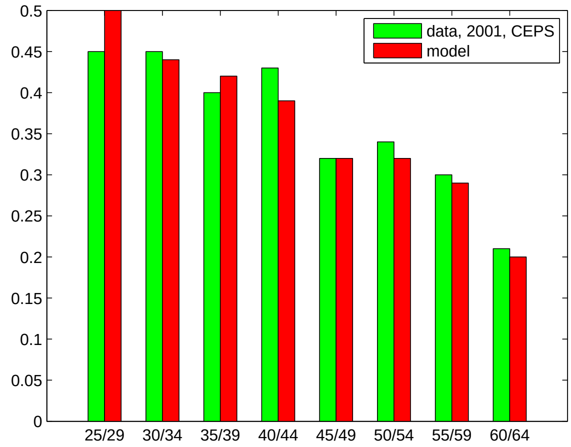

Figure 3: Share of cross-border commuters in total employment, across ages: data *vs.* model

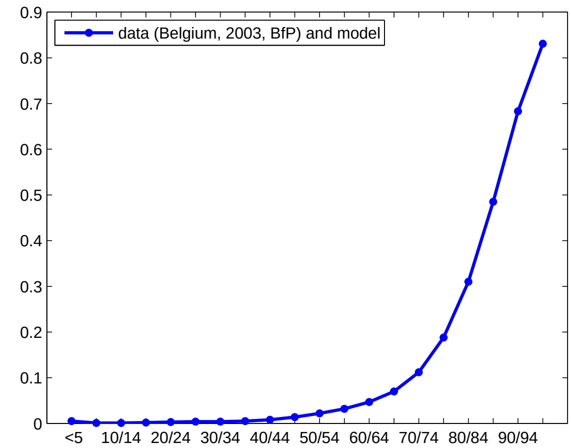

Figure 4: Death probability across ages: data and model

from these data and estimate early retirement inactivity rates at respectively $e^x_{6,0} = 0.4$ and $e^x_{7,0} = 0.8$. To obtain these values, we choose the parameters of the utility function derived from work and early retirement in equation (6). More precisely, we impose $d^n = 0.15$ (work disutility), $\phi = 0.2$ (concavity of early retirement utility) and we choose $d^e_6 = 0.102$ and $d^e_7 = 0.203$ (early retirement utility). We assume that the probability of finding a job is $p = 0.93$ and that the probability of a separation between the firm and the worker during a 5-year period is $\chi = 44\%$. Although our implied unemployment rates do not match the data perfectly (too high unemployment rates for young and old workers), Figure 5 shows that we nevertheless manage to capture the decreasing pattern observed across age groups.

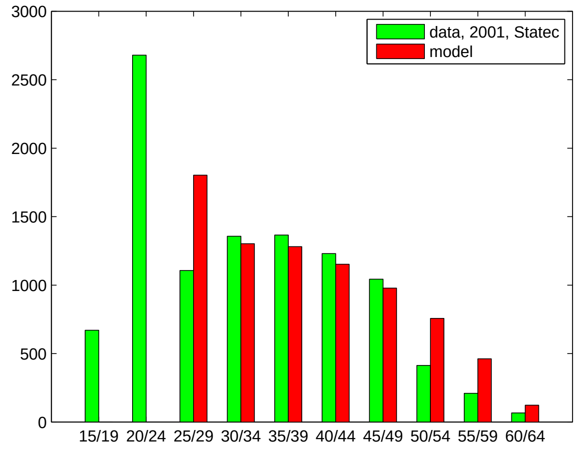

Figure 5: Unemployment level across ages: data *vs.* model

4.3 Vacancies and matching

We do not take a Cobb-Douglas matching function as in Pissarides (2000) but rather follow den Haan et al. (2000) using a function of the form:

$$M(V_t, \Omega_t) = \frac{V_t \, \Omega_t}{(V_t^\nu + \Omega_t^\nu)^{1/\nu}}.$$

The main advantage of this approach is that we always ensure that $0 < p_t, q_t < 1$ whatever the shock size. We choose $\nu = 9.55$ to obtain a labour market tightness equal to 1 (common in the search unemployment literature), *i.e.* that the probability of filling a vacancy is $q = p$.

4.4 Households: consumption, taxes, income and savings

As usual in OLG and DSGE models, households are risk-averse and we impose a logarithmic utility of consumption.[8] We fix the yearly household discount factor at 0.98 ($\beta = 0.904$ in our 5-year period model) and the yearly real interest rate (net of depreciation) at 2.8% ($\bar{R} = 15\%$ in our 5-year period model). Because agents discount the future less heavily than interest rate return ($1/\beta < 1 + \bar{R}$), we obtain a consumption growth of 4% every period (that is every 5 years). This is in line with what is usually observed in real data: consumption rises with age initially because of increasing income and then because of a progressive reduction in precautionary savings.[9]

In 2006, general government income from labour taxation (personal income taxes and social contributions) was €4679 mio (3224 mio in charge of employees and 1455 in charge of employers), whereas total gross remuneration was €15300 mio. From this, we infer (assuming identical proportional taxation across ages) an aggregate employee taxation $\tau^w = 21\%$ and employer taxation $\zeta = 10\%$. For simplicity, we suppose that all other government income comes from consumption taxation.[10] We set $\tau^c = 50\%$, involving a ratio of government consumption to GDP of 18%, close to what is observed in data (see Section 4.7).

Employment productivity $h^h_{a,0}$ is calibrated to match the observed evolution of wages across age groups, see Figure 6.[11] We assume no taxation on benefits [12] and fix the replacement ratio for unemployment benefit, early retirement benefit, and retirement benefit respectively at $\rho^u = 0.60$, $\rho^e = 0.55$ and $\rho^i = 0.55$. With an initial (25-30 year) gross wage normalized at 1000, this gives an average (25-65 year) net wage of 1080, an average net unemployment benefit of 820, a net early-retirement benefit for 55-60 of 910, a net early-retirement benefit for 60-65 of 1030, and a net retirement benefit of 1040. Unemployment benefits are available in Luxembourg for up to one year. Once this period has elapsed, the unemployed have access to the RMG (Revenu Minimum Garanti) which is kept in line with the minimum wage. The OECD (2006) computes a net replacement ratio between 85% and 90% in the initial phase of unemployment but this ratio is expected to fall if we take into account all phases of unemployment. Our net ratio of 75% should therefore be close to reality. The OECD (2008) also notes that net replacement rates

[8] See for instance de la Croix and Michel (2002) or King and Rebelo (1999).

[9] Consumption components also change across ages. For instance, we can expect a higher share for consumption of health services towards life end.

[10] This is equivalent to lump-sum taxation, and would be equivalent to firms' profit taxation in a closed economy. Also, we do not take into account taxation on consumption by non-residents. In practice, this would allow us to reduce τ^c and add a lump-sum non-resident income T^f, but wouldn't modify our results.

[11] Given the very weak participation rate of senior workers, the strong wage increase towards the end of the career might not be representative. Indeed, we expect that most senior workers remaining active occupy high level positions and therefore earn high salaries.

[12] Actually, there is taxation on benefits but lower than taxation on wages and the taxation depends on which kind of benefits. For simplicity, we assume no taxation at all.

for pensions are high in Luxembourg and may even exceed 100% in special cases. Again, with a net replacement rate around 95%, we should be close to reality. Our implied pension-related expenditures amount to 9% of GDP, slightly below the OECD estimation of 10%. If we include early retirement benefits (55-65), expenditures amount to 13% of GDP.

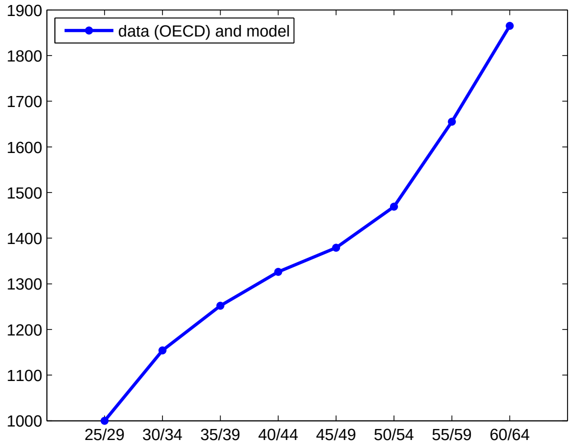

Figure 6: Gross wages across ages: data and model

Figure 7 shows the savings pattern implied by our calibration. During the first three periods (25-39), savings are negative, meaning that households borrow to finance their consumption (housing, children, ...). Then savings increase progressively until age 64 before falling again towards life end.

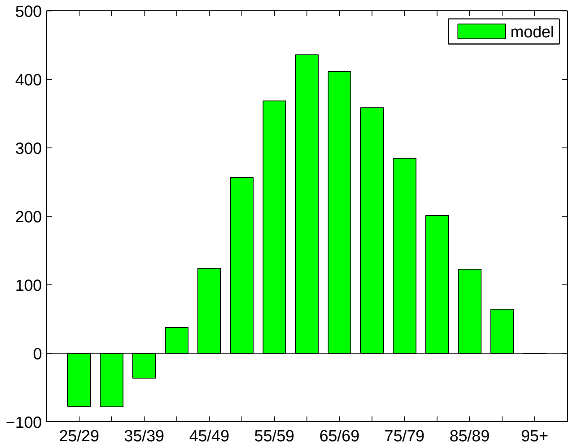

Figure 7: Savings across ages: model

4.5 Cross-border commuters

As already explained in Section 3.4, the cross-border commuters' behaviour is exogenous and we take it to be identical to home workers. More precisely, they have the same productivity ($h^f_{a,t} = h^h_{a,t}$, $\forall$ a) and they are paid the same wages ($w^f_{a,t} = w^h_{a,t}$, $\forall$ a).

4.6 Firms and production

Workers' bargaining power η_a, with $a \in \{0, 1, ..., 5\}$ is 0.5, as is common in the search unemployment literature, see for instance Pierrard and Sneessens (2008) for a calibration on Belgian data. However, in order to justify the very high wages of the 55-60 and 60-65 (see Figure 6) without assuming unrealistic levels of productivity, we increase the bargaining power of old workers to $\eta_6 = 0.8$ and $\eta_7 = 0.9$. As in de la Croix and Michel (2002) or King and Rebelo (1999), we use a Cobb-Douglas production function $F(N_t, K_t) = A_t \; K^{\mu}_t H^{1-\mu}_t$, the elasticity μ of output with respect to capital is set at 0.33 and the quarterly rate of capital depreciation is 2.5%, implying a 5-year depreciation rate $\delta = 40\%$. We normalize the TFP parameter $A = 300$. Our calibration implies that total vacancy costs represent 3.8% of GDP which is considered a reasonable value (see for instance Pissarides (2000) for similar values).

4.7 Implications

Finally, we compare in Table 1 the GDP demand decomposition implied by our calibration and the GDP demand decomposition obtained from average 2004-2008 data. The main difference is that we understate the importance of consumption presumably because in our model, private consumption is taxed highly to finance government consumption.

	data	model
GDP	1	1
private consumption	0.36	0.28
investment	0.19	0.24
public consumption	0.15	0.18
net exports	0.30	0.30

Table 1: Implied ratios (w.r.t. GDP, average 2004-2008): data *vs.* model

5 Policy experiments

For all the following simulations, we maintain the population distribution across ages at average levels observed in 2004-2008, that is we impose $X^x_{a,t+a} = X^x_{a,0+a}$, $\forall x$, $\forall a$ and $\forall t > 0$. The initial situation is displayed in Table 2. We successively look at the effects of a total factor productivity shock, a labour productivity shock, a cross-border shock and a bargaining power shock. All results are presented as percentage deviation from the initial situation, are displayed in Figures 8 and 9 and explained below.

resident employment	185 000
cross-border employment	110 000
resident employment 20-25	26 000
inactivity rate 55-60	40%
inactivity rate 60-65	80%
unemployment rate	4.5%
consumption	€ 27
savings	€ 5.2
capital stock	€ 63
labour share	65%
labour market tightness	1
GDP	€ 100
total pension expenses	€ 13
labour taxation receipts	€ 18
government consumption	€ 18
net exports	€ 30

Table 2: Initial values (average 2004-2008, GDP = € 100)

5.1 TFP shock

We introduce a permanent 1% level increase in TFP, that is $A_t = 1.01 \times A_0$, $\forall t > 0$. Activity and employment increases and unemployment falls (solid blue line with marker). Consumption, savings and exports are stimulated and GDP finally rises by about 2%. This increase is sligthly higher than what is usually observed with a closed economy model. Indeed, the interest rate goes up in the closed economy (which limits the positive effect on GDP) whereas the interest

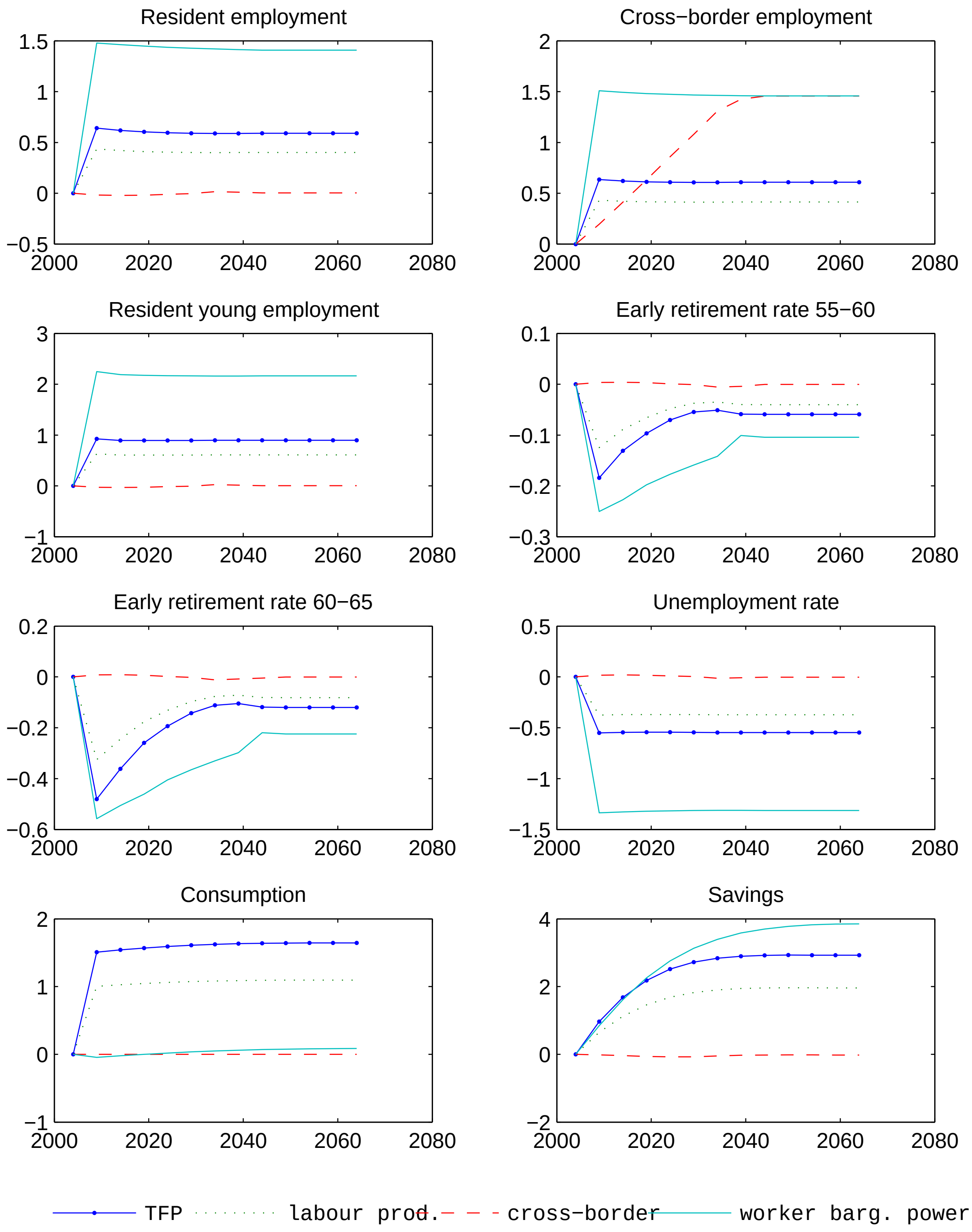

Figure 8: Different shocks with 2004-2008 calibration (% deviation from initial situation)

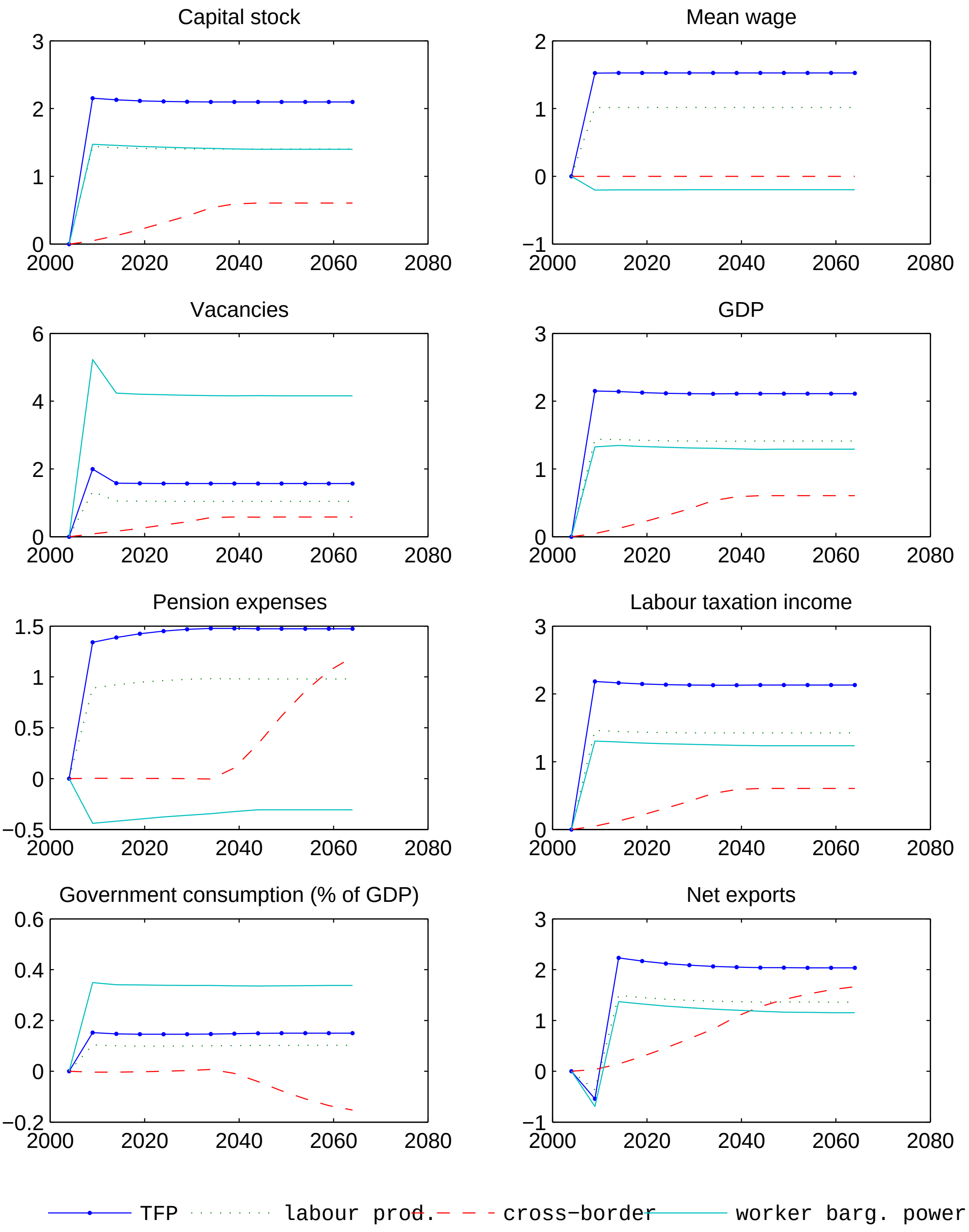

Figure 9: Different shocks with 2004-2008 calibration (ctd, % deviation from initial situation)

rate is fixed in our SOE. The increase in government receipts is stronger (higher employment and consumption) than the increase in government expenditures (higher pension level because of higher wages but less early retired) and this improves the government budget (about 0.2% of GDP).

5.2 Labour productivity shock

We introduce a permanent 1% level increase in employment productivity (for both cross-border commuters and residents, and for all ages), that is $h^x_{a,t+a} = 1.01 \times h^x_{a,0+a}$, $\forall x$, $\forall a$ and $\forall t > 0$. Effects are qualitatively equivalent to a TFP shock but weaker because only one of the two productive inputs is affected (dotted green line). As a result, GDP "only" rises by 1.4%. It is worth noting that after productivity shocks (both total factor and labour) net exports initially decrease (1st period). In a closed economy, capital (investment) supply cannot jump immediately after a positive productivity shock and therefore the interest rate rises. In our SOE, the interest rate is exogenous and investment demand is immediately filled by supply from the rest of the world (increase in imports) and so we have an initial net export contraction.

5.3 Cross-border job-seeker shock

We introduce a permanent 1% level increase in the cross-border population (both cross-border workers and job seekers), that is $x^f_t = 1.01 \times x^f_0$, $\forall t > 0$. Cross-border employment progressively increases and reaches its final level after 40 years, that is when the first generation of cross-border commuters hit by the shock (25 years in 2009) retires (65 years in 2049) (dashed red line). Pierrard (2008) shows that if cross-border commuters have comparative advantages (for instance lower wages and/or higher productivity), an increase in cross-border commuters will also be beneficial for resident employment (positive externality). Similarly, if cross-border commuters have comparative disadvantages (for instance higher wages and/or lower productivity), an increase in cross-border commuters will harm resident employment (negative externality). In our model, cross-border commuter and resident workers have exactly the same characteristics (same productivity and same wages) and it is therefore not surprising that resident employment and unemployment levels are almost unaffected by the commuting shock. The increase in cross-border employment has no effect on government expenses until they reach the age of 55 (possibility of early retirement in 2039). Then pension expenditures increase strongly and cannot be fully compensated by higher government income. In the long run, government consumption must fall by about 0.2% of GDP to keep the budget in equilibrium.

5.4 Employee bargaining power shock

We introduce a permanent 1 percentage point reduction in employee bargaining power (for all ages), that is $\eta_{a,t} = \eta_{a,0} - 0.01$, $\forall t > 0$. Although the implied reduction in wages is relatively limited (-0.2%), this has huge effects on vacancy openings (+4%) which in turn strongly stimulate activity and employment (solid cyan line).[13] Again, despite the relatively minor fall in early retirement and retirement benefits (because they are indexed to wages), there is a strong incentive for households to increase savings (rather than consumption). The fall in government expenses and the rise in receipts definitively improves the government budget (0.4% of GDP).

6 Ageing of the population

In most (if not all) European countries, the ageing of the population could have dramatic consequences for the sustainability of pay-as-you-go pension systems. In Luxembourg, strong employment growth of the last decades, mainly supported by young cross-border commuters, currently makes the pension system *sound* but the long-run prospects are uncertain. Will a continued strong growth in cross-border commuters be sufficient to ensure a sustainable pension system or are we heading into an inevitable collapse? In the latter case, it would be urgent to implement structural measures such as a reduction in pension benefits (or alternatively other government expenses), an increase in the *statutory* retirement age, an increase in the activity rate especially for the 55-65 group (*i.e.* an increase in the *effective* retirement age, which is one the main objectives of the "Lisbon strategy"), or an alternative financing of the pension system (shifting from from 1st to 2nd and 3rd pillars). We use our model to answer some of these questions. Our initial year is 2004-2008 (period $t = 0$) and the initial situation is displayed in Table 2. All results are presented as percentage deviation from the initial situation, are displayed in Figures 10 and 11 and explained below.

6.1 Benchmark

For all periods $t > 0$, we simply assume the natural ageing of the population (both resident and non-resident), that is we set $X^x_{a,t+a} = 0$, $\forall x$, $\forall a$ and $\forall t > 0$. It is worth noting that we keep the current (2004) survival probability $\beta^x_{a,t+a}$ because we don't have data about its - expected - evolution. Changing $\beta^x_{a,t+a}$ over time would reinforce the ageing effects.

The current age structure of cross-border commuters (mainly young) implies that their employment will increase in the future, which raises GDP by about 1.5% per year during the next 15 years (solid blue line with marker). However, the higher government receipts are not sufficient to pay pensions and the pension reserves (currently around 21% of GDP) are fully eaten up on a 13-year horizon.[14] Without any other growth/reform, in the long-run the government would have to reduce its consumption by about 15% of GDP to maintain a balanced budget.

[13]The implied elasticty of vacancies with respect to wages seems very high. This should checked in future versions of the model.

[14]We assume that reserves are not invested. An average positive return on reserves would of course slightly increase the sustainability of the pension system.

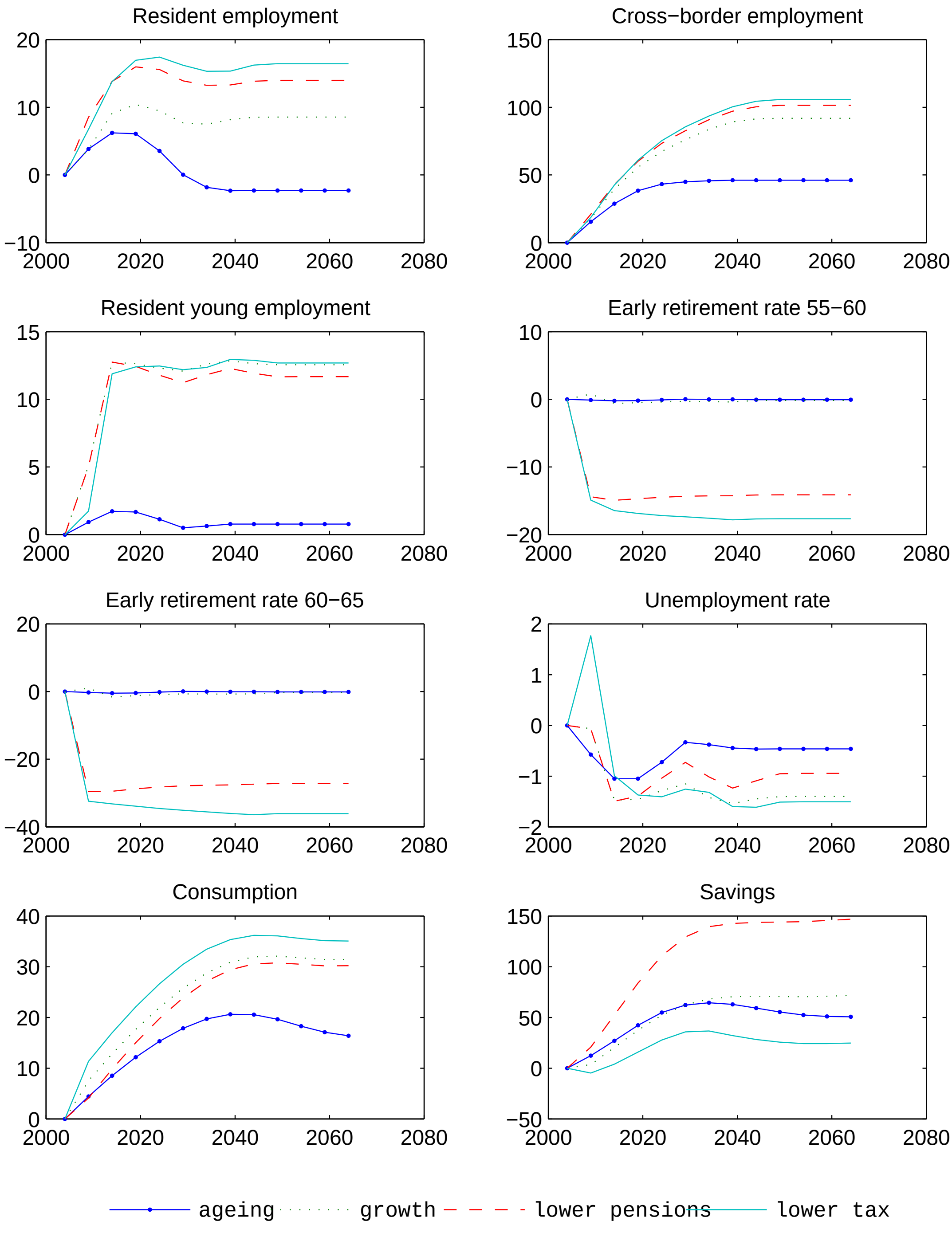

Figure 10: Population ageing with alternative assumptions (% deviation from initial situation)

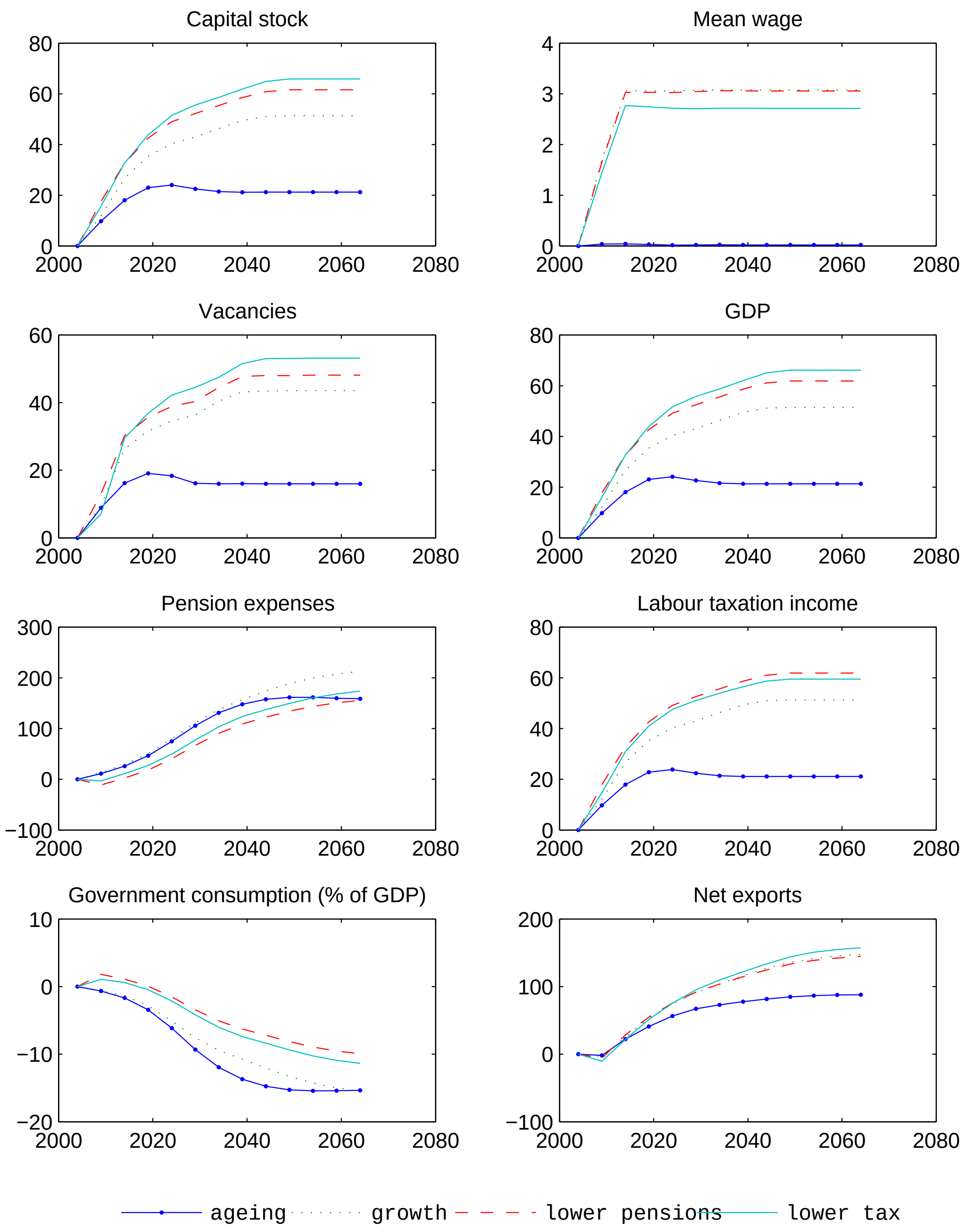

Figure 11: Population ageing with alternative assumptions (ctd, % deviation from initial situation)

6.2 Productivity and population growth

Apart from the impact of ageing, the first simulation keeps all other parameters at their 2004-2008 values (same productivity, same arrival of new cross-border commuters, ...). Following Guarda (2008), we now also assume a positive TFP (total factor productivity) shock on A_t of 0.2% per year during the next 10 years (from 2009 till 2018, that is for $t = 1, 2$). We also assume an immigration shock (or equivalently a fertility shock) on x_t^h of 1% per year and a cross border shock on x_t^f of 3% per year, both of them during the next 10 years. The last two shocks correspond to a lower bound relative to past observed values. We only introduce growth for the next 10 years because of the strong uncertainties surrounding the longer-term future. From 2019 onwards, we assume zero growth (that is A_t, x_t^h and x_t^f keep their 2018 values). This is obviously a rough way to introduce shocks but this nevertheless gives a broad idea of the effects of growth.

With respect to the previous simulation, we have a stronger increase in cross-border employment but also an increase in resident employment, and GDP now increases by 2.5% per year during the next 15 years (dotted green line). We also see that wages creep up in line with productivity gains and that the higher growth translates into higher consumption rather than higher savings (pensions are at a fairly high level and there is no need for further developing the third pillar). The worsening of the government financial position is slightly smoother than previously but in the long-run, the same problem remains.[15]

6.3 Lower retirement benefits

We keep the ageing of the population as well as the growth shocks, but we also add a decrease of 10% in the replacement ratio for retirement (and early retirement) benefits ρ^i and ρ^e, from 2009 onwards.

Lower pensions have two positive effects for the government budget: on the one hand this reduces expenses and on the other hand it increases activity and therefore receipts. As result, reserves increase further during the next 10 years and remain positive on a 27-year horizon (dashed red line). The long run problem is also reduced by almost 50%. These results are in line with Bouchet (2006), showing, using the same accounting model as in Bouchet (2003), that a gradual reduction of 20% in replacement ratio would help to generate a more sustainable

[15]Using an accounting model, Bouchet (2003) conducts similar simulations. He uses same assumptions about immigration and commuting. However, instead of a TFP shock, he assumes a labour productivity shock of 2% per year (roughly corresponds to a TFP growth of 1% per year). He also introduces real wage growth of 2% per year (that we do not have because this is an endogenous variable in our model). Moreover, his shocks last for 85 years instead of 10 in our model. Since his assumptions are less restrictive than ours, it is not surprising that he finds a long run cost of "only" 8% of GDP. However, when he introduces more restrictive assumptions such as a top-grading of the population, his costs increase to 14% of GDP.

pension system. Our model also show that higher activity for old workers is not detrimental to existing employment (unemployment rate is almost unchanged). That is, contrary to common wisdom, hiring old workers or keeping them at work does not sacrifice the job opportunities of the young. Finally, the lower legal pensions (first pillar) encourages savings (third pillar) which further stimulates GDP through the higher capital stock.

6.4 Lower "senior" employee taxation

The previous policy (lower pensions) might well be unpopular and politically difficult to implement. Alternatively, we consider replacing it by a reduction of 10 points in employee taxation τ^w for the 55-65 workers, from 2009 onwards.

Effects (higher activity, employment and GDP) are quite similar to the previous simulations (with lower pensions). The main difference is that the increase in savings is replaced by an increase in consumption, making this policy more acceptable to the population (solid cyan line). This policy is also auto-financed in the sense that the lower taxation rate is more than compensated by the increase in the taxable basis.

It is worth noting that we could imagine a policy combining both lower pension and lower taxation. This would maximize the positive effects on the budget while minimising the unpopular aspect of policy.

7 Conclusions

In this paper, we constructed and calibrated a SOE-OLG model for Luxembourg, and showed that the model can answer different economic questions. However, this paper is only a first step and new developments should be added in the future. For instance, in a version 1.1, we could improve the calibration (wages across generations too steep?, differentiation between residents and non-residents?, correct bargaining power?, ...), develop the public finance block (lower taxation on consumption, but lump-sum transfers from residents?, other type of taxation for firms?, ...), allow for public deficit financed by government bonds or introduce one more young generation (20-25). More ambitiously, in a version 2.0, a more realistic goods market (imperfections and demand from the rest of the world) would probably be welcome.

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Appendix: Household Optimization Problem

With initial and final financial wealth equal to zero (no bequests), the household's intertemporal budget constraint can be written as follows [16]:

$$\sum_{a=0}^{14} R_{t,t+a}^{-1}\, \beta_{a,t+a} \Big\{ \Big[(1-\tau_{a,t+a}^{w})\, w_{a,t+a}\, n_{a,t+a} + b_{a,t+a}^{u}\, u_{a,t+a} + b_{a,t+a}^{e}\, e_{a,t+a} + b_{a,t+a}^{i}\, i_{a,t+a} \Big] . q_{a,t+a} - (1+\tau_{t+a}^{c})\, c_{a,t+a} \Big\} = 0\,. \tag{20}$$

The discount factor $R_{t,t+a}$ is defined by $R_{t,t} = 1$ and $R_{t,t+j} = \Pi_{j=1}^{a} R_{t+j}$ for $j \geq 1$.
The values of $c_{a,t+a}$, $\lambda_{6,t+6}$ and $\lambda_{7,t+7}$ maximizing the household objective function (6) subject to (3) and (5) and the intertemporal budget constraint (20) can thus be obtained from the maximization of the following Lagrangean function:

$$\frac{W_t^H}{Z_{0,t}} = \max_{c_{a,t+a},\, \lambda_{6,t+6},\, \lambda_{7,t+7}} \sum_{a=0}^{14} \beta_{a,t+a} \Bigg\{ \beta^a \left(u(c_{a,t+a}) - d^n\, n_{a,t+a} . q_{a,t+a} + d_a^e\, \frac{(e_{a,t+a})^{1-\phi}}{1-\phi}\, q_{a,t+a} \right) + \mu_t\, R_{t,t+a}^{-1} \Bigg(\Big[b_{a,t+a}^{u} + \Big((1-\tau_{a,t+a}^{w})\, w_{a,t+a} - b_{a,t+a}^{u} \Big)\, n_{a,t+a} + (b_{a,t+a}^{e} - b_{a,t+a}^{u})\, e_{a,t+a} \Big] . q_{a,t+a} - (1+\tau_{t+a}^{c})\, c_{a,t+a} \Bigg) \Bigg\}$$

where μ_t is the Lagrange multiplier associated to the intertemporal budget constraint. The optimal values of $c_{a,t+a}$, $\lambda_{6,t+6}$ and $\lambda_{7,t+7}$ must satisfy the following first-order optimality conditions:

$$\beta^a\, \frac{u'_{c_{a,t+a}}}{1+\tau_{t+a}^{c}} = \mu_t\, R_{t,t+a}^{-1}\,; \tag{21}$$

$$\begin{aligned} &\left[d^n\, \frac{\partial n_{6,t+6}}{\partial \lambda_{6,t+6}} - d_6^e\, (e_{6,t+6})^{-\phi}\, \frac{\partial e_{6,t+6}}{\partial \lambda_{6,t+6}} \right] + \beta\, \frac{\beta_{7,t+7}}{\beta_{6,t+6}} \left[d^n\, \frac{\partial n_{7,t+7}}{\partial \lambda_{6,t+6}} - d_7^e\, (e_{7,t+7})^{-\phi}\, \frac{\partial e_{7,t+7}}{\partial \lambda_{6,t+6}} \right] \\ &\quad = \frac{u'_{c_{6,t+6}}}{1+\tau_{t+6}^{c}} \left[\Big((1-\tau_{6,t+6}^{w})\, w_{6,t+6} - b_{6,t+6}^{u} \Big)\, \frac{\partial n_{6,t+6}}{\partial \lambda_{6,t+6}} + \Big(b_{6,t+6}^{e} - b_{6,t+6}^{u} \Big)\, \frac{\partial e_{6,t+6}}{\partial \lambda_{6,t+6}} \right] \\ &\qquad + \beta\, \frac{\beta_{7,t+7}}{\beta_{6,t+6}}\, \frac{u'_{c_{7,t+7}}}{1+\tau_{t+7}^{c}} \left[\Big((1-\tau_{7,t+7}^{w})\, w_{7,t+7} - b_{7,t+7}^{u} \Big)\, \frac{\partial n_{7,t+7}}{\partial \lambda_{6,t+6}} + \Big(b_{7,t+7}^{e} - b_{7,t+7}^{u} \Big)\, \frac{\partial e_{7,t+7}}{\partial \lambda_{6,t+6}} \right]\,; \end{aligned} \tag{22}$$

$$\begin{aligned} &\left[d^n\, \frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} - d_7^e\, (e_{7,t+7})^{-\phi}\, \frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} \right] \\ &\quad = \frac{u'_{c_{7,t+7}}}{1+\tau_{t+7}^{c}} \left[\Big((1-\tau_{7,t+7}^{w})\, w_{7,t+7} - b_{7,t+7}^{u} \Big)\, \frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} + \Big(b_{7,t+7}^{e} - b_{7,t+7}^{u} \Big)\, \frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} \right]\,. \end{aligned} \tag{23}$$

[16] It is clear from equation (20) that consumption taxes have the same effect as income taxes.

In these expressions,

$$\frac{\partial e_{6,t+6}}{\partial \lambda_{6,t+6}} = 1\,; \qquad \frac{\partial e_{7,t+7}}{\partial \lambda_{6,t+6}} = (1-\lambda_{7,t+7})\,; \qquad \frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} = (1-\lambda_{6,t+6})\,;$$
$$\frac{\partial n_{6,t+6}}{\partial \lambda_{6,t+6}} = -\frac{n_{t+6}}{1-\lambda_{t+6}}\,; \qquad \frac{\partial n_{7,t+7}}{\partial \lambda_{6,t+6}} = -\frac{n_{t+7}}{1-\lambda_{t+6}}\,; \qquad \frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} = -\frac{n_{t+7}}{1-\lambda_{t+7}}\,.$$

The first optimality condition (21) is the usual Euler condition. It implies:

$$\frac{u'_{c_{a,t+a}}}{1+\tau^c_{t+a}} = \beta\, R_{t+a+1}\, \frac{u'_{c_{a+1,t+a+1}}}{1+\tau^c_{t+a+1}}\,.$$

The other two optimality conditions are specific to this model and determine the activity rate of senior workers. After substitution and rearrangements (where we also use (3)) and with the assumption that $u(c_{a,t+a})$ is logarithmic, these optimality conditions can be recast as follows:

$$\begin{aligned}
&\left[\frac{b^e_{6,t+6}-b^u_{6,t+6}}{(1+\tau^c_{t+6})\,c_{6,t+6}} + d^e_6\,\left(e_{6,t+6}\right)^{-\phi}\right]\,(1-e_{6,t+6}) \\
&\quad + \beta\,\frac{\beta_{7,t+7}}{\beta_{6,t+6}}\left[\frac{b^e_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} + d^e_7\,\left(e_{7,t+7}\right)^{-\phi}\right]\,(1-e_{7,t+7}) \\
&= \left[\frac{(1-\tau^w_{6,t+6})\,w_{6,t+6}-b^u_{6,t+6}}{(1+\tau^c_{t+6})\,c_{6,t+6}} - d^n\right]\,n_{6,t+6} \\
&\quad + \beta\,\frac{\beta_{7,t+7}}{\beta_{6,t+6}}\left[\frac{(1-\tau^w_{7,t+7})\,w_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} - d^n\right]\,n_{7,t+7}\,,
\end{aligned} \tag{24}$$

and

$$\left[\frac{b^e_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})c_{7,t+7}} + d^e_7\,\left(e_{7,t+7}\right)^{-\phi}\right](1-e_{7,t+7}) = \left[\frac{(1-\tau^w_{7,t+7})w_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})c_{7,t+7}} - d^n\right] n_{7,t+7} \tag{25}$$

The economic interpretation of these optimality conditions becomes easier if we notice that the unconditional probability of having a job is given by:

$$\pi_{a,t+a} = \frac{n_{a,t+a}}{n_{a,t+a}+u_{a,t+a}} = \frac{n_{a,t+a}}{1-e_{a,t+a}}\,,$$

so that the last optimality condition for instance can be written as follows:

$$\frac{b^e_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} + d^e_7\,\left(e_{7,t+7}\right)^{-\phi} = \pi_{7,t+7}\,\left[\frac{(1-\tau^w_{7,t+7})\,w_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} - d^n\right] + (1-\pi_{7,t+7})\,\left[\frac{b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}}\right]\,,$$

and similarly for the other optimality condition.

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