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ON-THE-JOB SEARCH AND CYCLICAL UNEMPLOYMENT: CROWDING OUT VS. VACANCY EFFECTS

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On-the-job Search and Cyclical Unemployment: Crowding Out vs. Vacancy Effects*

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Abstract

We incorporate on-the-job search (OTJS) into a real business cycle model in order to study whether OTJS increases the cyclical volatility of unemployment and vacancies. The increased search of employed workers during expansions has two effects on the unemployed: it induces firms to open more vacancies, but employed workers also crowd out unemployed workers in the job search. The overall effect of OTJS on unemployment volatility is thus ambiguous. We show analytically and numerically that the difference between the (employer's share of the) surplus of match with a previously employed versus a previously unemployed job seeker determines the degree to which OTJS increases unemployment volatility. We use this result to re-consider some related papers of OTJS and explain the amplification of volatility they obtain.

Keywords: on-the-job search, cyclical properties

JEL classification: E24, E32, J64

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Résumé non-technique

Le modèle d'appariement de Diamond-Mortensen-Pissarides est une manière simple de prendre en compte l'existence de frictions, les difficultés d'appariement et les problèmes de coordinations liés au déficit d'information que l'on peut rencontrer sur le marché du travail. Cette façon de modéliser le marché du travail est courante dans la littérature économique théorique. Il faut cependant noter que dans le modèle 'standard', seuls les chômeurs recherchent activement un emploi, c'est-à-dire que les travailleurs sont toujours satisfaits de leur emploi actuel. Cela est plutôt irréaliste car dans les données réelles, la majorité des transitions sont du type emploi vers emploi plutôt que du type chômage vers emploi. D'autre part, ce modèle 'standard' est incapable de reproduire la forte volatilité du taux de chômage que l'on peut observer dans les données.[1]

Plusieurs papiers récents montrent qu'en fait, introduire une intensité de recherche d'emploi non nulle et endogène pour les travailleurs permet également d'augmenter la volatilité du taux de chômage et donc de rendre le modèle plus réaliste. L'intuition est qu'une période de haute conjoncture stimule la recherche d'emploi des travailleurs (plus facile de trouver un nouveau job et salaires intéressants) et donc incite les firmes à ouvrir plus de postes vacants (plus facile de remplir ces postes vacants), ce qui *in fine* est également bénéfique pour les chômeurs (sortie du chômage plus rapide). Par contre, on peut objecter que les travailleurs cherchant un nouvel emploi sont en compétition avec les chômeurs et qu'une hausse de leur intensité de recherche sera préjudiciable aux chômeurs.

Dans ce papier, nous construisons un petit modèle afin de rationaliser ces deux types d'arguments. Nous décomposons les effets d'un choc conjoncturel positif en un 'vacancy effect' et un 'crowding out effect'. Le premier effet représente l'augmentation du nombre de postes vacants et augmente la probabilité de sortie du chômage. Le second effet représente la plus grande compétition entre travailleurs et chômeurs et ralentit la probabilité de sortie du chômage. Quand le premier effet domine, l'introduction d'une intensité de recherche non nulle et endogène pour les travailleurs permet effectivement d'augmenter la volatilité du taux de chômage. Quand le second effet domine, l'introduction d'une intensité de recherche non nulle et endogène pour les travailleurs ne permet pas d'augmenter la volatilité du taux de chômage mais la diminue. Dans ce papier, nous montrons analytiquement et numériquement que le premier effet domine quand les travailleurs (relativement aux chômeurs) sont suffisamment intéressants pour les entreprises (par exemple parce qu'ils ont une productivité plus élevée). Dans ce cas, les firmes réagissent fortement à cette offre de main d'oeuvre intéressante et créent suffisamment de nou-

[1] Ainsi, entre le sommet et le creux d'un cycle économique, le taux de chômage peut parfois varier du simple au double, et ce tant aux USA que dans de nombreux pays européens.

veaux postes, ce qui permet de plus que compenser l'effet compétition pour les chômeurs.

Le modèle que nous proposons permet d'être résolu analytiquement. Cependant, pour y arriver, nous devons introduire certaines hypothèses simplificatrices. Ainsi, nous supposons un 'random search', c'est-à-dire qu'une entreprise n'ouvre qu'un seul type de poste vacant et accepte le premier postulant qu'elle rencontre. Introduire du 'directed search' pourrait évidemment être intéressant mais rendrait le modèle plus complexe. L'introduction de rigidités dans le processus de formation des salaires serait également intéressante et renforcerait probablement les effets de recherche mais cela ajoute certaines difficultés (voir par exemple Shimer, 2006, et le problème de non-convexité). Nous laissons ces extensions pour de possibles recherches futures.

1 Introduction

As is well known, when a standard search-matching unemployment model such as Pissarides (2000) is embedded into a standard dynamic stochastic general equilibrium model of the macroeconomy, it generates too little volatility over the business cycle in the key labor market variables of unemployment and job vacancies (Shimer (2005)). A number of fixes have been proposed for this problem. One is to introduce some form of wage rigidity, by assumption (Gertler and Trigari (2009)), by calibration of the wage bargaining (Hagedorn and Manovskii (2008)) or by altering the bargaining mechanism (Hall and Milgrom (2008)). A second solution is the introduction of countercyclical vacancy costs as in Yashiv (2006) or Fujita and Ramey (2007). A third potential fix is to incorporate on-the-job search (OTJS) by currently employed workers for better jobs. While OTJS has been explored extensively in the partial equilibrium literature, to our knowledge only four papers to date have examined how OTJS increases the unemployment volatility in a DSGE context: Krause and Lubik (2010) and Van Zandweghe (2010) consider bifurcated labor markets in which workers with bad jobs search for good jobs, while Tasci (2007) and Nagypal (2007) construct models with imperfectly observed match quality, in which all employed workers search, but they only accept matches with a higher expected quality than the one they are currently in.[2]

It is worth noting these models of OTJS are quite different from one another and rely on specific assumptions as well as on different mechanisms to amplify the volatility of labor market variables. In this paper, we present a very simple model of OTJS, staying as close as possible to Pissarides (2000). First, we show that OTJS may increase *but also may* decrease the volatility of unemployment, depending on the difference between the match surplus of an experienced *vs.* an inexperienced match. Second, we adapt our model to re-consider some of above-mentioned papers of OTJS and we explain the amplification of volatility they obtain in the light of this result.

Incorporating OTJS is expected to increase the volatility of unemployment and vacancies over the business cycle through several mechanisms. First, even if employed workers search with less intensity than unemployed workers (hereafter experienced and inexperienced workers respectively), as is the case in this paper, OTJS smoothes the number of potential hires businesses face over the course of the business cycle, leading firms to post more vacancies during expansions than they otherwise would, which results in more matches with inexperienced workers and thus lower unemployment. This mechanism is common to all OTJS models. Second, if workers' gains from finding a better job are procyclical, experienced workers will expend greater search effort during expansions than during recessions, which serves to accentuate this

[2]See section 6 for more details.

first effect. This is the primary mechanism explored in Krause and Lubik (KL hereafter) and in van Zandweghe, as well as in this paper. Third, in models in which experienced workers are choosier about which jobs they will accept than are inexperienced workers, firms prefer to poach employees from one another rather than hiring the unemployed, because experienced workers are expected to stay at the new job longer than are inexperienced workers. Because a larger fraction of matches are with experienced workers during expansions, vacancy creation is more procyclical than it otherwise would be. This is the primary mechanism explored in Tasci and Nagypal.

In this paper, we follow Krause and Lubik (2010), using their calibration for OTJS activity. However, whereas KL assume a bifurcated labor market with good and bad jobs, we simplify things and assume a unified labor market. To motivate workers to search OTJ, we allow experienced workers to negotiate wages at their new job with their old job as a fallback position.[3] Each of the four OTJS papers above uses unemployment as the fallback position for all new matches, both those with experienced and inexperienced workers. Fujita (2011), however, reports empirical evidence supporting our position that, while workers rarely leverage outside job offers for higher pay at their old job, they do earn more at subsequent jobs simply from having been previously employed.[4] As would be expected, and as in KL, OTJS activity is procyclical in our model. The increased search of experienced workers during expansions has two effects on the unemployed: it induces firms to open more vacancies, but experienced workers also crowd out inexperienced workers in the job search. The overall effect of OTJS on unemployment is thus ambiguous. Gautier (2002) finds similarly ambiguous effects of the job search of high-skilled on low-skilled workers, and Pierrard (2008) of commuters' job search on residents.

When wages for experienced workers are higher due to their superior bargaining position, the match surplus with an experienced worker is smaller than with an inexperienced one. Under these circumstances,[5] the net effect of endogenous (procyclical) search intensity is to decrease unemployment volatility (i.e. unemployment varies less over the business cycle than in a model with constant, exogenous search intensity). To proxy for the fact that workers typically *trade up* in accepting a new job they found while searching OTJ, we give a productivity boost to experienced workers over inexperienced workers.[6] When this productivity boost is significant enough to make the surplus of an experienced match exceed that of an inexperienced match,

[3]As in the other papers, we allow wages to be renegotiated each period. Because of this, the advantage experienced workers enjoy in wages comes as a one-time hiring bonus in their first period at the new job.

[4]In section 6 we relax this assumption, forcing experienced workers to negotiate wages with unemployment as a fallback. Since this increases the surplus of an experienced match relative to an inexperienced one, we find that it increases the volatility of unemployment relative to our benchmark.

[5]When $\bar{x} = 1$ in our model, see section 2.

[6]Again, to allow for continually renegotiated wages, the value of this boost is realized and split between firm and worker in the first period of employment in a new job.

the job creation effect overpowers the crowding out effect, and endogenous OTJS results in greater unemployment volatility.

In experimenting with a number of alternative model specifications (unemployment as a fall-back for wage negotiation, a one-time hiring cost, allowing workers to switch jobs only once coupled with a permanent increase in productivity when they do so, a larger firm share in the wage bargain, and an endogenous search intensity of the unemployed – see section 6), we confirm that the difference between the (employer's share of the) match surplus of an experienced versus an inexperienced match largely determines the degree to which OTJS increases unemployment and vacancy volatilities. This paper presents what we believe is a simple and parsimonious method for varying the value of an experienced job match relative to an inexperienced one while staying as close as possible to the benchmark search and matching model of Pissarides (2000).

The remainder of the paper is organized as follows. Section 2 details the model. Section 3 proves the uniqueness of the steady state and determines under which conditions an increase in on-the-job search reduces steady state unemployment. Section 4 calibrates the model. Section 5 shows how, depending on the calibration, endogenous on the job search amplifies (or not) the volatility of the labor market variables. Section 6 compares our results to related literature. Section 7 concludes.

2 Model

The model embeds the search and matching framework *à la* Pissarides (2000) into a dynamic stochastic general equilibrium model. Unlike most of the related literature, we introduce on-the-job search. More precisely, both unemployed and employed workers search for a job. All unemployed workers search for a job with an intensity normalized to 1 whereas all employed workers search OTJ with an endogenous intensity $e_t \in (0, +\infty)$. An intermediate firm opening a vacancy may therefore match with an experienced (employed in the previous period) or inexperienced (unemployed in the previous period) worker. We assume that job vacancies enjoy an initial productivity, *i.e.* the productivity during the first period following the match, of 1 if filled with an experienced worker or of $\bar{x} \geq 0$ if filled with an inexperienced worker.[7] From the second period of the match onwards, we drop all distinction between workers who were employed or not before they started their current job; the productivity is 1 for all workers.

[7] *A priori*, we do not restrict the value of $\bar{x}$, although we may intuitively expect $\bar{x} > 1$ because unemployment depreciates skills, and because workers switch to jobs where they are more productive than the one they left.

2.1 Labor market flows

The labor market force is normalized to 1 and split between the employed n_t and the unemployed u_t:

$$u_t = 1 - n_t. \tag{1}$$

Intermediate firms open vacancies v_t. The number of new matches between job seekers and intermediate firms is generated by a standard Cobb-Douglas matching function:[8]

$$m_t = \bar{m}\, v_t^{1-\mu}\ (e_t n_t + u_t)^{\mu}\,, \tag{2}$$

where $\bar{m} > 0$ and $0 < \mu < 1$. The probability for an unemployed job seeker to find a job is $p_t = m_t/(e_t n_t + u_t)$, the probability for an employed job seeker to find a job is $p_t e_t$, and the probability for an intermediate firm to fill a vacancy is $q_t = m_t/v_t$. The labor market tightness is $\theta_t = v_t/(e_t n_t + u_t)$. Employment evolves according to :

$$n_{t+1} = (1-\rho)\ (p_t u_t + n_t)\,, \tag{3}$$

where $0 < \rho < 1$ is the exogenous job destruction rate.

2.2 Representative household

As in Merz (1995) or Andolfatto (1996), we assume a representative household pooling income between employed and unemployment workers. This household also owns intermediate firms and therefore receives their profits. It lives indefinitely and chooses the optimal consumption path, with preferences represented by a log-utility function.[9] As a result, intermediate firms and workers discount returns in the subsequent period according to $\beta C_t/C_{t+1}$, where C_t is consumption and $0 < \beta < 1$ is the household's exogenous discount factor.

2.3 Intermediate firms

A new job with an experienced worker has a productivity $\bar{x}$ the first period, whereas a new job with an inexperienced worker or an old job has a productivity normalized to 1. The asset values of the two types of jobs are respectively:

$$J_t^n = \bar{x} P_t - w_t^n + (1-\rho)\,(1-p_t e_t)\ E_t\left[\frac{\beta\, C_t}{C_{t+1}}\, J_{t+1}^o\right], \tag{4}$$

$$J_t^o = P_t - w_t^o + (1-\rho)\,(1-p_t e_t)\ E_t\left[\frac{\beta\, C_t}{C_{t+1}}\, J_{t+1}^o\right]. \tag{5}$$

[8] Cobb-Douglas matching functions are standard in the literature, although they imply that probabilities of matching in a given period can be greater than 1. This is not the case in our calibration.

[9] $\max_{\{C_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \ln(C_t)$, as in Krause and Lubik (2010).

Intermediate firms sell their goods at the competitive price P_t, w_t^n and w_t^o are the respective wages, and E_t denotes expectations. An intermediate firm opening a vacancy pays a cost $c > 0$ and the free entry condition implies:

$$c = q_t \, (1-\rho) \, E_t \left[\frac{\beta \, C_t}{C_{t+1}} \left(\frac{e_t n_t}{e_t n_t + u_t} J_{t+1}^n + \frac{u_t}{e_t n_t + u_t} J_{t+1}^o \right) \right]. \tag{6}$$

2.4 Workers

On the one hand, a worker may have a new job and be experienced. In this case, its asset value is W_t^n. On the other hand, a worker may have a new job and be inexperienced or may have an old job. In this case, the worker's asset value is W_t^o.

$$W_t^n = w_t^n - \mathcal{S}(e_t) + E_t \left[\frac{\beta \, C_t}{C_{t+1}} \, ((1-\rho) \, ((1 - p_t e_t) W_{t+1}^o + p_t e_t W_{t+1}^n) + \rho \, U_{t+1}) \right], \tag{7}$$

$$W_t^o = w_t^o - \mathcal{S}(e_t) + E_t \left[\frac{\beta \, C_t}{C_{t+1}} \, ((1-\rho) \, ((1 - p_t e_t) W_{t+1}^o + p_t e_t W_{t+1}^n) + \rho \, U_{t+1}) \right]. \tag{8}$$

$\mathcal{S}(e_t)$ is the on-the-job search cost, which we specify to be quadratic, that is $\mathcal{S}(e_t) = \bar{e} \, e_t^2/2$ with $\bar{e} > 0$. The asset value of an unemployed worker is:

$$U_t = z + E_t \left[\frac{\beta \, C_t}{C_{t+1}} \, (W_{t+1}^o \, p_t \, (1-\rho) + U_{t+1} \, (1 - p_t \, (1-\rho))) \right], \tag{9}$$

where $z > 0$ represents unemployment benefits. The first order condition for search intensity is:

$$\mathcal{S}'(e_t) = p_t (1-\rho) E_t \left[\frac{\beta \, C_t}{C_{t+1}} \, (w_{t+1}^n - w_{t+1}^o) \right]. \tag{10}$$

This equation means that in equilibrium, the marginal cost of OTJS is equal to the expected discounted marginal return.

2.5 Wages

Workers and intermediate firms negotiate wages at the beginning of every period through a Nash (1950) bargain over the surplus resulting from the match. Since workers and firms do not commit to future wages, the non-convexity problem discussed in Shimer (2006) does not arise.[10] If an OTJ searcher finds a new job, he negotiates with the new firm over the joint surplus (increase in asset values) of the match, with the surplus defined relative to his asset value in

[10] When workers and firms commit to future wages, the payoff set for workers and firms is nonconvex in the negotiated wage, thus violating the standard assumptions for Nash bargaining. The nonconvexity comes from the fact that workers paid higher wages are expected to stay with their employer longer than workers paid lower wages, and thus the present discounted value of the total gains to be split is increasing in the wage. See also Krause and Lubik (2007), section 6.3, for a discussion.

the previous job.[11] If an unemployed worker finds a new job, or if an OTJ searcher fails to find a new job and remains on the same job, they bargain the wage w_t^o with unemployment as fallback position. This gives:

$$\eta \ (J_t^n + W_t^n - W_t^o) \ = \ W_t^n - W_t^o, \tag{11}$$

$$\eta \ (J_t^o + W_t^o - U_t) \ = \ W_t^o - U_t, \tag{12}$$

where $0 < \eta < 1$ is the worker's bargaining power. After computation, we obtain $w^n = w^o + \eta(P - w^o)/(1 - \beta(1-\rho)(1-pe)) + \eta(\bar{x} - 1)P$ at the steady state. It is worth noting that as long as $\bar{x}$ is not too low, $W^n - W^o = w^n - w^o > 0$ and this is sufficient to justify the OTJS decision.[12] We define $n_{t+1}^n = (1-\rho)(p_t e_t n_t)$ and $n_{t+1}^o = (1-\rho)((1-p_t e_t)n_t + p_t u_t)$. The average wage in the economy is given by:

$$w_t n_t = (w_t^n n_t^n + w_t^o n_t^o). \tag{13}$$

2.6 Closing the model

A final good firm buys the goods produced by the intermediate firms at a price P_t and produces a final good using a constant return to scale technology:

$$y_t = n_t \, A_t + n_t^n \ (\bar{x} - 1) \ A_t, \tag{14}$$

where $A_t = A_{t-1}^a \, exp\,(u_t^a)$ is an aggregate productivity shock, $0 < a < 1$ and $u_t^a \sim N(0, \sigma)$. The price of the final goods serves as numeraire and profit maximization gives:

$$P_t = A_t \tag{15}$$

Finally, consumption is equivalent to net output:

$$y_t - v_t \, c - \mathcal{S}(e_t) \, n_t^o = C_t \tag{16}$$

When $e_t = 0 \; \forall t$, the model simplifies to the benchmark search and matching framework.

[11] In essence, we assume that if the worker failed to reach an agreement with the new firm, he would continue to work at his old job, though in practice they always reach an agreement. It is worth noting that we impose that the new bargained wage cannot be renegotiate until the next period. If not, the new employer would immediately renegotiate the wage once the worker breaks the relationship with the previous employer. We also impose that the previous employer cannot make a counteroffer in response to the new offer. Most other papers, e.g. Krause and Lubik (2010), with on-the-job search and Nash bargaining use unemployment as the fallback wage for simplicity. See the related literature section for more.

[12] Assuming that the fallback position for an OTJ seeker is unemployment would require $\bar{x} > 1$ to justify the OTJS decision.

3 Analytical properties

In this section, we look at the steady-state and the dynamic equilibrium, and we provide analytical results on the role of OTJS.

3.1 Steady state analysis

Using the asset values and the wage equations, we obtain:

$$
\begin{aligned}
J^o(\theta, pe) &= (1-\eta)\,\frac{P\,(1+\beta(1-\rho)(\bar{x}-1)\,\eta\,pe) - z - \mathcal{S}(\theta, pe)}{1-\beta(1-\rho)\,(1-pe(1-\eta)^2 - \eta p(\theta))},\\
J^n(\theta, pe) &= (1-\eta)\,\left(J^o(\theta, pe) + (\bar{x}-1)P\right),\\
\mathcal{S}(\theta, pe) &= \frac{\bar{e}(pe)^2}{2p(\theta)^2}.
\end{aligned}
$$

A usual condition is:

Assumption 1
$J^o > 0$ and $J^n > 0$.

The positive asset values $J^o > 0$ and $J^n > 0$ imply $W^n - W^o > 0$ and $W^o - U > 0$ and the Nash bargains always reach agreements. Equilibrium is determined by the free entry condition and the search equation:

$$
\left\{
\begin{aligned}
I(\theta, pe) &= 0\\
H(\theta, pe) &= 0
\end{aligned}
\right.
$$

where:

$$
I(\theta, pe) = \frac{c(\rho + pe(1-\rho))}{\beta q(\theta)(1-\rho)} - pe(1-\rho)J^n(\theta, pe) - \rho J^o(\theta, pe), \tag{17}
$$

$$
H(\theta, pe) = \mathcal{S}(\theta, pe) - \frac{\beta\, pe\, \eta\, (1-\rho)}{2(1-\eta)}\, J^n(\theta, pe). \tag{18}
$$

In the benchmark search and matching model (without OTJS), $e = 0$ and the H equation is irrelevant. As a result, the equilibrium is simply given by $I(\theta, 0) = I(\theta) = 0$, with $I'(\theta) > 0$. Since I represents the net cost of opening a vacancy, $I'(\theta) > 0$ ensures the uniqueness and stability of the equilibrium.

OTJS adds the H equation, which complicates the derivation of an equilibrium. Moreover, the sign of $dI/d\theta = I_\theta + I_{pe}\, dpe/d\theta$ is ambiguous.[13] To simplify the derivation of the results, we impose one restriction:

[13]In the subsequent analysis, $dI/d\theta$ represents the total (full) derivative of I with respect to θ, whereas I_θ represents the partial derivative I with respect to θ, that is $\partial I/\partial\theta$.

Assumption 2
$J^o > eJ^n$.

At a given pe, an increase in θ has two opposite effects on J^o. On the one hand, it improves the fallback position U of a worker and hence increases his wage. On the other hand, it reduces the search cost $\mathcal{S}$ and hence the bargained wage. Appendix A gives the partial derivatives of J^o and I with respect to θ and shows that when $J^o > eJ^n$, *i.e.* when e and/or J^n are not too high, the first effect dominates and $J^o_\theta < 0$. This, in turn, is a sufficient condition to ensure $I_\theta > 0$.

Note that with OTJS, $I_\theta > 0$ does not necessarily imply $dI/d\theta > 0$. Below, we add one extra condition for the uniqueness of a steady state equilibrium.

3.1.1 Uniqueness of a steady state equilibrium

Proposition 1
Assumptions 1 and 2, as well as $I_{pe} > -I_\theta/(dpe/d\theta)$ are sufficient conditions for the uniqueness of a steady state equilibrium.

Proof. Appendix B demonstrates that equation (18) can be written $A(\theta)(pe)^2 + B(\theta)pe + C(\theta) = 0$ with $C(\theta) < 0 < A(\theta)$ for all θ, implying that pe can be expressed as a function of θ. The implicit function theorem implies that $dpe/d\theta = -H_\theta/H_{pe}$. Appendix B shows that this is positive. Now we are able to write equation (17) as $I(\theta) = I(\theta, pe(\theta))$, and so $dI/d\theta = I_\theta + I_{pe}dpe/d\theta$. Appendix A gives the partial derivatives of I. Clearly, over any interval of θ where $dI/d\theta = I_\theta + I_{pe}\, dpe/d\theta > 0$, at most one solution to $I = 0$ exists. Since $I_\theta > 0$ through assumption 2, the condition for $dI/d\theta > 0$ can then be written as $I_{pe} > -I_\theta/(dpe/d\theta)$: I_{pe} cannot be too negative. ■

As already explained, I reflects the net cost of opening a vacancy. Holding pe constant, opening more vacancies has a direct effect on this cost (I_θ) that is always positive because of assumption 2; but as firms open more vacancies, they also induce more OTJS (higher pe). Provided that matches with experienced workers have a larger surplus to split, I_{pe} can be negative. This is precisely the amplifying effect we're interested in this paper (see proposition 2); however if this effect is too strong it can lead to multiple equilibria as opening more vacancies actually decreases the cost of opening additional vacancies simply because workers are taking jobs in order to switch to new jobs in the future. Note that requiring I_{pe} not to be too negative is equivalent to requiring that J^n not be too large relative to J^o.[14] Note as well that we prove the

[14]It helps to write I_{pe} as $-(pe(1-\rho)(1-\eta)+\rho)J^o_{pe} - \frac{\rho(1-\rho)}{\rho+pe(1-\rho)}(J^n - J^o)$, where the first term reflects the decreased expected value of a match as pe increases because matches dissolve faster, and the second value reflects the increased expected value of a match as pe increases because a larger fraction of matches are with job switchers who are -

uniqueness of a steady state equilibrium but not the existence. In fact, the existence of a solution depends on the parameter values in a complex way,[15] but our numerical results show that a solution does exist for our calibration.

3.1.2 Vacancy *vs.* crowding-out effects

The amount of OTJS activity, en, has two effects on unemployment. On the one hand, it increases the competition for jobs, making it more difficult for the unemployed to find them (crowding out effect). On the other hand, it stimulates the opening of additional vacancies and makes it easier for the unemployed to find a job (vacancy effect). The crowding out effect dominates when $d\theta/den < 0$ whereas the vacancy effect dominates when $d\theta/den > 0$. Proposition 2 states the conditions for each effect to dominate:

Proposition 2
Under the conditions of proposition 1: when $I_{pe} > 0$, the crowding out effect dominates; when $0 > I_{pe} > -I_\theta/pe_\theta$, the vacancy effect dominates.

Proof. Since we assume exogenous OTJS, we only consider equation (17). We replace equation (18) by making pe a function of (θ, en): $pe(\theta, en) = en(\rho + (1-\rho)p(\theta))/(1-\rho)$. The implicit function theorem implies $I_\theta\ d\theta + I_{pe}\ pe_{en}\ den + I_{pe}\ pe_\theta\ d\theta = 0$ where all derivatives are given in appendix A and C. This gives $d\theta/den = -I_{pe}\ pe_{en}/(I_\theta + I_{pe}\ pe_\theta)$. If $I_{pe} > 0$, then $d\theta/den < 0$ and the crowding out effect dominates. If $0 > I_{pe} > -I_\theta/pe_\theta$, then $d\theta/den > 0$ and the vacancy effect dominates. ■

We can easily show that

$$\begin{aligned} I_{pe} > 0 \quad &\Longleftrightarrow \quad \frac{c}{\beta q} > (1-\rho)J^n + (pe(1-\rho)(1-\eta)+\rho)J^o_{pe} \\ &\Longleftrightarrow \quad J^n < \underbrace{\left(1 + \frac{(1-\eta)\beta(\rho + pe(1-\rho))(\rho + pe(1-\rho)(1-\eta))}{\rho - \beta\rho(1-\rho)\,(1 - pe(1-\eta)^2 - \eta p)}\right)}_{\equiv\alpha} J^o \\ &\Longleftrightarrow \quad J^n < \alpha J^o \end{aligned}$$

with $\alpha > 1$. This means that the crowding out effect dominates when J^n is small. The vacancy effect dominates when J^n becomes sufficiently higher than J^o. The intuition is the following: I represents the expected net cost of opening a vacancy. As pe increases, both J^o and J^n decrease (because of the higher chance of leave), which has a direct positive effect on the expected net cost I. At the same time though, as pe increases, the fraction of matches with experienced

potentially - more productive. Only through this second term being very large relative to the first, that is only when J^n is very high with respect to J^o, do you get $I_{pe} << 0 \Rightarrow dI/d\theta < 0$ and hence multiple equilibria.

[15] For example, if ρ is very large, no nondegenerate equilibrium exists.

workers increases. Through this composition effect, I decreases if $J^n > J^o$ and I increases if $J^n < J^o$. If $J^n < J^o$, the two effects go in the same direction, I_{pe} is positive and crowding out dominates. If J^n is slightly higher than J^o, the two effects run contrary to one another but the first one dominates and I_{pe} is still positive. If J^n is sufficiently higher than J^o, I_{pe} becomes negative and the vacancy effect dominates. It is worth noting that when J^n becomes too high, we know from proposition 1 that we may have multiple equilibria. In the section 4, we calibrate the model and adjust the value of the parameter $\bar{x}$ to change the value of J^n.

3.2 Dynamic equilibrium

To compute the dynamic equilibrium, we must approximate equations. To do so, we choose to log-linearize the model and we define a "hat" variable as the proportionate deviation of that variable from its steady state level. $\mathbf{X}_t^s = [\hat{n}_t^n \ \hat{n}_t^o \ \hat{A}_t]'$ is a vector collecting the 3 state variables of the model, $\mathbf{X}_t^c = [\hat{C}_t \ \hat{J}_t^n \ \hat{J}_t^o]'$ is a vector collecting the 3 control variables of the model and $\mathbf{Y}_t$ is a vector collecting all the other hat variables. We can write the log-linearized model as:

$$\begin{bmatrix} \mathbf{X}_{t+1}^s \\ E_t \mathbf{X}_{t+1}^c \end{bmatrix} = \Psi \begin{bmatrix} \mathbf{X}_t^s \\ \mathbf{X}_t^c \end{bmatrix} + \Gamma \, u_{t+1}^a$$

$$\mathbf{Y}_t = \Xi \begin{bmatrix} \mathbf{X}_t^s \\ \mathbf{X}_t^c \end{bmatrix}$$

where Ψ, Γ and Ξ are matrices. We decompose $\Psi = Q \Lambda Q^{-1}$ where Q is a matrix of eigenvectors and Λ is a diagonal matrix of eigenvalues. The dynamic solution of the model is unique if the number of unstable eigenvalues is exactly equal to the number of control variables. This is the case with fair calibrations and hence in the subsequent analysis.

4 Calibration

We fully borrow the calibration from Krause and Lubik (2010). We fix $\mu = 0.4$, $\rho = 0.10$, $\beta = 0.99$, $\eta = 0.5$, $a = 0.90$, $u_t^a \sim N(0, \sigma)$ and $\sigma = 0.0049$. We determine $\bar{m}$, c, $\bar{e}$ and z to reproduce the steady states $q = 0.70$, $c\,v/y = 0.05$ (vacancy costs as a fraction of output), $u = 0.12$ and $e\,p = 0.06$ (probability to voluntary quit a job).

The only parameter specific to the model is $\bar{x}$. We compute $\bar{x}$ so that the vacancy and the crowding out effects cancel each other and we obtain $\bar{x} \cong 1.44$. This means that the productivity when you switch jobs is 44% higher during the first quarter of employment. Although it is difficult to directly observe this value, we can nevertheless compute the implications of the productivity parameter in terms of wage gains. We define wage gains as $((1-(1-\rho)(1-pe))w^n/w^o+(1-\rho)(1-pe)-1)\times 100$ and $\bar{x}=1.44$ implies wage gains of 6.1%.[16] Using the UK labor force survey, Fujita (2011) reports that when unsatisfied with their current jobs, workers enjoy wage gains between 6% and 10% upon job-to-job transitions. The productivity we use is therefore not unrealistic, although close to the lower bound of the estimations. We discuss this in more details in section 5.

If $\bar{x}$ affects the wage gains, it obviously also directly affects $J^n-\alpha J^o$. According to proposition 2, figures 1 and 2 show that when $\bar{x}<1.44$, $J^n<\alpha J^o$ and on-the-job search increases the unemployment level. When $\bar{x}>1.44$, $J^n>\alpha J^o$ and on-the-job search decreases the unemployment level. It is worth noting that $\bar{x}$ cannot be lower than 0.67 because J^n becomes negative (see section 3) and $\bar{x}$ cannot be higher than 1.71 because p becomes higher than 1.[17]

5 Dynamic simulations

In this section, economic fluctuations are driven by the productivity shock defined in section 2. We look at the cyclical properties of the model for different values of $\bar{x}$ when (i) on-the-job search $e_t n_t$ is constant and when (ii) OTJS $e_t n_t$ is endogenous.[18] A positive productivity shock obviously increases vacancies and tightness, and decreases unemployment. When OTJS is endogenous, $e_t n_t$ also increases and stimulates further the opening of vacancies. Is this opening of vacancies sufficient to counteract the crowding out effect? Figures 3 shows that when $\bar{x}<1.44$, the volatility of unemployment with exogenous $e_t n_t$ is higher than with endogenous $e_t n_t$. As a result, the crowding-out effect dominates. When $\bar{x}>1.44$, the volatility of unemployment with exogenous $e_t n_t$ is lower than with endogenous $e_t n_t$ and the vacancy effect dominates. We are therefore able to generalize proposition 2 to the dynamic setup.

In the Real Business Cycle literature, the volatility of unemployment is usually normalized with respect to the volatility of net output. Figures 4 shows that endogenous OTJS combined with a sufficiently high $\bar{x}$ increases the relative volatility of unemployment bringing it closer to real data. Table 1 shows selected statistics when $\bar{x}=1.0$ and $\bar{x}=2.4$ (with exogenous *vs.*

[16]This refers to average wage gains over a spell of employment. The average wage of an experienced worker is w^n the first period followed by w^o until the job is destroyed.

[17]The condition $p<1$ is stronger than the conditions stated in proposition 1. We could instead use alternative specifications for the matching function, see for instance den Haan et al. (2003), that would guarantee matching probabilities between 0 and 1. In this case, proposition 1 would determine the admissible values for $\bar{x}$.

[18]It is worth noting that in figures 1 and 2 in the previous section, a change in $\bar{x}$ modifies the steady state. For instance, $e\to 0$ when $\bar{x}$ is low and $p\to 1$ when $\bar{x}$ is high. In this section, we look at the numerical dynamic properties of the model and we want to keep similar steady states to get fair comparisons between simulations. This is why a change in $\bar{x}$ also implies changes in c, $\bar{e}$ and z to keep the same $c\,v/y=0.05$, $u=0.12$ and $e\,p=0.06$. With these changes, $\bar{x}>1.71$ does not imply $p>1$.

endogenous on the job search). We also compare these statistics to US and EA data, and to those obtained from the standard search and matching model (with the same calibration).[19] The main conclusion is therefore that OTJS may help generate realistic business cycle volatilities, provided that J_t^n is high enough, *i.e.* provided that OTJ searchers are enticing enough to firms. In our model, this is achieved through a sufficiently high $\bar{x}$.

Finally, figure 5 looks at the wage gains at the steady state for the different values of $\bar{x}$. Regarding the estimates of Fujita (2011) described in section 4, $\bar{x}$ should remain between 1.4 and 2.2 to generate plausible wage gains following job-to-job transitions. The value $\bar{x} = 2.4$ we need to generate enough amplification to reproduce US data is therefore slightly beyond the upper bound. In section 6, we discuss related OTJS approaches in the light of these results; that is we look at the relationships between J^n, the wage gains and the amplification effects.

6 Discussion

In the next subsections, we briefly review some related papers and we use our model to understand the mechanisms at work. Finally, we discuss endogenous search intensity for the unemployed, looking again at the role of $\bar{x}$ and hence the difference between J^n and J^o.

6.1 Related literature

OTJS has been proposed as a possible solution to the Shimer (2005) puzzle of insufficient volatility of unemployment and vacancies in RBC models. Three papers have investigated this directly, with different mechanisms for the effects:

Tasci (2007) constructs a model with imperfectly observed match quality following Pries and Rogerson (2005), where wages are determined each period as a simple split of the expected match surplus, with unemployment as the fallback even when switching jobs. All workers engage in costless OTJS with the same intensity, but an employed worker will only (and always) accept a new job if it has higher estimated productivity than his current job. This implies that firms strictly prefer experienced to inexperienced workers when filling a vacancy because the former have higher estimated productivity on average, which both directly increases output and implies the worker is expected to remain with the job longer. This increases the value of a vacancy during expansions because more of the contacted workers will be currently employed,

[19] We only report second moments for unemployment and vacancies because this is what we are interested to explain. Obviously we could extend the table to other labor market related variables as employment, tightness or job destruction.

and leads to more cyclical volatility in job openings and by extension unemployment.[20]

Nagypal (2007) proposes a similar approach. Upon matching with a firm, a worker draws a taste component. As a result, OTJ searchers will only accept jobs that are particularly attractive, *i.e.* with a higher taste component than their current job; whereas the unemployed will accept any job. Experienced workers are thus expected to have a higher taste component than inexperienced workers and so to stay with the job longer. It is worth noting that because of several assumptions about the bargaining protocol, the taste component has no effect on the bargained wages, and therefore it is more profitable for a firm to hire an OTJ searcher rather than an unemployed one. Nagypal shows that a positive productivity shock stimulates OTJS, which amplifies the volatility of labor market variables. Moreover, the mechanism is amplified further by the fact that firms must pay a sunk cost when hiring a worker. Since employment relationships formed with employed searchers last longer, firms are able to recoup this hiring cost over a longer period of time.

Krause and Lubik (2010), assume a bifurcated labor market for good and bad jobs, similar to Pissarides (1994), in which good jobs are strictly preferred to bad jobs and so workers with bad jobs engage in OTJS for good jobs. Unemployed workers must choose whether to search for a good or bad job, and in equilibrium are indifferent between the two; thus, the expected duration of unemployment is higher if searching for a good job than if searching for a bad job. Wages are renegotiated each period with unemployment as the fallback position, even for job switchers. OTJS is costly, and since the wage difference between good and bad jobs is procyclical, so is search effort, and thus so is vacancy creation.[21]

We depart from the above three models in allowing workers who successfully locate a new job through OTJS to negotiate their compensation at the new job with their previous job as a fallback position. We believe this comports with the evidence of Fujita (2011) This, along with a *possible* productivity advantage at the new job, is what motivates workers to search OTJ. To avoid a bifurcated labor market, or one with a distribution of job types, we assume that workers who switch jobs get a one-time productivity advantage on their new job (proportional to the aggregate macroeconomic shock), but in subsequent periods behave and are paid like other workers. Thus, a job located through OTJS is not expected to last any longer than one the worker found when unemployed.

[20]OTJS intensity in Tasci is fixed, unlike in Krause and Lubik where its procyclicity is responsible for the model's volatility amplification; nevertheless, in the standard MP framework, the overall search intensity of all workers is countercyclical since unemployment falls during expansions. Thus, there is a simple dampening effect of OTJS in the Tasci model as well.

[21]Van Zandweghe (2010) also look at OTJS in a bifurcated labor market in a DSGE model in order to study inflation propagation.

6.2 Comparison of different mechanisms

The benchmark model described in section 2 is simple and close to Pissarides (2000), chapter 4. In this section, we modify the model to introduce some of the features described in Tasci (2007), Nagypal (2007) and Krause and Lubik (2010). We investigate how the changes affect J^n, the wage gains, and the amplification mechanisms.

Simulation (a) uses the benchmark model described in section 2 with the calibration from section 4 ($\bar{x} = 1.44$). We see that endogenous on-the-job search weakly amplifies the relative volatility of unemployment by 5% (see also figure 4). Simulation (b) introduces a hiring cost $H > 0$ as in Nagypal (2007). The ratio J^n/J^o is unchanged and the amplification of unemployment fluctuations is due to a *sunk cost* mechanism similar to Fujita and Ramey (2007). This is therefore not directly related to an OTJS mechanism in our model. Simulation (c) replaces equation (11) by $\eta\ (J_t^n + W_t^n - U_t) = W_t^n - U_t$. We therefore have unemployment as fallback position, even when OTJS, as in Tasci (2007), Nagypal (2007) and Krause and Lubik (2010). Obviously, this reduces wage gains upon job-to-job transitions and therefore increases the difference between J^n and J^o, which amplifies unemployment volatility by 16%. Simulation (d) is the benchmark model described in section 2 but with $\bar{x} = 2.4$. We increase wage gains but also J^n/J^o which leads to a huge increase (+90%) in unemployment volatility (see also figures 4 and 5, and table 1). Simulation (e) introduces good jobs and bad jobs as in Krause and Lubik (2010). More precisely, we relax our initial assumption that workers who switch jobs get a one-time productivity advantage on their new jobs, and we instead assume that they enjoy this advantage until the job is destroyed. As a result, workers who have switched jobs once do not search OTJ anymore. This approach must be combined with using U as the fallback position for all workers when bargaining. We calibrate the *permanent* productivity advantage $\bar{x} = 1.20$ to obtain the same J^n/J^o ratio than in the simulation (d) and we see that, although the approach is somewhat different, we obtain very similar results and amplification.

These exercises underline again that a high J^n relative to J^o magnifies the vacancy effects and is therefore the key ingredient to amplify unemployment volatility through OTJS. As a last illustration, assuming a lower bargaining power η for the workers would reduce wage gains, increase the difference in the Js and therefore amplify further the unemployment volatility–see simulation (f) in table 2. A combination of some of the mechanisms described above would obviously push the effects further up still.

6.3 Endogenous search intensity of the unemployed

So far the search intensity of the unemployed is normalized to 1. However, they probably also search more or less intensively according to the business cycle situation – see for instance Merz

(1995) for such a mechanism but without OTJS. This should obviously increase the unemployment volatility but we can expect that the effects on the volatility of vacancies will depend on the match surplus of an experienced versus an inexperienced match.

To illustrate this, we consider k_t as the endogenous search intensity of an unemployed. Equation (2) becomes $m_t = \bar{m}\, v_t^{1-\mu}\, (e_t n_t + k_t u_t)^{\mu}$ and we also modify accordingly the subsequent equations. We define the search cost as $\mathcal{V}(k_t) = \bar{k}/2\, k_t^2$ and the first order condition with respect to k_t is $\mathcal{V}'(k_t) = p_t(1-\rho)E_t\left[\beta\, C_t/C_{t+1}\left(W_{t+1}^o - U_{t+1}\right)\right]$, where we choose $\bar{k}$ to obtain $k = 1$ at the steady state.

Figure 6 shows that, for any value of $\bar{x}$, the volatility of unemployment is higher when the unemployed search intensity is endogenous. However, figure 7 shows that the effects of unemployed search intensity on the volatility of vacancies depend on the value of $\bar{x}$, *i.e* depend again on the difference between J^n and J^o. When $\bar{x}$ is low (resp. high), firms prefer unemployed (resp. employed) job seekers and endogenous search of the unemployed therefore does (resp. does not) give firms the incentive to open more vacancies.

7 Conclusion

We present a very simple model of on-the-job search and show that unemployment volatility may increase or decrease, depending on the calibration of a single parameter. This parameter governs the difference between the match surplus of an experienced *vs.* an inexperienced match. Then we extend the model along several dimensions to reproduce the main features of related papers with OTJS, and confirm that the match surplus difference is the key ingredient to understand their results.

Another mechanism related to OTJS for increasing business cycle unemployment volatility not explored in this or any other paper to date to our knowledge would involve imposing some form of wage rigidity. With OTJS, the ability of firms to hire workers at a discount during recessions would be curtailed relative to benchmark models because such workers would be more likely to leave during subsequent expansions. This would reinforce the OTJS decision and, provided an adequate calibration, amplify further the unemployment volatility. Introducing wage rigidity into models with OTJS, however raises additional difficulties (see, e.g. Shimer (2006)) and often requires wage posting. In turn, introducing wage posing into a tractable DSGE model is not straightforward and requires a discretization of the wage possibilities. We expect to explore this in future research.

References

Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review 86*(1), 112–132.

Christoffel, K., K. Kuster, and T. Linzert (2009). The role of labor markets for euro area monetary policy. *European Economic Review 53*(8), 908–936.

den Haan, W., G. Ramey, and J. Watson (2003). Liquidity flows and fragility of business entreprises. *Journal of Monetary Economics 50*(6), 1215–1241.

Fujita, S. (2011). Reality of on-the-job search. The Federal Reserve Bank of Philadelphia, WP 10-34/R.

Fujita, S. and G. Ramey (2007). Job matching and propagation. *Journal of Economic Dynamics and Control 31*(11), 3671–3698.

Gautier, P. (2002). Unemployment and search externalities in a model with heterogeneous jobs and heterogeneous workers. *Economica 69*, 21–40.

Gertler, M. and A. Trigari (2009). Unemployment fluctuations with staggered nash wage bargaining. *Journal of Political Economy 117*(1), 38–86.

Hagedorn, M. and I. Manovskii (2008). The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review 98*(4), 1692–1706.

Hall, R. and P. Milgrom (2008). The limited influence of unemployment on the wage bargain. *American Economic Review 98*(4), 1653–1674.

Krause, M. and T. Lubik (2007). On-the-job search and the cyclical dynamics of the labor market. Deutsche Bundesbank WP 15-2007.

Krause, M. and T. Lubik (2010). On-the-job search and the cyclical dynamics of the labor market. The Federal Reserve Bank of Richmond, WP 10-12.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics 36*, 269–300.

Nagypal, E. (2007). Labor-market fluctuations and on-the-job search. mimeo.

Nash, J. (1950). The bargaining problem. *Econometrica 18*(2), 155–162.

Pierrard, O. (2008). Commuters, residents and job competition. *Regional Science and Urban Economics 38*(6), 565–577.

Pissarides, C. (1994). Search unemployment with on-the-job search. *The Review of Economic Studies 61(3)*, 457–475.

Pissarides, C. (2000). *Equilibrium unemployment theory*. MIT Press.

Pries, M. and R. Rogerson (2005). Hiring policies, labor market institutions, and labor market flows. *Journal of Political Economy 113*(4), 811–839.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *The American Economic Review 95*(1), 25–49.

Shimer, R. (2006). On-the-job search and strategic bargaining. *European Economic Review 50*, 811–830.

Tasci, M. (2007). On-the-job search and labor market reallocation. The Federal Reserve Bank of Cleveland, WP 0725.

Van Zandweghe, W. (2010). On-the-job search, sticky prices, and persistence. *Journal of Economic Dynamics and Control 34*, 437–455.

Yashiv, E. (2006). Evaluating the performance of the search and matching model. *European Economic Review 50*(4), 909–936.

A Partial derivatives of J^o, J^n, I

$$
\begin{aligned}
J^o_\theta &= -\frac{\beta(1-\rho)\eta p'}{1-\beta(1-\rho)\left(1-pe(1-\eta)^2-\eta p(\theta)\right)}\left(J^o-eJ^n\right)\\
J^o_{pe} &= -\frac{(1-\eta)\beta(1-\rho)}{1-\beta(1-\rho)\left(1-pe(1-\eta)^2-\eta p(\theta)\right)}J^o\\
J^n_\theta &= (1-\eta)\,J^o_\theta\\
J^n_{pe} &= (1-\eta)\,J^o_{pe}\\
\mathcal{S}_\theta &= -\frac{2p'\mathcal{S}}{p} = -\frac{\beta\eta(1-\rho)p'\,e}{1-\eta}J^n\\
\mathcal{S}_{pe} &= \frac{2\mathcal{S}}{pe} = \frac{\beta\eta(1-\rho)}{1-\eta}J^n\\
I_\theta &= -\frac{c(\rho+pe(1-\rho))q'}{\beta q^2(1-\rho)} - (pe(1-\rho)(1-\eta)+\rho)J^o_\theta\\
I_{pe} &= \frac{c}{\beta q} - (1-\rho)J^n - (pe(1-\rho)(1-\eta)+\rho)J^o_{pe}
\end{aligned}
$$

B Sign of $dpe/d\theta$

Restricting $0 < \theta, pe < \infty$, rewrite equation (17) as:

$$
\begin{aligned}
0 &= \bar{e}(1-\eta)pe - \beta\eta(1-\rho)p^2J^n\\
0 &= \left[\left(1+2\beta(1-\rho)(1-\eta)^2\right)\bar{e}p^{-1}\right](pe)^2\\
&+ \left[-2\beta^2(1-\rho)^2\eta(\bar{x}-1)P(1-\eta)^2p + 2\beta(1-\rho)\eta(\bar{e}-P(\bar{x}-1)) + 2\bar{e}(1-\beta(1-\rho))p^{-1}\right]pe\\
&+ \left[-2\beta(1-\rho)\eta\left(\beta(1-\rho)\eta(\bar{x}-1)Pp^2 + (1-\beta(1-\rho))(\bar{x}-1)Pp + (1-\eta)(P-z)\right)\right]
\end{aligned}
$$

As noted in the text, this is equivalent to $A(\theta)(pe)^2 + B(\theta)pe + C(\theta) = 0$ with $A > 0$ and $C < 0$ for all $\theta > 0$, implying the existence of a unique positive value for $pe(\theta)$. $dpe/d\theta$ is given by:

$$
dpe/d\theta = \beta\eta(1-\rho)p\frac{2p'J^n + pJ^n_\theta}{\bar{e}(1-\eta) - \beta\eta(1-\rho)p^2J^n_{pe}}
$$

The denominator is positive. The numerator is too:

$$
\begin{aligned}
& 2p'J^n + pJ^n_\theta \\
= \ & 2(1-\beta(1-\rho)(1-pe(1-\eta)^2-\eta p))p'J^n + p(1-\eta)\beta(1-\rho)\eta p'(J^o - eJ^n) \\
= \ & (2(1-\beta(1-\rho)) + \beta(1-\rho)(2pe(1-\eta)^2 + 2\eta p))J^n \\
+ \ & p\beta(1-\rho)\eta(1-\eta)J^o - pe\beta(1-\rho)\eta(1-\eta)J^n \\
= \ & (2(1-\beta(1-\rho)) + \beta(1-\rho)(pe(2-\eta)(1-\eta) + 2\eta p))J^n + p\beta(1-\rho)\eta(1-\eta)J^o \\
= \ & (2(1-\beta(1-\rho)) + \beta(1-\rho)(pe(2-\eta)(1-\eta) + 2\eta p))J^n + p\beta(1-\rho)\eta(1-\eta)J^o
\end{aligned}
$$

C Partial derivatives of pe

$$
\begin{aligned}
pe_\theta &= \frac{p'(1-\rho)pe}{\rho + (1-\rho)p} \\
pe_{en} &= \frac{\rho + (1-\rho)p}{1-\rho}
\end{aligned}
$$

	u		v	
	stdv	corr	stdv	corr
US data	6.67	-0.85	8.36	0.85
EA data	5.36	-0.85	14.3	0.71
standard MP	1.06	-0.76	1.48	0.87
exo OTJS, $\bar{x} = 1$	1.23	-0.79	1.99	0.91
endo OTJS, $\bar{x} = 1$	1.05	-0.77	2.61	0.95
exo OTJS, $\bar{x} = 2.4$	2.77	-0.85	3.91	0.98
endo OTJS, $\bar{x} = 2.4$	5.26	-0.94	9.26	0.99

Table 1: Selected statistics – second moments of EA and US data (1984Q1-2006Q4) [borrowed from Christoffel et al. (2009)]

		J^n/J^o	wage gains	stdv exo	stdv endo	Δ
(a)	$W^n - W^o$	1.20	+6.1%	1.53	1.60	+5.0%
(b)	$H = 0.20$	1.20	+6.1%	2.08	2.21	+6.3%
(c)	$W^n - U$	1.85	+3.5%	1.83	2.12	+16%
(d)	$\bar{x} = 2.4$	5.88	+12.0%	2.77	5.26	+90%
(e)	good/bad jobs	5.88	+12.2%	2.73	5.59	+105%
(f)	$\eta = 0.2$	2.34	+2.1%	4.12	4.42	+15%

Table 2: Alternative models and relative unemployment volatility

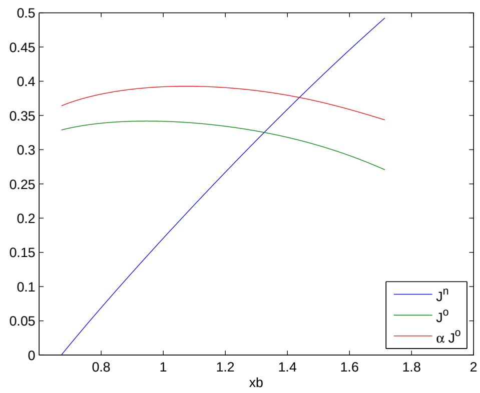

Figure 1: J^n, J^o and αJ^o for different values of $\bar{x}$

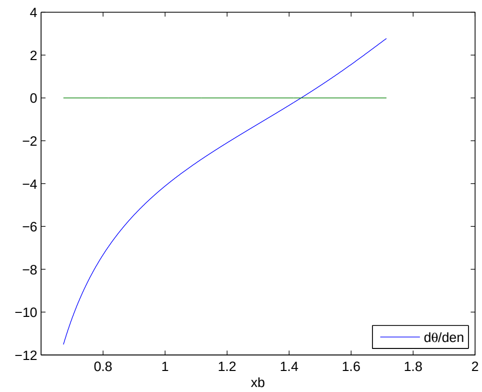

Figure 2: Vacancy *vs.* crowding out effects for different values of $\bar{x}$

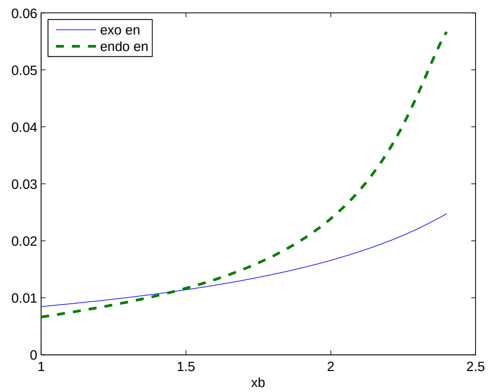

Figure 3: Absolute standard deviation of u_t for different values of $\bar{x}$

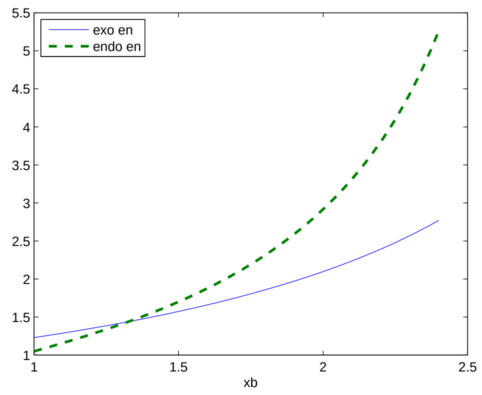

Figure 4: Relative standard deviation of u_t (w.r.t. the standard deviation of net output) for different values of $\bar{x}$

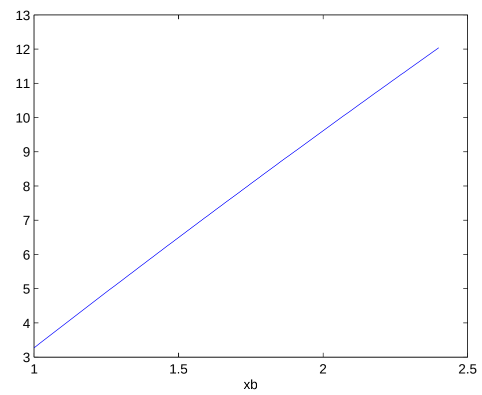

Figure 5: Wage gains for different values of $\bar{x}$

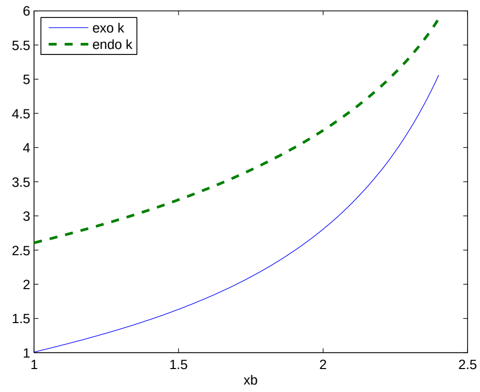

Figure 6: Relative standard deviation of u_t (w.r.t. the standard deviation of net output) for different values of $\bar{x}$

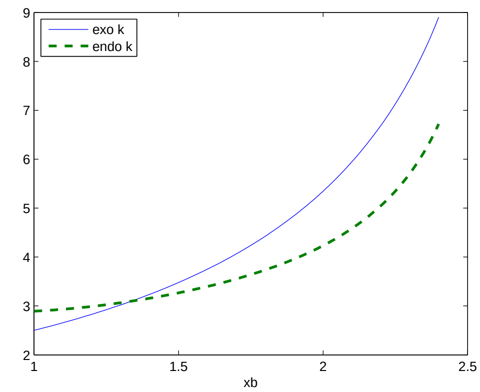

Figure 7: Relative standard deviation of v_t (w.r.t. the standard deviation of net output) for different values of $\bar{x}$

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