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Abstract

This paper estimates the natural real interest rate that is consistent with stable inflation and output at its potential for the euro area and Luxembourg. The natural interest rate provides a benchmark for assessing the monetary policy stance, as policy is contractionary when real interest rates rise above the natural rate and expansionary when real interest rates fall below this level. We follow Laubach and Williams (2003) in using a small backward-looking macroeconomic model to estimate the time-varying natural interest rate as an unobservable variable. For the euro area, our results suggest the natural interest rate has been fairly stable since 1970 and confirm its decline over the last decade. For Luxembourg, our estimate of the natural interest rate is much higher, reflecting higher potential growth. The results suggest that the single monetary policy may have had an expansionary impact in recent years, especially in Luxembourg.

Keywords: Kalman filter natural interest rate equilibrium real interest rate.

JEL Codes: C32, E43, E52.

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This paper was prepared while the first author was visiting the BCL. Ladislav Wintr acknowledges the support from the Ridgefield Foundation obtained through the Henry J. Leir Program at Clark University. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Banque centrale du Luxembourg. The authors are grateful to BCL colleagues for helpful comments.

Résumé non-technique

Le taux d'intérêt naturel est le taux d'intérêt réel compatible à la fois avec l'absence d'écart de production (le PIB est à son niveau potentiel) et avec la stabilité des prix. Cette étude fournit des estimations du taux d'intérêt naturel pour la zone euro et pour le Luxembourg. Théoriquement, le taux d'intérêt naturel est susceptible de servir de référence pour la politique monétaire. Quand le taux d'intérêt réel est plus élevé que le taux d'intérêt naturel, la politique monétaire est restrictive. A l'opposé, la politique monétaire est expansive quand le taux d'intérêt réel est inférieur au taux d'intérêt naturel. Ainsi, la banque centrale pourrait fixer le taux d'intérêt nominal de manière à faire évoluer les taux d'intérêt réels à court terme vers un niveau inférieur ou supérieur au taux d'intérêt naturel selon la nature des chocs déstabilisateurs. En pratique, le taux d'intérêt naturel est estimé avec un degré d'incertitude important et il est sujet à de multiples révisions dues à la publication de nouvelles données. Ces limitations réduisent l'utilité du taux d'intérêt naturel dans la conduite de la politique monétaire. Cependant, le taux d'intérêt naturel peut fournir une indication sur l'orientation de la politique monétaire relative aux périodes antérieures. De plus, le taux d'intérêt naturel estimé pour un pays à l'intérieur de la zone euro peut fournir des renseignements sur l'impact de la politique monétaire commune sur l'économie nationale en question.

Dans notre contribution, l'estimation du niveau du taux d'intérêt naturel pour la zone euro et pour le Luxembourg est basée sur l'approche semi-structurelle proposée par Laubach et Williams (2003). Cette approche s'appuie sur un petit modèle macroéconomique combinant une équation d'offre agrégée (courbe de Phillips) et une équation de demande agrégée (courbe IS). Le filtre de Kalman sert à estimer les variables inobservables, tels que le taux d'intérêt naturel, l'écart de production, et la croissance potentielle, en tenant compte de l'évolution des variables observées, en l'occurrence la production, l'inflation, et le taux d'intérêt réel. Ainsi, le taux d'intérêt naturel et la croissance potentielle sont estimés de manière simultanée.

Pour la zone euro, les résultats suggèrent une certaine stabilité du taux d'intérêt naturel depuis 1970 et confirment qu'il a baissé au cours de la dernière décennie. Pour le Luxembourg, le taux d'intérêt naturel estimé est beaucoup plus élevé, signe d'une croissance potentielle plus importante. Les résultats laissent présager que la politique monétaire commune a eu un impact expansif au cours des périodes récentes, particulièrement au Luxembourg.

Non-technical summary

The natural interest rate is the level of the real interest rate that is consistent with stable inflation and output at its potential. This paper estimates the natural interest rate for the euro area and Luxembourg. In theory, the natural interest rate could provide a benchmark for monetary policy. Monetary policy is contractionary when the real interest rate is above the natural interest rate. On the other hand, policy is expansionary when real interest rates fall below this level. It follows that the central bank should set nominal interest rates to push the real rate above or below the natural interest rate depending on the nature of the shocks hitting the economy. In practice, however, estimates of the natural interest rate are subject to significant uncertainty and can be revised substantially following new data releases. These limitations reduce the usefulness of the natural interest rate in day-to-day monetary policymaking. However, the natural interest rate can still provide a useful indicator of the policy stance over past periods. Furthermore, the natural interest rate estimated for a member country within a monetary union can help understand the impact of the single monetary policy on the national economy considered.

This study estimates the natural interest rate for the euro area and for Luxembourg using the semi-structural approach in Laubach and Williams (2003). This is based on a small backward-looking macroeconomic model combining an aggregate supply relation (Phillips curve) and an aggregate demand relation (IS curve). The Kalman filter is used to estimate the time-varying unobservable variables (natural interest rate, output gap and potential growth) based on the observed variables (output, inflation and the ex-post real interest rate). Thus, the natural interest rate and potential growth are estimated simultaneously.

For the euro area, our results suggest the natural interest rate has been fairly stable since 1970 and confirm its decline over the last decade. For Luxembourg, our estimate of the natural interest rate is much higher, reflecting higher potential growth. The results suggest that the single monetary policy may have had an expansionary impact in recent years, especially in Luxembourg.

I. Introduction

This paper aims to contribute to the growing literature estimating the natural real rate of interest. Our focus is on the euro area and Luxembourg. The concept of the natural (or neutral) real interest rate is commonly ascribed to the Swedish economist Knut Wicksell who wrote in 1898:[1]

> *There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them.*[2]

The modern interpretations of Wicksell's idea focus on the interest rate that is consistent with a neutral monetary policy stance, i.e. the real interest rate that stabilizes inflation (rather than the general price level). Today's definitions of natural interest rate fall into two broad categories depending on the time dimension they consider.

1. From the medium-term perspective, the natural interest rate abstracts from short-run price and output fluctuations. In this way, it is analogous to potential output, also an unobservable variable and long-run benchmark. Current output can deviate from potential output for prolonged periods, much as the real interest rate may differ from the natural rate of interest in the short run. However, over longer periods, the observed variables and their unobservable counterpart are supposed to take the same value on average.

   Following this line of reasoning, we can define the natural real rate of interest as the level of the real short-term interest rate that is consistent with stable inflation and output at its potential level. In other words, it is the short-term real interest rate that would prevail once the effects of demand and supply shocks have completely faded away. This definition is widely cited, see Laubach and Williams (2003), Cuaresma et al. (2003), Basdevant et al. (2004) and ECB (2004), and is further discussed in sections 2.1 and 2.4 below.

2. A different strand of literature focuses on the short-term perspective, defining the equilibrium interest rate as that which yields period-by-period price stability. Here, the natural interest rate is seen as the level of the real rate at the flexible price equilibrium, i.e. the equilibrium in the absence of any price rigidities. See for instance Neiss and Nelson (2001), Smets and Wouters (2002), Giammarioli and Valla (2003). More details of this approach can be found in section 2.2 (structural models).

The natural interest rate concept seemed to die a natural death for nearly a century after its invention.[3] The recent resurgence of interest is linked to the popularity of interest rate policy rules that grew out of the time inconsistency literature. Taylor (1993) proposed a simple monetary policy rule determining the short-term nominal interest rate as a linear function of a baseline equilibrium value (i.e. the natural interest rate) and two gaps: that between inflation and its desired target and that between output and its potential level. The success of the Taylor rule, more precisely the finding in Taylor (1993) that it could accurately describe actual policy

[1] Humphrey (1986) argues that most of the ingredients of Wicksell's model were described already 70 years earlier by two British economists–Henry Thornton and Thomas Joplin.
[2] Wicksell: *Interest and Prices*, cited by Williams (2003).
[3] The Stockholm School of economic thought represents a notable exception, most prominently through the work of Gunnar Myrdal who further developed Wicksell's ideas.

decisions in the US from 1987 to 1992, prompted the return of the natural interest rate concept to mainstream economics. While Taylor assumed the natural interest rate was constant, the more recent revival of the Wicksellian framework of monetary analysis (e.g. Woodford, 2003, chapter 4) emphasizes its nature as a time-varying variable. This allows for changes in its fundamental determinants including consumers' rate of time preference, risk premia, and the institutional structure of financial markets.

Much of the current research on natural interest rates originated in central banks (Williams, 2003, Laubach and Williams, 2003, Giammarioli and Valla, 2003, Basdevant et al., 2004, Mésonnier and Renne, 2004). This reflects the need for new benchmarks for monetary policy given the declining information content of monetary aggregates and exchange rates following episodes of financial innovation. If the authorities push the real short-term interest rate below the natural rate, this raises investment and consumption, lowering incentives to save and creating inflationary pressures in the economy. By definition, such a monetary policy is expansionary.

Unfortunately, the natural interest rate is unobservable and difficult to estimate. In addition, the real-time estimates of the natural interest rate may not be reliable enough to aid policymakers in the everyday decision making process. However, it may still be useful for ex-post monetary policy evaluation (as suggested in the previous paragraph). Moreover, in the context of the euro area, estimates of the natural interest rate for an individual country can serve to assess the impact of the common monetary policy on that particular country. Since euro area countries have different levels of inflation (and presumably different potential growth rates), the natural interest rate will differ across individual countries. However, ECB policy only takes into account inflation in the euro area as a whole, so it need not be consistent with the requirements of individual countries. For instance, a country with high inflation may need a restrictive policy to return to price stability, but it might actually face negative real interest rates (because high inflation lowers real interest rates) actually adding to inflationary pressures.

The remainder of the paper is organized as follows. Section 2 briefly summarizes the most common approaches to estimation of the natural interest rate. Section 3 describes our model and shows how it relates to the relevant literature. Sections 4 and 5 introduce the data and present the estimates for the euro area and Luxembourg, respectively. Section 6 concludes.

2. Methodology

There are several approaches to estimate the natural interest rate. A survey of these methods is beyond the scope of this paper but in what follows we will briefly consider the four most popular methods.

2.1. Averaging actual short-term real interest rates

The most common way to estimate natural interest rates is simple averaging of actual real short-term interest rates. Williams (2003) illustrates that this method gives good estimates in periods of stable inflation and output growth but tends to be grossly misleading when inflation changes substantially.

2.2. Structural models

In part, averaging is unsatisfactory because it disregards other relevant information on the state of the economy. Neiss and Nelson (2001) incorporate such information by building a dynamic stochastic general equilibrium model with sticky prices. This allows them to derive a time-varying natural interest rate whose evolution can be given a structural interpretation. The natural interest rate is estimated by constructing a 'flexible-price scenario' and calculating the required equilibrium real interest rate. However, estimates of natural interest rates based on such structural models are typically quite volatile and sensitive to the assumptions concerning the structure of the economy and the structural shocks affecting it, see e.g. Giammarioli and Valla (2003), Smets and Wouters (2002).

2.3. Financial-market indicators

Another alternative is to infer the natural interest rate from the real yield on inflation-indexed securities. Bomfim (2001) argues that the best prediction of long-term equilibrium interest rate is market participants' forecast of the short-term interest rate 5 to 10 years ahead. This approach has the advantages that it is simple, forward-looking and available in real time. However, it also suffers from significant disadvantages. The yields on inflation-linked bonds may be distorted by relative liquidity premia, by term premia, and by irrational expectations among market participants. Moreover, inflation-linked securities were not introduced in the euro area until November 2001.

2.4. Unobservable component models

Laubach and Williams (2001) took a time series approach to estimating time-varying natural interest rates using 'unobservable component' models. Starting with a simple macroeconomic model (consisting of a reduced-form Phillips curve and an IS curve) they use the Kalman filter to simultaneously estimate the time-varying natural interest rate and potential output. This strand of the literature includes Orphanides and Williams (2002), Cuaresma et al. (2003), Basdevant et al. (2004) and others. The relevant definition of the natural interest rate focuses on the medium term when the effects of transitory shocks on the output gap and inflation fade away. The advantage of this semi-structural approach lies in its tractability and robustness to the particular structural assumptions. It can also account for large changes in the structural variables and hence may perform better in data sets with large shocks and/or many structural changes.

3. Model

In this paper we adopt the strategy of Laubach and Williams (2001) and estimate potential output and the natural interest rate as unobservable variables. This approach builds on the rich literature on estimating potential output using unobservable component models. Watson (1986) and Clark (1989) proposed a decomposition of the observed series, y_t (output), into two unobserved components—a trend (potential output), y_t^*, and a transitory component, x_t. Potential output is assumed to follow a random walk with drift, while the transitory component follows a stationary AR(2) process. Formally, the model of Clark (1989) consists of the following four equations

$$y_t = y_t^* + x_t$$
$$y_t^* = \mu_t + y_{t-1}^* + u_t$$
$$x_t = \theta_1 x_{t-1} + \theta_2 x_{t-2} + w_t$$
$$\mu_{t+1} = \mu_t + v_{t+1}$$

where u_t, w_t, v_t ~ iid$(0,\Omega)$[4]. This purely statistical approach was criticized for ignoring other economic variables which economic theory may link to the output gap. Kuttner (1994) complements Watson's (1986) model with a Phillips curve equation in which the lagged growth rate and the inflation rate, π_t, are related to the lagged growth rate and the output gap

$$\Delta\pi_t = \eta_1 + \eta_2 \Delta y_t + \eta_3 x_{t-1} + \gamma(L)\varepsilon_t$$

with ε_t ~ iid$(0,\sigma_4)$. Gordon (1997), Laubach (2001) and Fabiani and Mestre (2001) replaced the output gap in the Phillips curve with the unemployment gap, estimating the NAIRU instead of potential output as an unobservable variable. The Phillips curve was also combined with a reduced form IS curve in a small macro model along the lines of Rudebusch and Svensson (1999) and was estimated by Gerlach and Smets (1997) on US data and by Peersman and Smets (1999) on EU data. Laubach and Williams (2003) generalized this model to estimate a time-varying natural interest rate along with potential output. While Laubach and Williams worked with US data, Mésonnier and Renne (2004) did a similar study for the euro area.

The model consists of the following six equations

$$y_t = y_t^* + \sum_{k=1}^{2} a_k (y_{t-k} - y_{t-k}^*) + \frac{a_r}{200}[(r_{t-1} - r_{t-1}^*) + (r_{t-2} - r_{t-2}^*)] + \varepsilon_t^y \quad (1)$$

$$\pi_t = \sum_{i=1}^{4} b_i \pi_{t-i} + 100 b_y (y_{t-1} - y_{t-1}^*) + \sum_{l=1}^{2} b_i^m (\pi_{t-1}^m - \pi_{t-1}) + \varepsilon_t^\pi \quad (2)$$

$$r_t^* = 4cg_t + z_t \quad (3)$$

$$z_t = z_{t-1} + \varepsilon_t^z \quad (4)$$

$$y_t^* = y_{t-1}^* + g_{t-1}/100 + \varepsilon_t^{y*} \quad (5)$$

$$g_t = g_{t-1} + \varepsilon_t^g \quad (6)$$

where the ε^j represent independently normally distributed innovations.

[4] In Watson (1986), the trend growth rate μ_t is assumed to be constant, i.e. $\mu_t = \mu$.

Equation (1) is a reduced form IS curve that relates the output gap, $100(y_t - y_t^*)$, measured as the log deviation of real GDP from its potential, to its own lags and to lags in the interest rate gap, i.e. the difference between short-term real interest rate r_t and the natural interest rate r_t^*.[5] If r_t is equal to the natural interest rate and there are no demand shocks in the system ($\varepsilon_t^y = 0$), then the output gap converges to zero.

The second equation, a backward-looking Phillips curve, explains inflation π_t as a function of its own lags, the lagged output gap, and lags of relative import price inflation, $(\pi_t^m - \pi_t)$. We restrict the coefficients on lagged inflation terms to sum to one (i.e. $b_4 = 1 - \sum_{j=1}^{3} b_j$).[6] Together, equations (1) and (2) constitute the measurement equations linking the observed variables (y_t and π_t) to the unobserved variables (y_t^* and r_t^*).

Equation (3) determines the natural interest rate as a linear combination of the growth rate of potential output, g_t, and other random factors, denoted z_t, which for simplicity follow a pure random walk, see equation (4). This could be motivated by economic agents' intertemporal maximization. For instance, the Solow model implies that the equilibrium real interest rate will change with fluctuations in the rate of technological progress, in population growth, and in the discount rate of the representative agent. In this case, changes in population growth or the discount rate would be captured by the variable z_t.

Potential output, y_t^*, follows a random walk with a time-varying drift (growth rate), g_t, which is in turn modeled in equation (6) as a pure random walk. Hence, equations (3)–(6) form the transition equations of the state-space model, specifying the data generating processes for the unobservable variables.

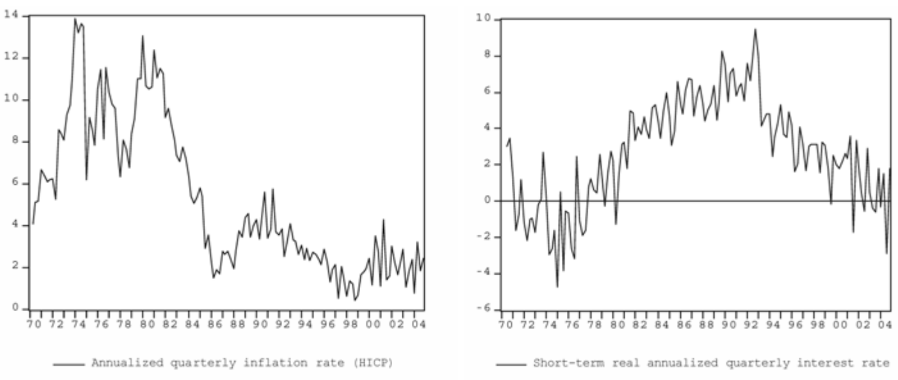

Fig. 1. Annualized HICP inflation and the real short-term interest rate.

[5] The choice of two lags of output and interest rate gaps corresponds to that of Laubach and Williams (2003). The restriction that the two lags of the interest rate gap take the same coefficient is not rejected by a Wald test.

[6] This restriction is not rejected by a Wald test. Four lags of inflation and two lags of relative oil price inflation were selected by the Akaike information criterion. In line with Laubach and Williams (2003), we included only one lag of the output gap in equation (2).

4. Euro area estimates

4.1. Data

Estimation of the model requires four observed variables: real GDP, inflation, the interest rate and import price inflation. The data were obtained from the AWM (Area-Wide Model) database, more precisely its September 2004 update (for details see Fagan et al. (2000) and the EABCN website www.eabcn.org). The AWM data set includes quarterly observations for the period 1970Q1 to 2003Q4, however, not all series are covered from the very beginning of the sample. The data for 2004 come from ECB's monthly bulletin (February 2005).

Our measure of inflation is based on the harmonized index of consumer prices (HICP), more precisely it is the annualized quarterly growth rate of HICP. The HICP series provided by AWM is not seasonally adjusted (unlike the other variables), so the Tramo/Seats procedure for seasonal adjustment was applied. The decomposition of the HICP series into energy and non-energy prices is only available since the last quarter of 1987, so it could not be exploited in our estimations.

The short-term (ex-post) real interest rate is the real annualized interest rate deflated using the HICP. Output, denoted y_t above, is simply the log of real GDP. Import price inflation is calculated using the 'imports of goods and services deflator'.

4.2. Estimation

Appendix A shows that the baseline model specified in equations (1)–(6) can be rewritten in state-space form, which means that the vector of unobserved variables (including the natural interest rate) can be estimated by maximum likelihood using the Kalman filter.[7]

Table 1. Different specifications of the baseline model in equations (1)–(6)

Specification	g_t	z_t	Log Likelihood
A	RW	RW	297.44
B	RW	AR(2)	296.25
C	AR(2)	RW	296.71

One potential problem with maximum likelihood estimates of the class of models we consider here is the 'pile-up problem'. If the variation in trend growth rate, g_t, or in the natural interest rate, r_t^*, is small (relative to the overall variability of the model), the estimated variance of their innovations (ε_t^z, ε_t^g) will be biased towards zero. Stock and Watson (1998) propose a median-unbiased estimator of the signal-to-noise ratio $\hat{\lambda}_g = \sigma^g / \sigma^y$ and $\hat{\lambda}_z = \frac{\sigma^g}{\sigma^y}\frac{a_r}{\sqrt{2}}$, where σ^j is the

[7] Initial conditions for the z_t variable were set to zero. For the remaining state variables, the Hodrick-Prescott filter was applied to the observed counterpart of the series (i.e. *y* for y^* and r for r^*) and the filtered series was used to initialize the state vector. The variance-covariance matrix was initialized at 0.2 times the identity matrix, as in Laubach and Williams (2001). The parameters were initialized at coefficient values obtained from single-equation OLS regressions using the HP trends.

standard deviation of the corresponding error term ε_t^i.[8] Their method is based on sequential tests for a break in the regression of the actual growth rate on a constant. Appendix B provides technical details concerning estimation of these ratios and their standard errors.

We have considered three versions of the state-space model (1)–(6) with different specifications of transition equations (4) and (6). For both the z_t variable and the growth rate of potential output, g_t, we considered either a random walk or a second order auto-regressive process. The log-likelihood values obtained from models combining these two assumptions are presented in table 1. None of the models performed clearly better (based on the likelihood ratio test) and hence we chose our preferred specification according to the plausibility of the output gap estimates. Our preferred model is specification `A`. The estimated natural interest rate and the key coefficient values turned out to be very similar across the three specifications in table 1.

For specification `A` (g_t and z_t modeled as a random walk) we run sensitivity analysis with respect to the values of $\hat{\lambda}_g$ and $\hat{\lambda}_z$. Table 2 summarizes the combinations of these values that were considered, each implying a restriction on the ratios of variances of the innovations. The confidence interval for λ_g and λ_z was obtained from 10 000 Monte Carlo simulations (see Laubach and Williams, 2001), with the 'low' and 'high' values standing respectively for the 5th and 95th quantile of the simulated distribution of the particular series. The column 'Log Likelihood' in table 2 reveals that the models provide equivalent fit to the data, so we decided to proceed with the median unbiased estimates of both λ_g and λ_z (this model is denoted `A.1` in table 3). The estimated coefficient values turned out to be only marginally sensitive to changes in the values of $\hat{\lambda}_g$ and $\hat{\lambda}_z$.

Table 2. Sensitivity analysis of specification `A`, as defined in table 1

Model	$\hat{\lambda}_g$	$\hat{\lambda}_z$	Log Likelihood
1	median* $\hat{\lambda}_g$ = 0.057	median* $\hat{\lambda}_z$ = 0.033	297.43
2	low $\hat{\lambda}_g$ = 0.000	median* $\hat{\lambda}_z$ = 0.033	297.31
3	high $\hat{\lambda}_g$ = 0.061	median* $\hat{\lambda}_z$ = 0.033	297.31
4	median* $\hat{\lambda}_g$ = 0.057	low $\hat{\lambda}_z$ = 0.000	298.38
5	median* $\hat{\lambda}_g$ = 0.057	high $\hat{\lambda}_z$ = 0.077	296.85

Note: * Median unbiased estimator using Stock and Watson (1998) procedure.

The parameter estimates of our preferred model `A.1` and the remaining two specifications considered in this paper (`B.1` and `C.1`) are presented in table 3 together with the results of

[8] The ratio λ_g is restricted only if the potential growth rate is assumed to be non-stationary and analogously λ_z is restricted only if z_t is assumed to be non-stationary.

Laubach and Williams (2003) for the United States (1961Q1 to 2002Q2) and those of Mésonnier and Renne (2004) for the euro area (1979Q1 to 2002Q4), denoted `LW` and `MR` respectively.

The estimates of λ_g and λ_z are similar in magnitude to those in Laubach and Williams. In model `B.1`, z_t is assumed to follow a stationary AR(2) so the value of λ_z was not restricted. Analogously, λ_g was not restricted in specification `C`. The sum of autoregressive coefficients in the IS equation is estimated below unity in all models and close to the point estimate in Laubach and Williams. The semi-elasticity of the output gap with respect to the interest rate gap, a_r, has the expected sign in all our models but it is not significant in specifications `A.1` and `B.1`. The coefficient is statistically different from zero at 10% significance level in model `C.1` but only reaches about half the size of the effect estimated by Laubach and Williams and by Mésonnier and Renne. The slope of the Phillips curve, b_y, is significant in all specifications and has the expected positive sign. The values obtained in models `A.1` and `B.1` are close to those estimated by Mésonnier and Renne while the coefficient estimate in model `C.1` lies just between the point estimate of Laubach and Williams and that of Mésonnier and Renne. All models confirm that lagged relative import price inflation is an important determinant of the overall inflation rate.

Laubach and Williams find that the natural interest rate varies about one-for-one with the growth rate of potential output. For the euro area this link is confirmed only in model `C.1` because the c coefficient is statistically insignificant in the other two specifications. Nevertheless, in all cases the estimated value of c is close to one.

Table 3. Parameter estimates, euro area models

Parameter	Model A.1	Model B.1	Model C.1	LW	MR
λ_g	0.057	0.057	na	0.042	
λ_z	0.033	na	0.000	0.058	
$a_1 + a_2$	0.936	0.942	0.951	0.945	0.80
a_r	-0.025	-0.023	-0.045	-0.098	-0.12
	(0.02)	(0.02)	(0.03)†	(0.03)	
b_y	0.222	0.245	0.106	0.043	0.19
	(0.10)‡	(0.11)‡	(0.06)†	(0.03)	(0.1)
b_1^m	0.104	0.106	0.119		
	(0.06)†	(0.06)†	(0.05)‡		
b_2^m	-0.182	-0.186	-0.192		
	(0.05)‡	(0.05)‡	(0.04)‡		
c	1.336	1.060	1.042	1.068	
	(1.17)	(1.56)	(0.62)†		
σ^y	0.227	0.227	0.299	0.387	0.37
σ^π	1.065	1.059	1.089	0.731	0.97
σ^z	0.419	0.001	0.000	0.323	
σ^{y*}	0.004	0.004	0.004	0.605	
σ^g	0.000	0.000	0.000	0.102	
Standard error (sample average)					
$r*$	1.776	1.137	0.472	1.88	
g	0.041	0.040	0.056	0.48	
$y*$	0.009	0.008	0.011	3.02	
Standard error (last observation)					
$r*$	2.653	1.151	0.443	4.22	
g	0.041	0.040	0.036	0.59	
$y*$	0.012	0.011	0.012	3.01	
Log Likelihood	297.43	296.25	296.71		

Note: Models A.1, B.1 and C.1 are defined in tables 1 and 2. The baseline results from Laubach and Williams (2003) and Mésonnier and Renne (2004) are presented in columns LW and MR, respectively. Numbers in brackets are standard errors, 'na' means not applicable, † significant at 10% level, ‡ significant at 5% level.

The smoothed (two-sided) estimate of the natural interest rate obtained from model `A.1` is plotted in figure 2 together with the HP trend of the actual real short-term interest rate. The point estimate of natural interest rate ranges from 0.5% to 3.0%, which is comparable with the results for the US of Laubach and Williams. On the other hand, Mésonnier and Renne obtain a more volatile natural interest rate for the euro area that fluctuates from 1% to 7%.

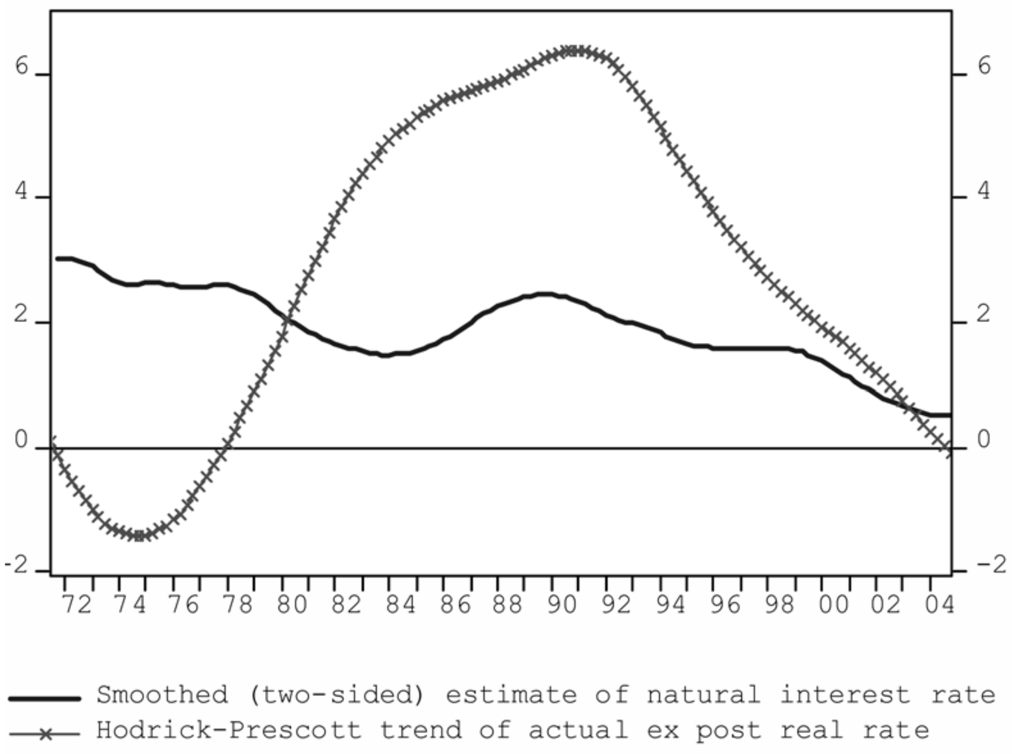

Fig. 2. Natural (real) interest rate obtained from model `A.1`.

Based on the relative position of the real short-term interest rate, inflation, and the output gap, we can distinguish three periods. The first period includes the 1970's and ends in 1980 when the observed short-term real interest rate first exceeds the point estimate of the natural interest rate. This period is characterized by the move towards more flexible exchange rate systems and the two oil price shocks. For most European countries, this is a period of high price and wage inflation. The upsurge of oil prices together with the collapse of the Bretton-Woods system led to loose monetary policies in most countries. Figure 1 documents the sudden rise of inflation, which was not matched by the rise of nominal interest rates, resulting in the abrupt drop in real interest rates. This suggests that during this period the natural interest rate lay above the ex-post real short-term interest rate. In other words, monetary policy was too loose to control inflation given the slowdown in potential output growth (see figure 3 for estimates from the AWM data set). Our estimated natural interest rate is consistent with this claim, as it decreases slowly from 3% in 1971 to 2% in 1980, remaining above the observed values of short-term real interest rates over the whole period.

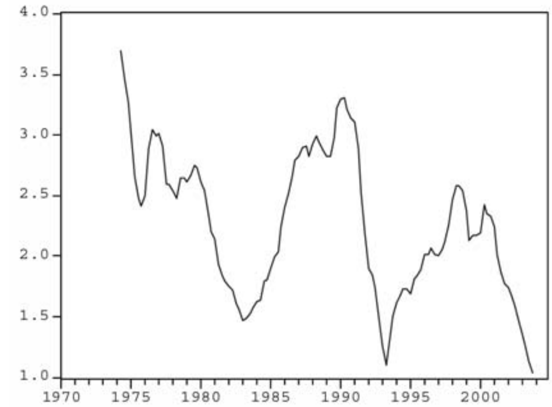

Figure 3. Growth rate of potential output per annum (AWM data set).

The second period ends in the year 1991, at which point there was an abrupt break in the trend of short-term real interest rates. These rose through the 1980's along with government deficits in most countries and the rise in inflation risk premia following the 1970's. At the same time, monetary policies of European countries were rather restrictive to bring inflation down from two-digit levels. Therefore, it would be reasonable to assume that the natural interest rate was below the observed real short-term interest rate during this period. This intuition is again confirmed by figure 2, which reveals an increasing interest rate gap at the end of this period.

The last period from 1991 to 2004 can be characterized by a reduction of the real short-term interest rate, reflecting stable inflation at low levels and the process of fiscal consolidation required by the Maastricht criteria for EMU. Other factors that may have lowered the natural interest rate during the 1990's include the disappearance of exchange rate risk premia within the euro area, the slowdown in population and productivity growth and the decrease in inflation risk premia. This is consistent with figure 2 which suggests that the interest rate gap was closed in 2003.

In table 3, the section labeled 'Standard error (sample average)' refers to the average standard error surrounding the estimate of the smoothed (two-sided) unobserved variable. The uncertainty surrounding the estimates of potential output and its growth rate is smaller in all our models than that reported in Laubach and Williams (2003). However, the uncertainty surrounding the natural interest rate reaches nearly the same level in models `A.1` and `B.1` and implies relatively imprecise estimates. As in Laubach and Williams (2003), the imprecision of the estimates further increases when we consider the filtered (one-sided)[9] estimates in model `A.1` but remains approximately the same in models `B.1` and `C.1` (see the final observation standard errors in table 3). Both the one-sided and two-sided estimates of the natural interest rate are depicted in figure 4. This illustrates that the one-sided estimate based only on data up to time t substantially overshoots and undershoots the two-sided estimate based on all data up to the sample endpoint. This source of mismeasurement, which is likely to be even greater in real-time estimates (where coefficient estimates are also affected), can reach up to 2.3 percentage points and casts serious doubts on our ability to use one-sided estimates of natural interest rate as a guide for monetary policy. Similar conclusions were reached by Laubach and Williams (2003) and other researchers in this area (e.g. see Clark and Kozicki, 2004, or Orphanides and Williams, 2002).

[9] One-sided (filtered) estimates at time *t* use only information up to time *t* although the model parameters are estimated using the whole sample.

So far, we have ignored the uncertainty surrounding the estimates of λ_g and λ_z. Figure 8 in the appendix C shows the sensitivity of the estimated natural interest rate and output gap to the values of $\hat{\lambda}_z$. As found by Laubach and Williams (2003), lower values of $\hat{\lambda}_z$ imply smaller variability of the smoothed natural interest rate and vice versa. The estimates of output gap are relatively insensitive to changes of $\hat{\lambda}_z$ within the 95% confidence interval. Figure 9 in the appendix C illustrates the sensitivity of our estimates with respect to stationarity assumptions of the two unobserved components in the model, namely the z_t variable and the potential growth rate, g_t. Model `A.1` yields the most volatile estimate of natural interest rate while the estimate obtained from model `B.1` is nearly constant. Finally, the bottom panels of figure 8 and 9 (in the appendix) demonstrate the plausibility of the output gap estimates, as most are close to the conventional HP estimate.

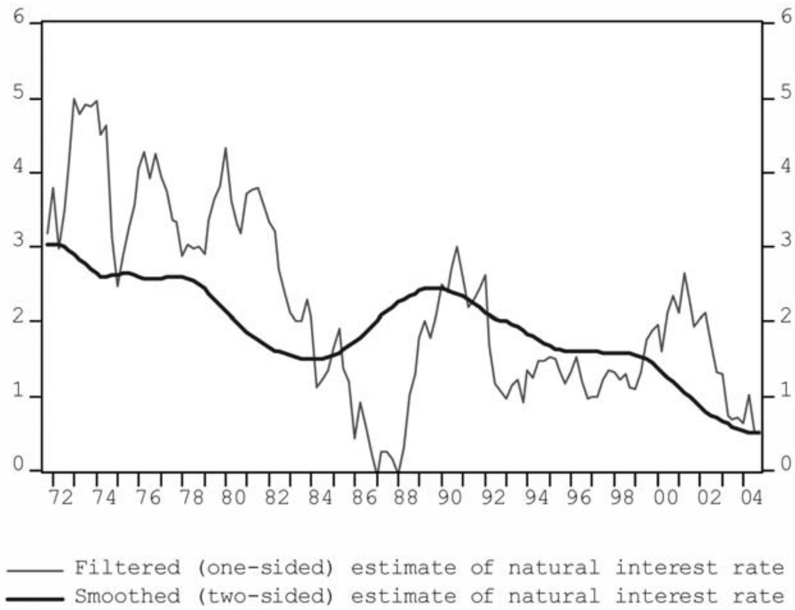

Fig. 4. One-sided and two-sided estimates of the natural interest rate (model `A.1`***).***

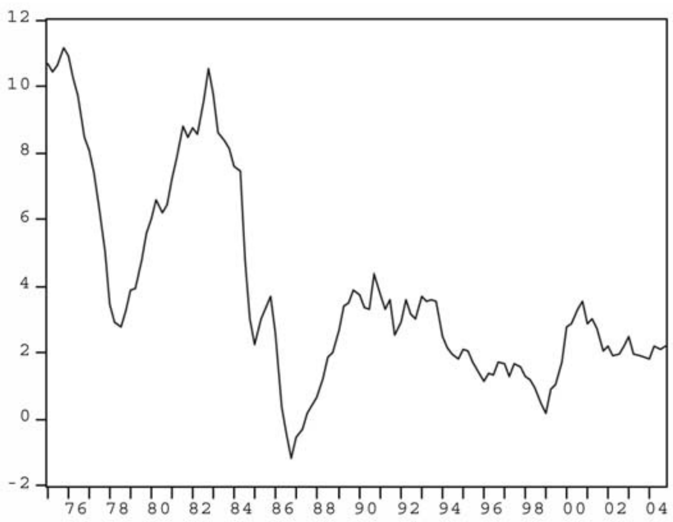

Figure 5. Inflation rate in Luxembourg (CPI based).

5. Natural Interest Rate in Luxembourg

The model used to obtain estimates of the natural interest rate in Luxembourg extends the one for the euro area (equations (1)–(6)). Given that Luxembourg is a small and very open economy, we have considered a range of additional supply shocks in the Phillips curve (2), including shocks in world oil prices, labour productivity, unit labour costs and terms of trade (see Gordon, 1997). Most of these shocks (and their lags) turned out insignificant and even deteriorated our estimates of the remaining parameters and/or state variables. For reasons of parsimony, we have included in the final specification only two lags of shocks in labour productivity and oil prices (which replace relative import price inflation).

5.1. Data

The model was estimated on a quarterly data set spanning 1975Q1 to 2004Q4. When earlier periods are included, the model exhibited structural instability, possibly due to the poor quality of Luxembourg's data in the early sample.

The inflation rate is obtained from the monthly series of the national consumer price index published by Statec (the national statistical office of Luxembourg). We apply the Tramo/Seats seasonal adjustment procedure to the monthly series, aggregate to obtain quarterly values and then calculate the annualized growth rate. The resulting inflation rate is depicted in figure 5. To measure the short-term interest rate in Luxembourg, we use the three-month money market rate in Belgium till 1998Q4.[10] Since European monetary union in 1999, the quarterly average of daily three-month EURIBOR rate was used. The series was deflated with the inflation rate defined above. Our quarterly measure of real GDP is interpolated from the annual series published by Statec (quadratic interpolation).[11] The oil price shocks, oil_t, were calculated using a simple formula $oil_t = \max(0, \ln poil_t - \ln poil_{t-1})$, where $poil_t$ is the brent crude oil price per barrel expressed in euro. The series in US dollars and the Euro/USD exchange rate were retrieved from the IMF's International Financial Statistics database. The labour productivity shocks are the log-ratio of real GDP per worker and its Hodrick-Prescott trend.

5.2. Estimation

For Luxembourg, we considered only the random walk specification of both g_t and z_t with the median unbiased estimates of λ_g and λ_z. The key coefficient estimates are presented in table 4.

The sum of autoregressive coefficients in the IS equation is higher than in the euro area but still less than one and implies a stationary output gap in Luxembourg. The semi-elasticity of the output gap with respect to the interest rate gap, a_r, has the anticipated negative sign, it is significant at 10% level, and its value is comparable to the euro area estimate. The estimated negative slope of the Phillips curve (b_y) is counterintuitive, however the coefficient is not statistically different from zero. The coefficient c linking the natural interest rate with potential growth rate is close to its anticipated value of one.

The supply shocks turned out insignificant in this specification except for the second lag of labour productivity shocks.

[10] Luxembourg was in a monetary association with Belgium at the time and had no independent monetary policy.

[11] Quarterly data on real GDP for 1995Q1 to 2004Q4 was published in May 2005. However, use of this data led to parameter instability and unrealistic results, proabably due to high volatility of the quarterly GDP growth rate in this preliminary release.

Table 4. Coefficient estimates for Luxembourg

	Coefficient	Std. error	Smoothed states	
λ_g	0.000		Standard error (sample average)	
λ_z	0.013		$r*$	2.057
$a_1 + a_2$	0.975		g	0.081
a_r	-0.038	(0.03)†	$y*$	0.042
b_y	-0.013	(0.03)	Standard error (last observation)	
b_1^{lp}	-0.299	(0.22)	$r*$	3.024
b_2^{lp}	0.451	(0.23)‡	g	0.081
b_1^{oil}	-2.214	(2.17)	$y*$	0.053
b_2^{oil}	2.074	(3.15)		
c	1.212	(2.30)	Log Likelihood	175.59

Note: 'na' means not applicable, † significant at 10% level, ‡ significant at 5% level.

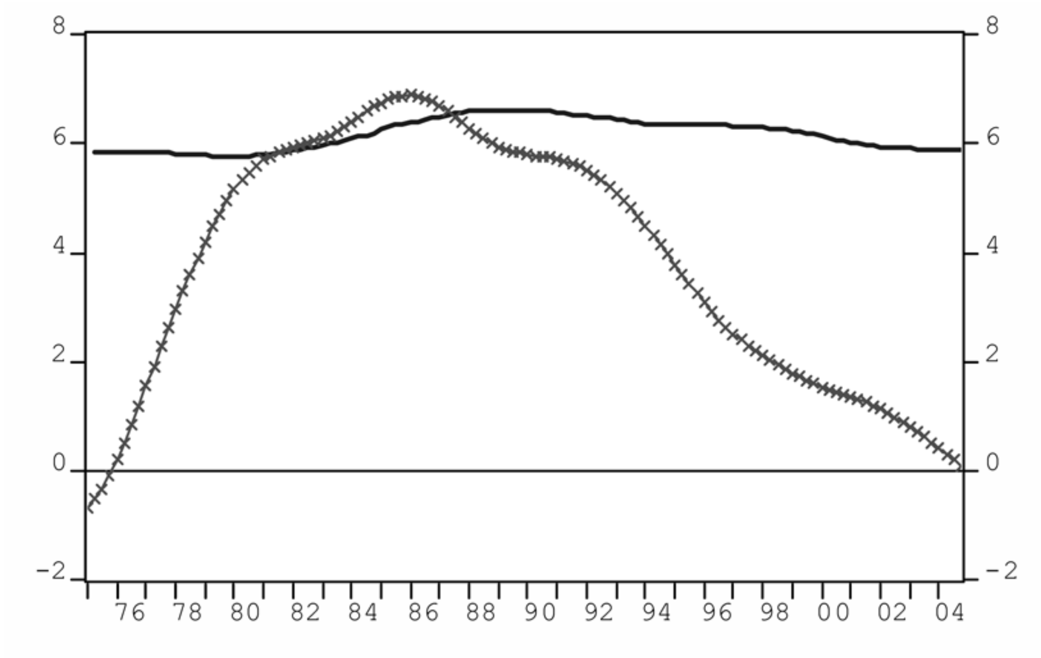

Fig. 6. Natural (real) interest rate in Luxembourg.

Figure 6 shows both the natural interest rate for Luxembourg and the HP trend of the real short-term interest rate. From the beginning of our sample in 1975 to the second quarter of 1981, the estimated natural interest rate lies above the trend of the actual ex-post real interest rate, which implies monetary policy had an expansionary effect in Luxembourg. This may have contributed to the relatively high levels of inflation during this period, see figure 5. The trend of the real interest rate slightly exceeds the values of the natural interest rate between 1981 and 1987 indicating neutral or modestly restrictive monetary policy. This was a period of declining inflation in Luxembourg as in other European countries. Finally, since the third quarter of 1987, the natural interest rate remains more or less constant around 6% while the trend real interest rate declines to zero. This widening interest rate gap is not yet reflected in the evolution of inflation in Luxembourg, which shows no signs of an increasing trend up to 2004. The decreasing real interest rate trend reflects decreasing interest rates in the whole euro area, while the relatively stable natural interest rate for Luxembourg reflects its relatively high potential output growth (estimated at 4.8% per annum).

The uncertainty surrounding the smoothed estimates of unobserved variables in the model for Luxembourg is greater than that found for the euro area (cf. sections 'Standard error (sample average)' in tables 3 and 4). As expected, the standard errors of state variables further increase if we consider one-sided estimates (an upper bound is given by the last observation's standard error in table 4).

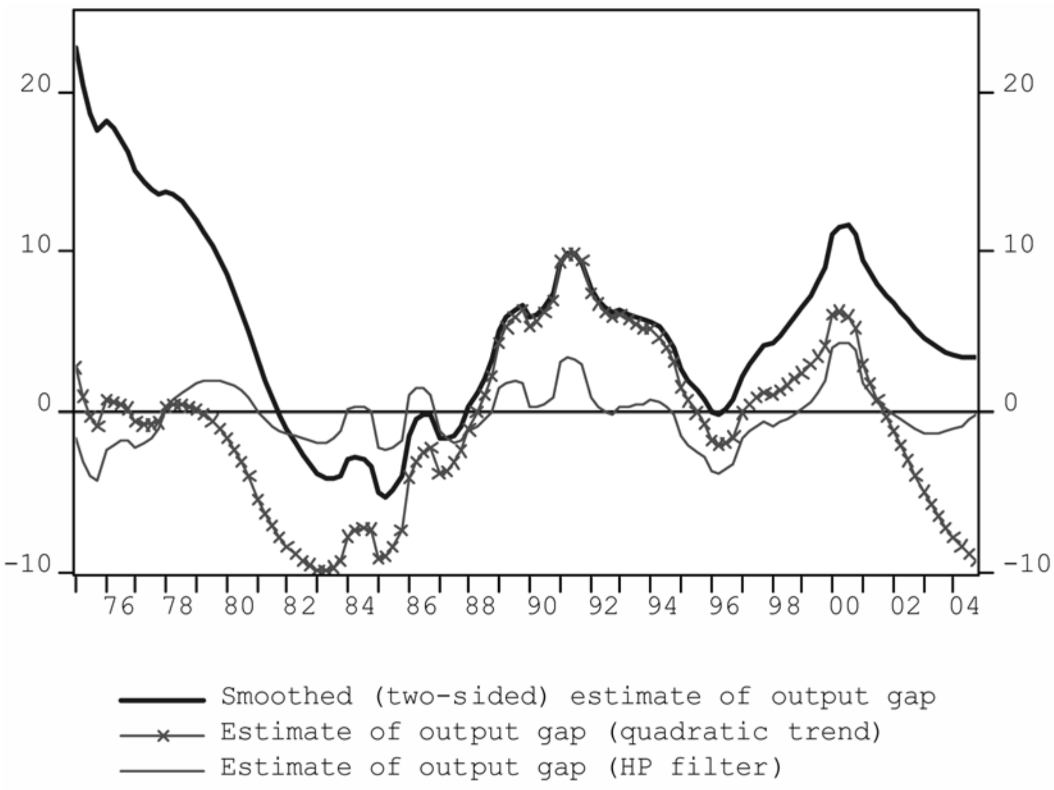

Fig. 7. Estimates of output gap in Luxembourg.

Finally, the plausibility of our estimate of the output gap can be assessed from figure 7 which plots the smoothed (two-sided) estimate of the output gap together with two common univariate alternatives. The smoothed estimate of the output gap lies within the interval delimited by the HP filter and quadratic trend estimates, except for the beginning and end of the sample. This finding may be due to the recursive estimation by the Kalman filter.

6. Conclusion

This paper provides estimates of the time-varying natural interest rate in the euro area over the period 1972 to 2004 and in Luxembourg between 1975 and 2004. The methodology chosen is that from the seminal paper on the US by Laubach and Williams (2003) and is also partly comparable to Mésonnier and Renne (2004) who estimate the natural interest rate in the euro area.

For the euro area, our results are broadly consistent with those found by other authors and with the basic stylized facts regarding growth and inflation. However, our results seem more plausible than those of Mésonnier and Renne (2004) as we obtain a natural interest rate with substantially lower variability (ranging from 0.5 to 3 percent) which seems more consistent with conventional estimates of potential growth. Based on the relative position of the natural interest rate and observed real short-term interest rate, we distinguished three periods. First, the 1970's when the natural interest rate was above the observed real short-term interest rate, suggesting that monetary policy accommodated inflationary pressures. Second, the 1980's and early 1990's with the opposite situation, suggesting that the monetary policy stance turned contractionary to get inflation under control. Finally the period since 1993 with a decreasing natural interest rate and monetary policy returning to a neutral or even expansionary stance. The recent decline in the natural interest rate may reflect slower growth in productivity and population, falling inflation risk premia, the disappearance of exchange rate risk premia, or the period of fiscal consolidation that preceded monetary union. In Luxembourg, the natural interest rate is less volatile, fluctuating at slightly less than 6%, but the impact of monetary policy follows a broadly similar trajectory over the sample considered.

Compared to the euro area, the estimated natural interest rate in Luxembourg was significantly higher. This probably reflects the difference in potential growth rates. According to our estimates, the growth rate of potential output reaches 4.8% per annum in Luxembourg, which is more than twice the level in the euro area. This gap suggests that since the common monetary policy will be driven by the euro area growth rate, it will generally be expansionary for Luxembourg, which could contribute to relatively higher levels of inflation in Luxembourg. This could have a negative impact on national competitiveness unless fiscal policy is appropriate and flexibility is increased through progress with structural reforms.

One has to bear in mind the significant uncertainty surrounding the estimates for both the euro area and Luxembourg. For policy purposes, the level of uncertainty suggested in this study is probably understated because data on future periods is unavailable. This means that the variability of the filtered (one-sided) estimate is more relevant for day-to-day policymaking than that of the smoothed (two-sided estimate). In addition, the most recent data observations, which play a crucial role for determining policy, are generally preliminary releases that are subject to substantial later revision. The present paper does not even begin to assess this additional source of uncertainty in real-time estimates of the natural rate (see Clark and Kozicki, 2004). In conclusion, while the natural rate estimates may provide a very imprecise guideline for policy, they do allow an *ex post* assessment of the impact of monetary policy on the economy.

7. References

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Appendix A—State-Space Representation

In order to estimate the Kalman filter specified in equations (1)–(6), the model must be rewritten in state-space form

$$x_t = \boldsymbol{H}\beta_t + \boldsymbol{A}z_t + e_t \quad (7)$$

$$\beta_t = \boldsymbol{F}\beta_{t-1} + v_t \quad (8)$$

where x_t is a n x 1 vector of observed variables at time t; β_t is a k x 1 vector of unobserved state variables;, $\boldsymbol{H}$, $\boldsymbol{A}$, $\boldsymbol{F}$ are matrices of coefficients of the n x k, n x r and k x k respectively; and finally, z_t is an r x 1 vector of exogenous (predetermined) variables. We also need to assume that $e_t \sim iid\mathrm{N}(0,R)$, $v_t \sim iid\mathrm{N}(0,Q)$ and $\mathbf{E}[e_t\, v_s'] = 0$. Equation (7) is the measurement equation and (8) the corresponding transition equation.

It is straightforward to see that the model `A.1` as defined in tables 1 and 2 can be represented as equations (7) and (8) with the following parametrization

$$H = \begin{bmatrix} 1 & -a_1 & -a_2 & \frac{-a_r}{200} & \frac{-a_r}{200} & 0 & 0 \\ 0 & -100b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} a_1 & a_2 & \frac{a_r}{200} & \frac{a_r}{200} & 0 & 0 & 0 & 0 & 0 & 0 \\ 100b_y & 0 & 0 & 0 & \left(b_1 - b_1^m\right) & \left(b_2 - b_2^m\right) & b_3 & \left(1 - \sum_{i=1}^{3} b_i\right) & b_1^m & b_2^m \end{bmatrix}$$

$$\beta_t' = \begin{bmatrix} y_t^* & y_{t-1}^* & y_{t-2}^* & r_{t-1}^* & r_{t-2}^* & z_t & g_t \end{bmatrix}, \quad x_t' = \begin{bmatrix} y_t & \pi_t \end{bmatrix}$$

$$z_t' = \begin{bmatrix} y_{t-1} & y_{t-2} & r_{t-1} & r_{t-2} & \pi_{t-1} & \pi_{t-2} & \pi_{t-3} & \pi_{t-4} & \pi_{t-1}^m & \pi_{t-1}^m \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1/100 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Given initial values for the state mean and covariance, and for the system matrices, the Kalman filter can be used to compute the contemporaneous or filtered mean of the state vector and its variance.

Appendix B—Estimating λ_g, λ_z and their confidence bands

This section describes the way we deal with the 'pile-up' problem using the methodology of Stock and Watson (1998). To obtain the point estimate of λ_g we first estimate a simplified version of the state-space model (1)–(6) that does not exhibit the pile-up problem. We omit the real rate gap from equation (1), which in turn eliminates the state equations for r_t^* and z_t, and we assume that the growth rate of potential output is constant. From this model we obtain an estimate of potential output as a smoothed state series using the Kalman filter.

In the next step, we test for an intercept shift with unknown date in the first difference of the estimated potential output. Formally, we regress the first difference of the smoothed potential output on a constant. In terms of equation (5), this is equivalent to testing whether g_t can be approximated by a constant or not. The regression is run over the entire sample (*T* observations) and sequentially over the sub-samples defined by splitting the sample at every potential break date *Ts*, where $0.05T \leq Ts \leq 0.95T$.[12] In the sequential process we calculate the Chow F-statistics, $F_T(s)$, testing for a structural break at all dates Ts as defined above. Denoting the sum of squared residuals from the OLS regression over observations $t_1 \leq t \leq t_2$ as $SSR_{t1,t2}$, the Chow F-statistic can be written as

$$F_t(s) = \frac{(SSR_{1,T} - SSR_{1,Ts} - SSR_{Ts+1,T})/k}{(SSR_{1,Ts} + SSR_{Ts+1,T})/(T-k)}$$

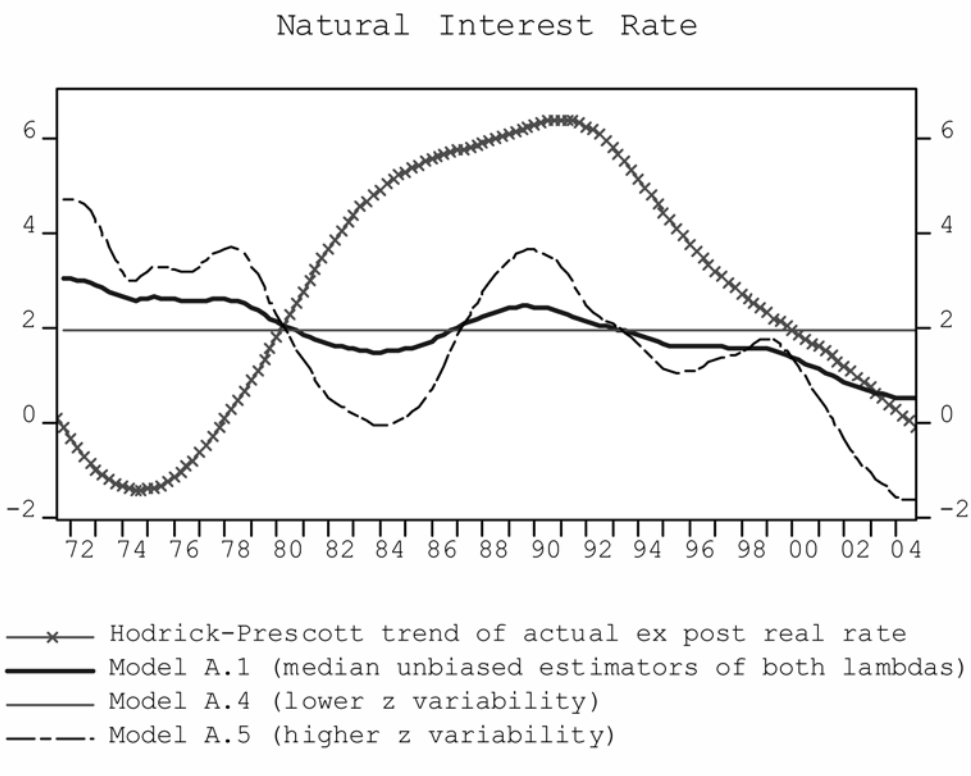
To obtain the median-unbiased estimator of λ_g, we calculate the exponential Wald statistic, defined as $EW_T = \ln\left\{(0.8T)^{-1}\sum_{r=0.1T}^{0.9T}\exp(0.5F_T(r))\right\}$, and use the lookup table provided by Stock and Watson (1998) (table 3, page 354) to retrieve $T\,\hat{\lambda}_g$.

The confidence interval for the point estimate of λ_g is computed from 10 000 Monte Carlo simulations using the model object in EViews.

The estimation procedure for λ_z and its confidence interval follows the same logic and steps as outlined for λ_g. In this case we calculate the exponential Wald statistic for an intercept shift in the output gap equation at an unknown break date.

12 This is a standard 5% trimming. If *Ts* is not an integer, we take the largest lesser integer.

Appendix C—Sensitivity of Smoothed Estimates in the euro area



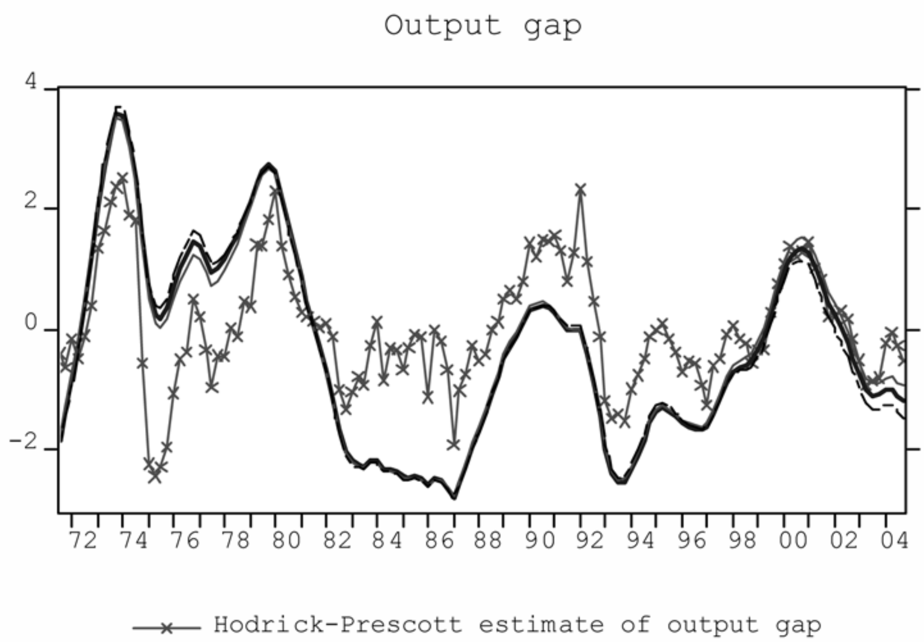


Fig. 8. Sensitivity of smoothed estimates to values of $\hat{\lambda}_z$

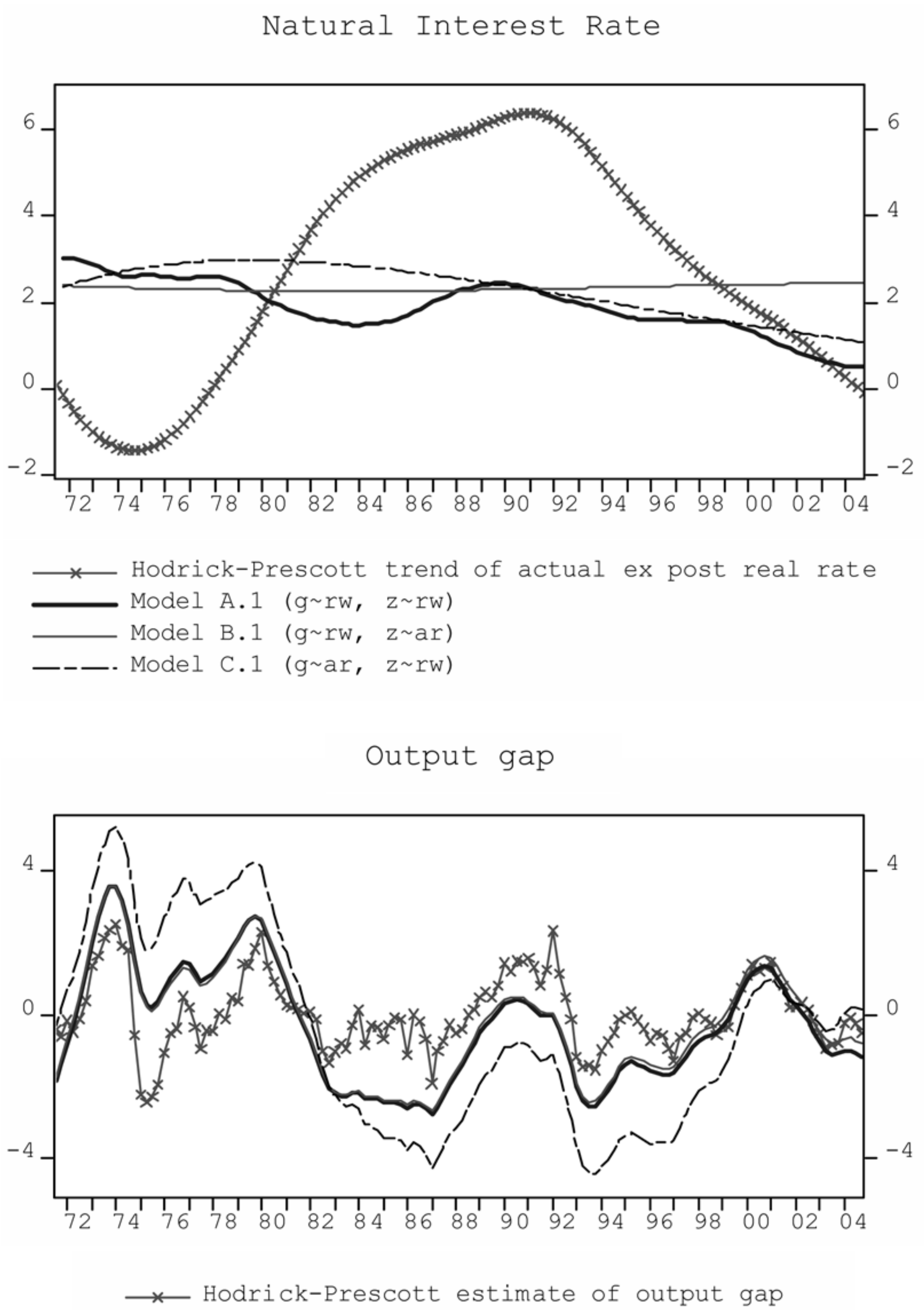


Fig. 9. Sensitivity of smoothed estimates to model specification.