CAHIER D'ÉTUDES
WORKING PAPER

N° 204

Public Capital Stocks in Dynamic Fiscal Competition

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FEBRUARY 2026

BANQUE CENTRALE DU LUXEMBOURG

EUROSYSTÈME

Public Capital Stocks in Dynamic Fiscal Competition*

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Abstract

We study dynamic fiscal competition when mobile capital responds to the stock of public infrastructure, not contemporaneous expenditure flows. Two governments choose taxes and public investment in a differential game with gradual accumulation and investment frictions. In the unique Nash equilibrium, long-run dynamics depend on discounting, erosion of advantages, the share of public investment that effectively adds to infrastructure, the private productivity effects of infrastructure, and adjustment/acquisition costs on public investment. The model delivers convergence to a symmetric steady state, and "invest–then–stop" behavior. Large initial gaps can rationally delay the laggard's investment, generating distinct transition paths and persistent asymmetries.

Keywords: Fiscal competition; public capital stock; infrastructure investment; differential games; mobile capital.

JEL classification: F21; F23; H25; H54; H73; C73.

*This paper should not be reported as representing the views of the BCL or the Eurosystem. The views expressed are those of the authors and may not be shared by other research staff or policymakers in the BCL or the Eurosystem. We are grateful to Paolo Guarda and Jean-Charles Rochet for their insightful suggestions. We thank Pablo Garcia Sanchez and Alban Moura for their comments during the ER Seminar at the Banque centrale du Luxembourg.

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Résumé non technique

Le rapport Draghi (2024) souligne le risque de décrochage de l'Europe si celle-ci ne réalise pas des dépenses publiques conséquentes dans les technologies de pointe nécessaires à préserver sa compétitivité. Même à l'intérieur de la zone euro, les États membres se font concurrence pour attirer des entreprises et des ressources mobiles. Or, cette concurrence internationale repose moins sur des mesures budgétaires de court terme que sur l'accumulation d'un stock de capital public (infrastructures, réseaux, capacités administratives, institutions légales et réglementaires) capable de façonner l'attractivité territoriale et donc sa base fiscale.

Cette étude propose un cadre théorique pour répondre à la question suivante. Lorsque le capital privé est mobile au niveau international, les politiques nationales consistant à investir stratégiquement en infrastructures publiques suffisent-elles à réduire les écarts technologiques entre pays (convergence) ou au contraire, sont-elles susceptibles de creuser des écarts existants (divergence) ? Afin d'approfondir cette question, nous proposons une analyse dynamique dans laquelle deux pays se livrent concurrence pour attirer du capital international. Pour ce faire, ils utilisent deux types d'instruments. D'une part, l'impôt sur les sociétés et d'autre part des investissements publics qui contribuent à augmenter l'infrastructure nationale. L'infrastructure améliore la productivité des entreprises qui s'implantent au pays et peut donc attirer du capital étranger, augmentant l'activité économique et donc les recettes fiscales. Cependant, l'infrastructure publique est sujette à la détérioration à travers le temps, et la diffusion technologique peut éroder les avantages nationaux en la matière.

Le modèle développé dans notre papier permet de tirer plusieurs enseignements. Premièrement, la convergence entre pays n'est pas assurée. La dynamique concurrentielle favorise la réduction des écarts et donc la convergence lorsque la dépréciation de l'infrastructure est rapide, lorsque les avantages liés à l'infrastructure se diffusent plus rapidement (diffusion des standards, interopérabilité, transferts de savoir-faire), ou lorsqu'il existe des coûts d'ajustement de l'investissement public. À l'inverse, la divergence technologique entre pays peut persister du fait de la course à l'investissement lorsque les autorités publiques sont patientes, lorsque l'infrastructure contribue significativement à la productivité du secteur privé, ou lorsque les investissements publics augmentent efficacement le stock d'infrastructure. Néanmoins, dans notre modèle la convergence des économies n'implique pas nécessairement l'amélioration du bien-être social. Afin de répondre à cette question, une analyse normative reste nécessaire. Deuxièmement, les conditions initiales ont un impact sur le résultat. Si l'écart en infrastructure entre pays est modéré, les deux gouvernements investiront en infrastructure. Par contre, si l'écart initial est important, le pays ayant un retard technologique peut rationnelle-

ment différer l'investissement, en attendant que l'avantage de l'autre pays s'atténue à travers la diffusion technologique. Troisièmement, la course à l'investissement public peut être abandonnée au profit d'une concurrence exclusivement fiscale lorsqu'il existe d'importantes frictions liées à l'ajustement de l'investissement public (permis de construction, procédures administratives, coûts de financement, aléas de mise en œuvre des infrastructures, etc.). Enfin, un pays richement doté en infrastructures peut maintenir un impôt sur les sociétés à un niveau élevé, même si la base imposable est mobile.

Du point de vue de la politique économique, trois messages se dégagent. D'abord, réduire les frictions qui augmentent le coût de l'investissement public (permis, procédures, marchés publics, capacité d'exécution, financement) peut éviter l'abandon des programmes d'infrastructure lorsque l'investissement est socialement souhaitable. Ensuite, l'importance des conditions initiales plaide pour des aides ciblant les régions à très faible niveau d'infrastructure publique, afin de limiter les délais d'entrée dans la dynamique d'investissement. Enfin, lorsque l'infrastructure est un levier puissant et que les avantages sont persistants, la dynamique peut être « fragile » et conduire à des écarts durables. Cela plaide en faveur de mécanismes de coopération et d'intégration (projets transfrontaliers, standards communs, interopérabilité, intégration des marchés de l'énergie, du transport et du numérique) facilitant la diffusion et la transférabilité des avantages, sans nécessairement, dans l'esprit du rapport Draghi (2024), affaiblir le rôle productif des infrastructures. Ceci afin de contribuer à enrayer le décrochage de l'Europe face aux grandes économies que sont les États-Unis et la Chine.

1 Introduction

When the literature on tax competition considers public expenditure, it usually assumes international attractiveness depends directly on the flow of contemporaneous public spending. Contributions such as Zissimos and Wooders (2008), Hindriks et al. (2008), Pieretti and Zanaj (2011), and Han et al. (2023) emphasize the strategic role of public inputs in shaping tax competition, showing that jurisdictions do not only compete on taxes but also through public investment and the provision of public goods. However, these analyses implicitly assume that spending translates immediately into attractiveness, neglecting the time required to accumulate infrastructure that increases a country's long-run appeal. Moreover, empirical evidence consistently suggests that accumulated infrastructure stocks are more important than contemporaneous expenditure flows. Röller and Waverman (2001) show that telecom *stocks* raise output once critical mass is achieved. Calderón et al. (2015) estimate economically and statistically significant long-run output elasticities of roughly 0.07–0.10 with respect to the infrastructure *stock*, while flows misstate dynamics. These results are consistent with the meta-study by Bom and Ligthart (2014), which reports a similar long-run elasticity. Infrastructure quality also shapes location choices of mobile firms: poor services depress FDI at the firm level (Kinda, 2010). Bénassy-Quéré et al. (2007) highlight infrastructure and institutions as key determinants of foreign direct investment.

To the best of our knowledge, very few papers address this distinction between flows and stocks of public expenditure. An exception is Han et al. (2018), who showed that tax competition can amplify or reduce disparities in infrastructure depending on capital mobility, initial endowments, and country size. However, because their analysis remained static, it did not trace how disparities evolve over time, whether steady states exist, or how they are reached.

The present paper develops a dynamic framework where the stock of public infrastructure, not the flow of public expenditure, is the central state variable determining the international allocation of private capital. By formulating the interaction between jurisdictions as a differential game, we are able to capture transitional dynamics, identify the conditions for a stable equilibrium, and analyze the role of forward-looking government strategies. Recent policy debates underscore the relevance of a stock-based view of public capital. In Europe, the competitiveness agenda emphasizes persistent productivity gaps and calls for large-scale investment in energy, and digital infrastructure, supported by deeper market integration and joint instruments, with an explicit convergence objective across regions and relative to global leaders (Draghi, 2024). At the same time, US–China competition in digital infrastructure (computing capacity, data centers, and energy supply for AI) illustrates a setting in which patient policy and strong network effects can sustain long-lasting strategic rivalry (US-China Economic and Security Review Commission, 2024). These narratives about gap-closing programs versus persistent races map naturally into the regimes characterized by our model.

First, we find that the existence of a stable equilibrium in this framework depends on the government's effective discount factor, the depreciation rate on infrastructure, the share of public investment that effectively adds to infrastructure, the private productivity effects of infrastructure and any adjustment costs in public investment. The model predicts catch-up, or convergence toward a symmetric steady state, when discounting is sufficiently high, when infrastructure depreciates rapidly, when private productivity effects of infrastructure are low, when only a low share of public investment effectively adds to infrastructure, or when adjustment costs to public investment are high.[1] However, when infrastructure delivers large productivity gains, convergence is less likely unless discounting is high.

Second, we also find that higher public-sector capital stocks support higher taxes on mobile capital. While internationally mobile private capital may be sensitive to tax differences, elasticities may be low and heterogeneous[2] Hence, where public infrastructure or amenities raise productivity, higher taxes may be sustainable, as illustrated by our comparative statics.

Third, we find that the initial infrastructure gap plays a decisive role. When the gap is small, both jurisdictions will invest in public infrastructure from the outset, though at different intensities. When the gap is very large, the lagging country will rationally delay public investment, waiting for the leader's advantage to erode before beginning to catch up. This generates distinct patterns of adjustment and helps explain why some jurisdictions postpone costly infrastructure programs despite long-run benefits. This delay mechanism is consistent with evidence from EU Cohesion policy that limited absorptive capacity can slow the transformation of transfers into effective investment and growth (Becker et al., 2013; Crescenzi and Giua, 2020).[3]

Fourth, we find an important role for acquisition costs, reflecting frictions in the process of building infrastructure, for example, obstacles to land acquisition, administrative delays, or financing difficulties, credit constraints, but also time required to staff policy institutions to disburse public investment. When these costs are negligible, countries compete through both infrastructure and tax. When acquisition costs are very high, jurisdictions may abandon infrastructure competition to focus on tax competition to attract

[1]Empirically, short political horizons and myopic incentives are well documented (Brender and Drazen, 2005; Healy and Malhotra, 2009; Veiga and Veiga, 2007).

[2]Inventors respond to tax differentials (Akcigit et al., 2016; Kleven et al., 2014), and establishments reallocate across borders after state tax changes (Giroud and Rauh, 2019).

[3]Absorptive capacity refers to administrative and implementation capacity to turn funds into effective investment.

internationally mobile private capital. When acquisition costs are moderate, investment often follows an "invest-then-stop" trajectory: governments begin competing on infrastructure but the laggard eventually drops out, followed by the leader. Such acquisition frictions may be linked to permitting, procurement, and renegotiation, and can generate stop–go patterns consistent with our "invest–then–stop" dynamics (Bajari et al., 2014; Decarolis and Palumbo, 2015; Guasch et al., 2008).

From a policy perspective, our results carry three main implications. First, reducing frictions that raise the cost of building infrastructure, through streamlined land-use regulation, better administration, or cheaper financing, can prevent governments from abandoning infrastructure competition when investment is socially desirable. Second, the role of initial conditions suggests that structural funds or transitional aid should prioritize very large gaps, to encourage lagging regions not to delay improving public infrastructure. Third, the model highlights a source of fragility. When infrastructure strongly attracts mobile capital and advantages are persistent, equilibrium gaps can widen and asymmetries may persist. Shifts into the convergent regime typically arise when the strategic return to infrastructure weakens or when advantages diffuse/erode faster. This is a positive characterization rather than a normative recommendation. Policies to support balanced regional development should promote technological diffusion while limiting distortions from strategic competition that can depress infrastructure accumulation.

The remainder of the paper unfolds as follows. Section 2 presents the model and describes the strategic interaction between jurisdictions. Section 3 analyzes the dynamic equilibrium, the role of initial conditions, and rational investment behavior. Section 4 discusses the role of frictions in the process of building infrastructure. Section 5 concludes.

2 The Model

Consider a world economy with two countries indexed by $\ell = i, j$. A representative multinational company (MNC) wants to decide on the allocation an amount of capital k each time t among two countries.[4] Capital invested in country ℓ is $k_\ell(t)$. So $k_1(t)+k_2(t) = k$. We can assume that $k = 1$ for simplicity.

In each location, the MNC has a great number of subsidiaries of equal size all producing the same good. Therefore, we consider the aggregate production in location $\ell = i, j$ is given by $F(k_\ell)$ that depends on the stock of private capital k_ℓ and on the stock of public

[4] We abstract from labor mobility and focus on mobile capital, as is standard in the tax-competition literature. Allowing for worker migration could reinforce fiscal competition because public infrastructure may also attract workers (especially skilled workers) in addition to firms. For related evidence and analysis on skilled bilateral migration, see Pieretti et al. (2024).

infrastructure G_ℓ available in location $\ell = i, j$. Assume that

$$F(k_\ell(t)) = (Q + \alpha G_\ell(t))\, k_\ell(t) \ , \tag{1}$$

Q is a private productivity parameter that is the same across countries and time. Infrastructure G_ℓ has the nature of a local public good in ℓ, which increases the productivity of all companies located in that country. The parameter $\alpha > 0$ determines the private sector productivity effects of public sector infrastructure. We assume that the cost of private investment increases more than proportionally with the level of capital. For the MNC, the cost of investing in jurisdiction ℓ is $C(k_\ell) = \frac{1}{2}k_\ell^2$. The convexity of this cost function reflects that it is increasingly difficult to acquire additional capital.

We assume that the government in each country levies a tax on capital. The price of a unit of capital is $\beta > 0$, so the value of capital is $\beta k_\ell(t)$. The tax on each unit value of capital in country ℓ is τ_ℓ. In the following, we normalize β to one.

The total profit (net of taxes) of the MNC is

$$\Pi(k_i(t), k_j(t)) = \sum_\ell \left[F(k_\ell(t)) - k_\ell(t)\tau_\ell(t) - \frac{k_\ell^2(t)}{2} \right].$$

$G_\ell(t)$ represents the stock of accumulated public infrastructure/institutions in country $\ell = 1, 2$. For example, this includes transportation systems, energy grids, water supply, sanitation, and digital networks. However, note that the quality and scope of such infrastructure vary drastically between countries. Disparities in infrastructure also reflect an uneven distribution of technological knowledge.[5]

The evolution of the stock of public infrastructure $G_\ell(t)$ depends on public investment decisions. Each period, the government in country ℓ decides to invest I_ℓ in infrastructure expenditure that is added to the infrastructure stock, which evolves according to the following dynamic equation:

$$\dot{G}_\ell(t) = \phi I_\ell(t) - \delta G_\ell(t) + \eta(G_{-\ell}(t) - G_\ell(t)), \qquad \ell = i, j, \tag{2}$$

where $I_\ell(t)$ is the public investment made at time t, ϕ is the share of public investment that is effectively converted into public infrastructure, while δ represents the depreciation rate of the public infrastructure, and η the erosion of a jurisdiction's infrastructural advantage through knowledge and technology diffusion. A higher η thus reflects faster

[5] Technological knowledge includes engineering and design skills, project management expertise, digital tools, as well as institutional knowledge (e.g., policy frameworks, regulation standards). Countries that possess or can access advanced technological knowledge are better equipped to plan and implement efficient infrastructure projects, and train a skilled workforce to manage complex systems.

spillovers of infrastructure-related know-how to competitors. This depreciation/diffusion channel, where a jurisdiction's advantage erodes as technologies and standards spread, is consistent with measured spatial spillovers from infrastructure: the productivity effects of highways, airports, and other networks extend across borders, reducing the persistence of local advantages and pushing regions toward diffusion-driven convergence (Cohen and Paul, 2004).

In each period, the MNC maximizes its profit by deciding on the share of capital allocated to each country. The first order condition for k_i can be derived

$$\frac{\partial \Pi(k_i(t))}{\partial k_i(t)} = 0 \qquad \text{yields} \qquad k_i^*(t) = \frac{1}{2} + \frac{\Big[\alpha(G_i(t) - G_j(t)) - (\tau_i(t) - \tau_j(t))\Big]}{2},$$

and

$$k_j^*(t) = 1 - k_i^*(t) \ .$$

Therefore, capital allocation depends on the public infrastructure gap between countries $(G_i(t) - G_j(t))$, as well as the tax differential $(\tau_i(t) - \tau_j(t))$. We assume that governments can modify the tax differential every period, but that the infrastructure gap follows the differential equation,

$$\dot{y}(t) = \phi(I_i(t) - I_j(t)) - \gamma y(t), \tag{3}$$

where $y(t) \equiv G_i(t) - G_j(t)$ for shorter notation, while $\gamma \equiv \delta + 2\eta$ represents the effective depreciation rate of the public infrastructure, combining not only physical or organizational decay (δ) but also knowledge and technology diffusion (η).

To lighten notation, we generally omit the explicit time dependence and write for example y, I_i, k_i instead of $y(t), I_i(t), k_i(t)$ unless the time argument is needed for clarity.

The Governments' Problem

We follow the standard approach in the tax competition literature and assume that each government chooses tax and infrastructure investment to maximize the discounted value of net fiscal revenues (see Kanbur and Keen, 1993, Zissimos and Wooders, 2008 or Pieretti and Zanaj, 2011). This objective can be interpreted as a proxy for residents' demand for publicly financed goods. The optimal control problem of the government in country ℓ is

$$J_\ell = \max_{\tau_\ell, I_\ell} \int_0^\infty e^{-\rho t} \left[\tau_\ell k_\ell - \left(bI_\ell + c\frac{I_\ell^2}{2}\right)\right] dt \ , \qquad \ell = i, j \tag{4}$$

subject to the law of motion equation (3) and a non negative investment constraint $I_\ell \geq 0$, where, $b \geq 0$ measures the unit acquisition cost, $c > 0$ measures the unit adjustment cost,

$\rho \geq 0$ measures the government discount factor,[6] and the capital allocation is derived from the MNC problem,

$$k_i = \frac{1}{2} + \frac{\alpha y - (\tau_i - \tau_j)}{2}, \qquad k_j = 1 - k_i \ .$$

The last term in the objective function $\left(bI_\ell + c\frac{I_\ell^2}{2}\right)$ represents the cost of public investment in infrastructure, which takes the linear-quadratic form for simplicity (Dawid et al., 2010). This cost term is meant to capture two different aspects. The linear part $(b\, I_\ell)$ is an acquisition cost reflecting that every extra unit of public capital requires permits, planning, political effort, and basic administrative capacity. Even if investment is spread over time, a country still pays the same acquisition cost. The quadratic part $(c\, I_\ell^2/2)$ is an adjustment cost reflecting that investing a given amount in one period is more expensive than spreading this investment over several periods. This can be due to construction bottlenecks, limited engineering capacity, inflated prices in the construction sector, or coordination problems when countries try to implement too many projects at once. Economically, b contributes to determining whether it is worth entering the infrastructure race at all, while c determines the speed of the race, meaning that investment will be more gradual when c is high and more aggressive when c is low.

3 The Dynamic Nash Equilibrium

As a first step, we simplify the problem (4) by setting the infrastructure acquisition cost to zero $b = 0$. We will address the case $b > 0$ in a later section. In this scenario, there is no acquisition cost associated with investment and governments only face adjustment costs. This simplified framework helps build intuition about the nature of the government's optimization problem.

To find an equilibrium path, define the Hamiltonian for government i, and derive optimality conditions with respect to taxes and public investment,

$$H_i(\tau_i, I_i; y, \lambda_i; \mu_i) = \tau_i k_i - \frac{cI_l^2}{2} + \lambda_i[\phi(I_i - I_j) - \gamma y] + \mu_i I_i,$$

where λ_i is the co-state variable and μ_i is the Kuhn-Tucker multiplier.

The first order conditions with respect to τ_i and I_i yield,

$$\tau_i = \frac{1}{2}\left(1 + \alpha y + \tau_j\right), \tag{5}$$

[6] In quantitative macro settings it is standard to set $\rho \in (0, 1)$ to match real interest rates, but our theoretical results do not rely on an upper cap.

$$I_i = \frac{\phi\lambda_i + \mu_i}{c}. \tag{6}$$

The Kuhn-Tucker conditions yield,

$$\mu_i \geq 0, \qquad I_i \geq 0, \qquad \text{and} \qquad \mu_i I_i = 0\ . \tag{7}$$

Accordingly, either $\mu_i > 0$ and $I_i = 0$, or $\mu_i = 0$ and $I_i > 0$. If $I_i = 0$, from equation (6), $\mu_i = -\phi\lambda_i > 0$, which is true if and only if $\lambda_i < 0$.

Therefore, the government maximizes its objective function by setting infrastructure investment along the path given by,

$$I_i^* = \begin{cases} 0, & \lambda_i < 0, \\ \dfrac{\phi\lambda_i}{c}\,(>0)\,, & \lambda_i \geq 0. \end{cases} \tag{8}$$

The costate variable obeys the following dynamic equation,

$$\dot{\lambda}_i = (\rho + \gamma)\lambda_i - \frac{\alpha}{2}\,\tau_i \tag{9}$$

with corresponding transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda_i(t)y(t) = 0.$$

Similarly, for government j, the following first order conditions hold:

$$\tau_j = \frac{1}{2}\left(1 - \alpha y + \tau_i\right), \tag{10}$$

$$I_j^* = \begin{cases} 0, & \lambda_j > 0, \\ -\dfrac{\phi\lambda_j}{c}(>0), & \lambda_j \leq 0. \end{cases} \tag{11}$$

$$\dot{\lambda}_j = (\rho + \gamma)\lambda_j + \frac{\alpha}{2}\,\tau_j \tag{12}$$

with transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda_j(t)y(t) = 0.$$

Solving the system of best responses, equilibrium tax policies are given by:

$$\begin{cases} \tau_i = 1 + \frac{\alpha y}{3}, \\ \tau_j = 1 - \frac{\alpha y}{3}. \end{cases} \tag{13}$$

Note that equilibrium taxes, τ_i and τ_j can be solved in terms of the infrastructure gap alone.[7]

Moreover, equation (13) shows that taxes tend to be higher for the country with higher infrastructure. This is because the higher infrastructure provides an edge for attracting private capital, which makes it possible to set higher taxes. This effect increases with private sector productivity effects of public infrastructure (α).

The dynamics of the state variable y and the costate variables $\lambda_i(t)$ and $\lambda_j(t)$ are governed by a system of differential equations. These equations are obtained by substituting the policy rules (8), (11), and (13) into the state and costate equations, under the assumption of positive investment in both countries. The resulting system is as follows:

$$\begin{cases} \dot{y} = \phi(I_i - I_j) - \gamma y = \frac{\phi^2}{c}(\lambda_i + \lambda_j) - \gamma y, \\ \dot{\lambda}_i = (\rho+\gamma)\lambda_i - \frac{\alpha}{2}\,\tau_i = (\rho+\gamma)\lambda_i - \frac{\alpha^2}{6}y - \frac{\alpha}{2}, \\ \dot{\lambda}_j = (\rho+\gamma)\lambda_j + \frac{\alpha}{2}\,\tau_j = (\rho+\gamma)\lambda_j - \frac{\alpha^2}{6}y + \frac{\alpha}{2}, \end{cases} \tag{14}$$

which is a linear system in terms of y, λ_i, λ_j.

Proposition 1 *(Existence of an equilibrium) The dynamic game described above generates one Nash equilibrium* $((\tau_i^*(t), I_i^*(t)), (\tau_j^*(t), I_j^*(t)))$, *which is given by*

$$\tau_i^*(y(t)) = 1 + \frac{\alpha y(t)}{3}, \qquad \tau_j^*(y(t)) = 1 - \frac{\alpha y(t)}{3}$$

and

$$I_i^*(t) = \begin{cases} 0, & \text{if } \lambda_i(t) < 0, \\ \frac{\phi\lambda_i(t)}{c}, & \text{if } \lambda_i(t) \geq 0, \end{cases} \qquad I_j^*(t) = \begin{cases} 0, & \text{if } \lambda_j(t) > 0, \\ -\frac{\phi\lambda_j(t)}{c}, & \text{if } \lambda_j(t) \leq 0. \end{cases}$$

where $y(t), \lambda_i(t), \lambda_j(t)$ *are given by the unique solution of the linear system (14) with the corresponding transversality conditions.*

[7] In the model τ_ℓ is a tax per unit of capital, not a tax on profits. We normalize technology and prices in such a way that the benchmark tax level happens to be unity. There is no theoretical reason why τ_ℓ must be below 1. What matters is that it remains below the gross marginal product of capital, otherwise firms would simply not invest. In equilibrium, for the parameter range and gaps we look at, taxes are always below that threshold, so $\tau_\ell > 1$ is perfectly consistent with firms still finding it profitable to produce.

Proof. In the text. ■

3.1 Steady State and Convergence Properties

Setting all time derivatives to zero, the unique steady state of the linear system (14) is

$$\overline{y} = 0, \qquad \overline{\lambda_i} = \frac{\alpha}{2(\rho+\gamma)}, \qquad \overline{\lambda_j} = -\frac{\alpha}{2(\rho+\gamma)}.$$

In the steady state, the two countries have the same level of infrastructure, with non-zero costate values, that are equal in absolute terms, reflecting positive public investment.

The stability of the equilibrium for the above dynamic system (14) can be analyzed by calculating its eigenvalues (see Appendix). Considering the transversality conditions, there is only one, potentially negative, eigenvalue (ν), governing the convergence of the system:

$$\nu = \frac{1}{2}\left(\rho - \sqrt{(\rho+2\gamma)^2 - \frac{4\alpha^2\phi^2}{3c}}\right). \tag{15}$$

For monotonic convergence of the system, ν must be negative,

$$\nu < 0 \qquad \text{iff} \qquad \rho > \frac{\phi^2\alpha^2}{3c\gamma} - \gamma\ .$$

We can identify two sub-cases of non-convergence, again depending on the eigenvalue ν. First, ν is real and positive when $\frac{2\alpha\phi}{\sqrt{3c}} - \gamma < \rho < \frac{\alpha^2\phi^2}{3c\gamma} - \gamma$, indicating monotonic divergence. Second, ν becomes complex when $\rho < \frac{2\alpha\phi}{\sqrt{3c}} - \gamma$, which indicates non-convergence with oscillations.[8]

The conditions above indicate that convergence requires the government discount factor (ρ) to exceed a function of several parameters, including the share of public investment that effectively adds to infrastructure (ϕ), the private sector productivity effect of public infrastructure (α), the effective depreciation rate (γ) and the adjustment cost parameter for public investment (c).

The government discount factor, ρ, plays a fundamental role because it takes time for public investment to raise the stock of infrastructure. Governments that are sufficiently patient (ρ small) tend to internalize the future effects of a change in their current investment, making the system unstable. This is because there will always be an incentive to change investment to attract a larger share of private capital. On the contrary, for

[8] We see the oscillatory regime mainly as a theoretical curiosity. If the infrastructure-attractiveness channel is extremely powerful and erosion is very slow, the system predicts unstable cycles around the symmetric steady state. In practice, we view that region as implausible and focus on the convergence and divergence regimes.

sufficiently impatient enough governments (ρ large enough), the present value of future benefits will be low enough and the system converges to the steady state.

When private sector productivity effects of infrastructure (α) are high, there is a greater incentive to invest in infrastructure. The same is the case if the share of public investment that effectively adds to infrastructure (ϕ) is high. While with low effective depreciation (γ), the infrastructure gap decays slowly and the leader's advantage is more persistent, preventing convergence. With lower adjustment costs on public investment (c), it is also easier to prevent convergence.

Proposition 2 *(Steady state and stability) The linear dynamic system defined by (14) admits a unique symmetric steady state , given by*

$$\overline{y} = 0, \qquad \overline{\lambda_i} = \frac{\alpha}{2(\rho+\gamma)}, \qquad \overline{\lambda_j} = -\frac{\alpha}{2(\rho+\gamma)}.$$

Furthermore, the steady state is globally and monotonically stable[9] *if and only if*

$$\rho > \frac{\alpha^2\phi^2}{3c\gamma} - \gamma \tag{16}$$

Proof. The first part is proven in the text, and the second part in Appendix A.1. ∎

3.2 Mapping different regimes

Proposition 2 implies that the long-run dynamics of the infrastructure gap are governed by the government discount rate ρ, effective depreciation γ, and the composite parameter

$$E \equiv \frac{\alpha\phi}{\sqrt{3c}}, \tag{17}$$

introduced as a convenient normalization to explore different regimes. The parameter α measures the private productivity effects of infrastructure, ϕ measures how much of public investment effectively adds to infrastructure, and c governs the marginal (convex) cost of adjusting investment. Importantly, E does not measure a country's overall location attractiveness (which depends on taxes and on the current state $y(t)$), nor does it by itself pin down equilibrium investment levels. Instead, intertemporal forces, such as government discounting ρ and erosion/diffusion γ, determine the present value and persistence of any infrastructure advantage.

[9] Although the steady state is a saddle point of the linear system, the transversality conditions select the unique stable trajectory consistent with optimality, effectively setting the coefficients of unstable modes to zero.

In these terms, the convergence condition (16) becomes

$$\rho \;>\; \frac{E^2}{\gamma} - \gamma. \tag{18}$$

Figure 1 plots the regime map in the (ρ, E) plane for a given γ. On the y-axis, greater values of ρ correspond to more impatient governments, while on the x-axis, greater values of E correspond to higher public infrastructure effects on private productivity (α), a higher share of public investment effectively adding to infrastructure (ϕ) or lower adjustment costs on public investment (c). For a fixed (ρ, E), a higher diffusion/erosion rate γ shifts the boundary downward, making convergence more likely.

For readers interested in an informal bridge to current policy narratives, Appendix C provides a deliberately illustrative mapping of the policy "races" between the EU and the US or between the US and China onto a zoomed version of Figure 1 in the (E, ρ) plane.

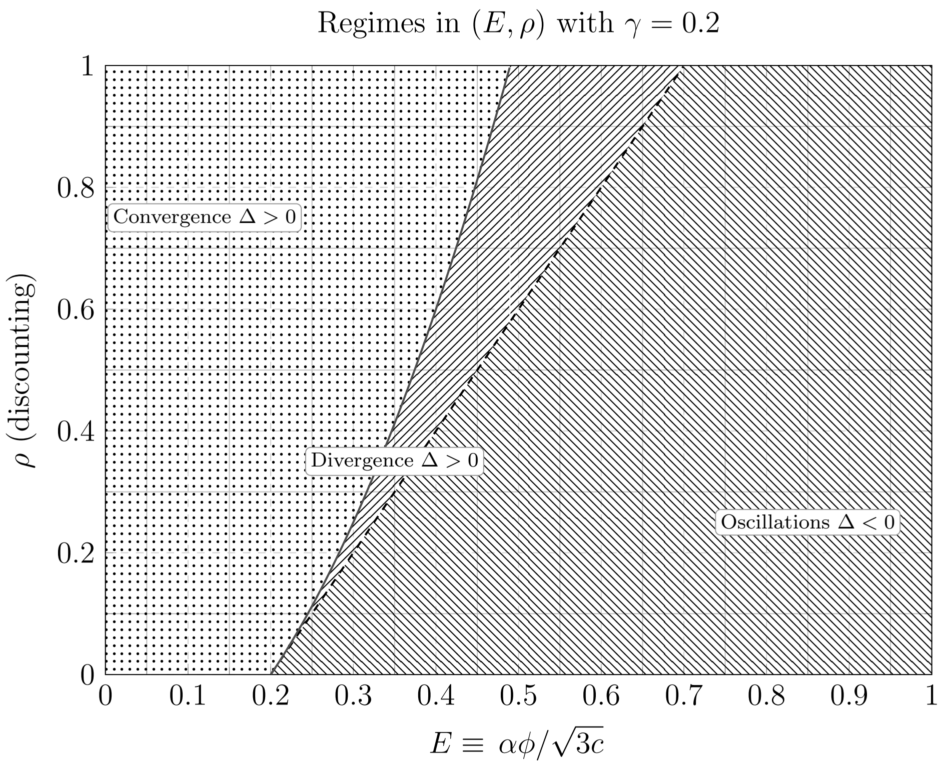

Figure 1: Regime map in the (ρ, E) plane for a given γ. Appendix C provides a zoomed view and illustrative interval placements.

In the following analysis, we focus only on the case in which condition (16) holds, i.e., the system converges to a stable steady state.

3.3 Public Investment Behavior

This section analyzes the transitional dynamics implied by the linear system (14).

Specifically, we solve for the time paths of the infrastructure gap $y(t)$ and the co-state variables $\lambda_i(t)$, $\lambda_j(t)$, under the condition that governments are sufficiently impatient (16) to ensure convergence.

The resulting trajectories characterize how jurisdictions adjust their tax and investment policies over time, and how the infrastructure gap evolves toward zero. These dynamics are critical for understanding the strategic incentives behind convergence, delay, and eventual symmetry in public capital stocks.

Under the convergence condition in Proposition 2, we can derive equilibrium tax and investment paths by solving the system (14).

The unique solution of linear system (14), whose derivation is provided in Appendix A.1, is

$$\begin{cases} y(t) = y_0 e^{\nu t}, \\ \lambda_i(t) = \dfrac{y_0 \alpha^2}{6(\rho+\gamma-\nu)} e^{\nu t} + \dfrac{\alpha}{2(\rho+\gamma)}, \\ \lambda_j(t) = \dfrac{y_0 \alpha^2}{6(\rho+\gamma-\nu)} e^{\nu t} - \dfrac{\alpha}{2(\rho+\gamma)}, \end{cases} \tag{19}$$

where $y_0 = G_i(0) - G_j(0)$, and $\nu = \frac{1}{2}\left(\rho - \sqrt{(\rho+2\gamma)^2 - \frac{4\alpha^2\phi^2}{3c}}\right) < 0$ given that condition (16) holds.

The first equation in (19) specifies that the infrastructure gap decays exponentially over time, as the system evolves toward the steady state. This reflects catch-up dynamics, where the lagging country gradually closes the gap with the leader. The other two equations describe how the shadow values evolve to guide public investment balancing the discount rate ρ, depreciation rate γ, and the marginal return to extra public capital.

The equilibrium public investment policy is thus given by

$$\begin{cases} I_i^*(t) = \dfrac{\phi}{c}\left[\dfrac{\alpha}{2(\rho+\gamma)} + \dfrac{\alpha^2 \, y_0 e^{\nu t}}{6(\rho+\gamma-\nu)}\right], \\ I_j^*(t) = \dfrac{\phi}{c}\left[\dfrac{\alpha}{2(\rho+\gamma)} - \dfrac{\alpha^2 \, y_0 e^{\nu t}}{6(\rho+\gamma-\nu)}\right]. \end{cases} \tag{20}$$

From equation (20) we see that public investment paths include two different terms. The first term is the optimal level of public investment in the absence of an infrastructure gap.[10] The second term is the effect of the infrastructure gap when it deviates from zero.[11] Notice that for any time $t \geq 0$,

$$I_i^*(t) - I_j^*(t) = \frac{y_0 \alpha^2}{3c(\rho + \gamma - \nu)} e^{\nu t} > 0,$$

provided $y_0 > 0$. In other words, the initially advanced country always undertakes higher public investment. The intuition is that, along the convergent steady-state path, the infrastructure gap closes, but the leader needs to invest more to offset the effect of technological diffusion. Hence, the laggard closes the gap while investing less in absolute terms than the leader, but more relative to its existing infrastructure stock (i.e., a higher investment rate as a share of (G)).

The equilibrium investment policy in equation (20) requires that investment in both countries is always positive. This is not generally the case. In fact, while the more advanced country will always invest ($I_i > 0$, $\forall t > 0$ since $\rho + \gamma - \nu > 0$), the lagging country might only start investing later. In fact, the shadow value of the advanced country, $\lambda_i(t)$, is always positive, providing a positive incentive to invest, while that of the lagging country, $\lambda_j(t)$, may initially be positive (discouraging investment) (see (19)). This depends on the size of the initial infrastructure gap y_0: if the gap is small ($y_0 < \frac{3(\rho+\gamma-\nu)}{\alpha(\rho+\gamma)} \equiv \bar{y}_0$) investment is always positive in both countries, but this is not always the case.

From Propositions 1 and 2 we know that steady-state investment is positive for both countries ($\overline{\lambda_i} > 0, \overline{\lambda_j} < 0$), therefore the lagging country will eventually start investing, even if the initial gap is large.

When the initial gap is large $y_0 > \bar{y}_0$, the equilibrium investment policy is given by,

$$\begin{cases} I_i^*(t) = \begin{cases} \dfrac{\phi\, \lambda_i^I(t)}{c}, & \text{if } 0 \leq t < T, \\[2ex] \dfrac{\phi}{c}\left(\dfrac{\alpha^2\, \overline{y_0}\, e^{\nu(t-T)}}{6(\rho+\gamma-\nu)} + \dfrac{\alpha}{2(\rho+\gamma)}\right), & \text{if } t \geq T; \end{cases} \\[8ex] I_j^*(t) = \begin{cases} 0, & \text{if } 0 \leq t < T, \\[2ex] \dfrac{\phi}{c}\left(\dfrac{\alpha}{2(\rho+\gamma)} - \dfrac{\alpha^2\, \overline{y_0}\, e^{\nu(t-T)}}{6(\rho+\gamma-\nu)}\right) > 0, & \text{if } t \geq T. \end{cases} \end{cases} \tag{21}$$

[10] This level of public investment is "optimal" relative to the government's objective (net fiscal revenue), not in the sense of "socially optimal".

[11] The public investment strategies in a Markov-perfect equilibrium are qualitatively the same. Tax policies are exactly the same (see Appendix B).

where the switching time $T(>0)$ is unique and positive. Detailed calculations are provided in Appendix A.2.

Equation (21) describes a *wait-then-invest* strategy: when the initial infrastructure gap y_0 is large, the lagging country delays investment until the gap has eroded enough through depreciation. The unique switching time T is endogenous and depends on the initial gap and model parameters. Even with a small initial infrastructure gap, when both countries always invest, the convergence condition requires that investment in the advanced country decreases over time, while it increases in the lagging country ($\dot{I}_i < 0, \dot{I}_j > 0$), until convergence as $t \to \infty$: $I_i^*(t) - I_j^*(t) \to 0$, when the infrastructure gap disappears $\overline{y} = 0$.

Therefore, equation (21) allows for a two-regime scenario when the initial gap in public input stock is large ($y_0 > \bar{y}$). At $t = t_0$, the lagging country has no incentive to invest and catch-up, due to a positive shadow value. However, as time goes by, infrastructure gap diminishes to a point at which investing becomes economically relevant (λ_j becomes negative). This could be seen as a two-stage game with an endogenous switching time. Once the gap erodes and the lagging country begins investing in infrastructure, the advanced country will rationally reduce infrastructure investment.

When there are no acquisition costs ($b = 0$), at least one country will be investing in infrastructure, as demonstrated in Appendix A.3.[12]

Proposition 3 *(Public Investment Behavior) The lagging country has an incentive to delay investment if the initial infrastructure gap is sufficiently large ($y_0 > \bar{y}_0$). However, it will begin investing once depreciation has shrunk the infrastructure gap below the critical level $\bar{y}_0$. This switch occurs after a finite period ($t \geq T$).*

Proof. See Appendix A.2. ■

Since infrastructure investment usually involves multi-year commitments (procurement, permitting, time to build) through which governments credibly signal a stable environment to firms, we adopt an open loop (OL) representation that provides a more natural framework than continuously re-optimized rules. If governments could flexibly retune infrastructure investment with minimal adjustment frictions, then a Markov-perfect (feedback) formulation would be more relevant. To verify that our results do not hinge on OL, Appendix B solves the Markovian benchmark for (b=0) and derives interior investment rules.[13] We find that taxes coincide with OL, while investment policies are qualitatively similar but imply milder reactions and faster convergence.

[12]The following section shows that this remains true even for positive but moderate acquisition costs.
[13]For richer transitional dynamics see Chen et al. (2025).

4 Public Investment Behavior with Acquisition Costs

In this section, we consider positive acquisition costs ($b > 0$) in infrastructure investment. Public investment carries a direct per-unit cost, that must be paid regardless of the speed of adjustment. When $b > 0$ resources must be bought or mobilized, such as hiring staff for policy institutions to disburse public investment, or acquiring any necessary structures, equipment and machinery on the market. When acquisition costs are small, we retrieve the same results as in the previous section. However, as acquisition costs become large, both countries eventually stop investing or do not invest ever.

When $b > 0$, all optimal conditions[14] derived in the previous section are still valid, but (8) and (11) are replaced by the following Kuhn-Tucker conditions:

$$I_i^b = \begin{cases} 0, & \text{if } \lambda_i < \dfrac{b}{\phi}, \\ \dfrac{\phi\lambda_i - b}{c}, & \text{if } \lambda_i \geq \dfrac{b}{\phi}, \end{cases} \qquad I_j^b = \begin{cases} 0, & \text{if } \lambda_j > -\dfrac{b}{\phi}, \\ -\dfrac{\phi\lambda_j + b}{c}, & \text{if } \lambda_j \leq -\dfrac{b}{\phi}. \end{cases} \tag{22}$$

Equation (22) implies a threshold rule on the shadow values. Investment occurs only if the shadow values exceed the effective acquisition cost, b/ϕ. We use the superscript b to distinguish the case of $b > 0$.

We check under what conditions neither country chooses to invest in infrastructure. Recall that such a situation does not occur when $b = 0$ (see Appendix A.3).

First, there is positive investment in the steady state only if the acquisition cost b is not too large, $\bar{\lambda}_\ell = \frac{\alpha}{2(\rho+\gamma)} > \frac{b}{\phi}$ $(\ell = i, j)$. This can be seen by combining Kuhn-Tucker conditions (22) with steady state solutions in Proposition 2.

To simplify the analysis of the transitional dynamics, we focus on the case when the initial infrastructure gap is not too large, i.e., $y_0 \leq \bar{y}_0$. The case of a large gap, i.e., $y_0 > \bar{y}_0$, will be briefly discussed later.

We can identify three regimes (see Figure 2), depending on the level of the acquisition cost b, the share of public investment effectively adding to infrastructure (ϕ). Higher values of ϕ reduce the impact of b. When effective acquisition costs $\left(\frac{b}{\phi}\right)$ are low, i.e., $\left(\frac{b}{\phi}\right) < \bar{\lambda}_\ell$, countries always invest in infrastructure (see case (a) in Figure 2). For large effective acquisition costs, $\left(\frac{b}{\phi} > \bar{b} \equiv \bar{\lambda}_\ell + \frac{\alpha^2 y_0}{6(\rho+2\gamma)}\right)$, countries only compete in taxes and never invest in infrastructure (see case (b) in Figure 2). Finally for moderate effective acquisition costs $\left(\bar{\lambda}_\ell < \frac{b}{\phi} < \bar{b}\right)$, there are three sub periods (see case (c) in Figure 2). Countries start with positive investment, but eventually stop investing, with the advanced

[14] These conditions relate to governments' optimal control problem. They are not necessarily socially optimal.

country investing for longer.

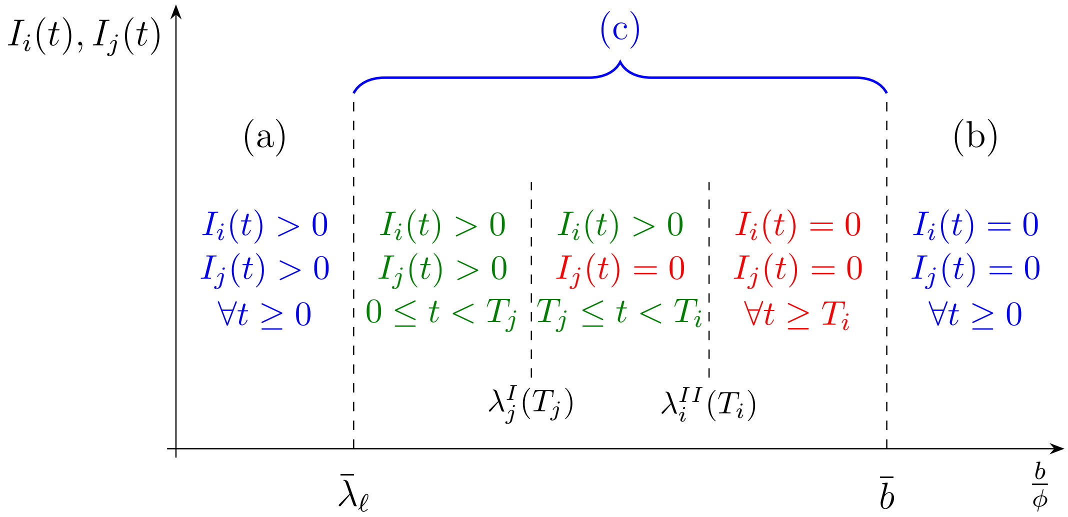

Figure 2: Public Investment with $b > 0$

For a large infrastructure gap ($y_0 > \bar{y}_0$), case (a) would have two subcases, similar to when $b = 0$, in which initially only the advanced country invests, the other country starts investing later. Case (c) may show initial investment from the advanced country only, while case (b) would remain unchanged.

Proposition 4 *In presence of acquisition costs ($b > 0$), countries may eventually stop investing in public infrastructure, or never invest at all. Specifically:*

- *When acquisition costs are sufficiently small $\left(\frac{b}{\phi} < \bar{\lambda}_\ell\right)$, both countries invest in public infrastructure as well as competing on taxes.*
- *When acquisition costs are too large $\left(\frac{b}{\phi} > \bar{b}\right)$ countries never invest in infrastructure, and rely solely on tax competition.*
- *For intermediate acquisition costs $\left(\bar{\lambda}_\ell < \frac{b}{\phi} < \bar{b}\right)$, both countries adopt an "invest-then-stop" strategy, with the laggard country j stopping first.*

Proof. See Appendix A.4. ■

Proposition 4 evokes the possibility of an "invest-then-stop" pattern, with the laggard country j stopping first (Case c in Figure 1). In fact, with positive effective acquisition costs $\frac{b}{\phi} > 0$, public investment is determined by the (discounted) marginal gain ($|\lambda_\ell|$) from investing and the shrinking infrastructure gap y. Initially both countries invest (Period I, in Figure 1), but for the lagging country j, the marginal value of catching up, $-\lambda_j$, declines. This is because the gap erodes passively at rate γ and any investment by

i offsets investment by j. Therefore, λ_j rises until it reaches the stopping threshold $-b/\phi$ and j stops investing at T_j. Once country j stops (Period II), i gradually reduces public investment: its shadow value λ_i declines toward b/ϕ as the remaining gap narrows and the anticipated no-investment phase reduces the payoff to current spending, so $I_i \to 0$ at T_i. In the terminal phase (Period III), with $I_i = I_j = 0$, the gap decays exogenously.

5 Conclusions

This paper models fiscal competition using stocks of public infrastructure as the state variable. This can explain both convergence and divergence in public investment. The central stability condition is based on the government's discount factor, infrastructure effects on private productivity, the share of public investment that effectively adds to infrastructure, public investment adjustment costs, and the rate at which advantages erode. These parameters determine whether trajectories converge globally to the symmetric steady state. Otherwise, dynamics diverge monotonically or oscillate. Taxes are higher where public infrastructure is higher, reflecting the increase in private sector productivity created by public infrastructure.

Initial asymmetries are decisive. With a small infrastructure gap, both countries invest; with a large gap, the follower rationally waits until the leader's advantage decays before engaging public investment, producing distinct transitional paths and a unique switching time. These results rationalize observed delays in infrastructure investment by lagging jurisdictions despite positive long-run returns.

Acquisition costs in public investment transform the game: moderate acquisition costs generate "invest-then-stop" dynamics (the lagging country exits first), while high acquisition costs shut down infrastructure competition altogether in favor of pure tax competition.

Our results suggest several policy recommendations: (i) reducing acquisition and adjustment costs for public investment can help sustain infrastructure competition when this is socially desirable; (ii) where initial infrastructure gaps are large, targeting support to lagging jurisdictions can make public investment attractive sooner; and (iii) policies to support balanced regional development should promote technological diffusion while limiting distortions from strategic competition that can depress infrastructure accumulation.

Finally, the analysis also speaks to coordination. In the present setup, in which infrastructure primarily reallocates a given mobile tax base, a fully cooperative benchmark is not well-defined without additional structure (e.g., an extensive margin for aggregate capital or a productivity channel through which infrastructure raises total surplus). Introducing such ingredients would allow a meaningful comparison between non-cooperative and cooperative outcomes, and could in principle generate incentives for specialization in infrastructure provision combined with transfers. Whether coordination yields convergence, specialization, or sustained asymmetries would then hinge on the curvature of returns to infrastructure, investment frictions, and the feasibility and credibility of cross-jurisdictional transfers. We leave this extension for future work.

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A Appendix

A.1 Proof of Proposition 2 (second part) and the system solution in equation (19)

The dynamic system (14) in matrix form is

$$\begin{pmatrix} \dot{y} \\ \dot{\lambda}_i \\ \dot{\lambda}_j \end{pmatrix} = \begin{pmatrix} -\gamma & \frac{\phi^2}{c} & \frac{\phi^2}{c} \\ -\frac{\alpha^2}{6} & \rho+\gamma & 0 \\ -\frac{\alpha^2}{6} & 0 & \rho+\gamma \end{pmatrix} \begin{pmatrix} y \\ \lambda_i \\ \lambda_j \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{\alpha}{2} \\ \frac{\alpha}{2} \end{pmatrix}$$

Denote the eigenvalues of the above coefficient matrix as ν_l $(l = 1, 2, 3)$ and the corresponding eigenvectors as $\overrightarrow{v_l} = (v_{l1}, v_{l2}, v_{l3})^T \neq 0$, with superscript T representing transpose of matrix.

One checks that the three eigenvalues are

$$\begin{cases} \nu_1 = \rho + \gamma > 0, \\ \nu_2 = \frac{1}{2}\left(\rho + \sqrt{(\rho+2\gamma)^2 - \frac{4\alpha^2\phi^2}{3c}}\right), \\ \nu_3 = \frac{1}{2}\left(\rho - \sqrt{(\rho+2\gamma)^2 - \frac{4\alpha^2\phi^2}{3c}}\right). \end{cases}$$

Depending on the sign of the discriminant

$$\Delta = (\rho+2\gamma)^2 - \frac{4\alpha^2\phi^2}{3c}$$

the eigenvalues ν_2 and ν_3 can be real or complex. When

$$\rho > \frac{2\alpha\phi}{\sqrt{3c}} - 2\gamma,$$

both eigenvalues ν_2 and ν_3 are real. Moreover, when (16) is satisfied, i.e.,

$$\rho > \frac{\alpha^2\phi^2}{3c\gamma} - \gamma,$$

ν_2 and ν_3 have opposite sign with $\nu_3 < 0$, indicating that equation (16) is a necessary condition for convergence. This is because when equation (16) is not satisfied, ν_2 and ν_3 are either *(i)* both positive (if $\frac{2\alpha\phi}{\sqrt{3c}} - 2\gamma < \rho < \frac{\alpha^2\phi^2}{\sqrt{3c\gamma}}$ and $\Delta > 0$), or *(ii)* both complex (if $\rho < \frac{2\alpha\phi}{\sqrt{3c}} - 2\gamma$ and $\Delta < 0$) with positive real part, thus there is no convergence. In the former case, the system never converges, and the gap dynamics exhibits oscillations. In the latter, the system diverges with an infrastructure gap increasing over time.

The following part shows that equation (16) is also a sufficient condition for global convergence under the transversality conditions. To this end, consider the explicit solution of system (14),

$$\begin{cases} y(t) = c_1 v_{11} e^{\nu_1 t} + c_2 v_{21} e^{\nu_2 t} + c_3 v_{31} e^{\nu_3 t}, \\ \lambda_i(t) = c_1 v_{12} e^{\nu_1 t} + c_2 v_{22} e^{\nu_2 t} + c_3 v_{32} e^{\nu_3 t} + \overline{\lambda}_i, \\ \lambda_j(t) = c_1 v_{13} e^{\nu_1 t} + c_2 v_{23} e^{\nu_2 t} + c_3 v_{33} e^{\nu_3 t} + \overline{\lambda}_j, \end{cases} \tag{23}$$

where eigenvector $\overrightarrow{v_l} = (v_{l1} \quad v_{l2} \quad v_{l3})^T \neq 0$, $l = 1, 2, 3$, are determined by the coefficient matrix above, and c_1, c_2 and c_3 are constants to be determined.

Following the definition of eigenvectors, and taking into account the stability of steady state, the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t}\lambda_l(t) y(t) = 0 \qquad l = i, j$$

hold if and only if $c_1 = c_2 = 0$.

Then, the solution simplifies to

$$\begin{cases} y(t) = c_3 e^{\nu_3 t}, \\ \lambda_i(t) = c_3 v_{32} e^{\nu_3 t} + \overline{\lambda}_i, \\ \lambda_j(t) = c_3 v_{33} e^{\nu_3 t} + \overline{\lambda}_j, \end{cases} \tag{24}$$

with

$$v_{32} = v_{33} = \frac{\alpha^2}{6(\rho + \gamma - \nu_3)}.$$

From the initial condition $y(0) = y_0$, it follows that $c_3 = y_0$. Substituting into (24), we obtain the solution reported in equation (19). For simplicity in the main text, we write $\nu = \nu_3$.

That completes the proof.

A.2 Proof of Proposition 3

Both players always invest in infrastructure if and only if $\lambda_i(t) > 0$ and $\lambda_j(t) < 0$ when $t \to 0^+$. Given equation (19), $\lambda_i(t) > 0$ $\forall t \geq 0$, while $\lambda_j(t) < 0$ requires $y_0 < \overline{y_0} \equiv \frac{3(\rho+\gamma-\nu)}{\alpha(\rho+\gamma)}$.

This means that if $y_0 > \overline{y}_0$, for some time $t \in [0, T]$ with $T \geq 0$, the lagging country will not invest ($I_j^* = 0$) and the dynamic system is reduced to

$$\begin{cases} \dot{y} = \phi I_i - \gamma y = \frac{\phi^2}{c}\lambda_i - \gamma y, \\ \dot{\lambda}_i = (\rho+\gamma)\lambda_i - \frac{\alpha^2}{6}y - \frac{\alpha}{2}, \\ \dot{\lambda}_j = (\rho+\gamma)\lambda_j - \frac{\alpha^2}{6}y + \frac{\alpha}{2}, \end{cases} \tag{25}$$

where the first two equations are independent of λ_j. Thus, we first focus on the first two equations and then solve the last equation as a linear differential equation alone.

The steady state of linear system (25) is

$$\widehat{y} \equiv \frac{\alpha\phi^2}{2c\left[\gamma(\rho+\gamma) - \frac{\alpha^2\phi^2}{6c}\right]}, \qquad \widehat{\lambda_i} = \frac{\gamma c}{\phi^2}\widehat{y}, \qquad \text{and} \qquad \widehat{\lambda_j} = \frac{\alpha^2\,\widehat{y} - 3\alpha}{6(\rho+\gamma)}.$$

Therefore, country j invests in the infrastructure if and only if $\widehat{y} < \frac{3}{\alpha}$, which is equivalent to condition (16). This condition also implies convergence of system (25).

In fact, considering the first two equation of the linear system (25) yields the two eigenvalues,

$$\begin{cases} \mu_1 = \frac{1}{2}\left(\rho - \sqrt{(\rho+2\gamma)^2 - \frac{2\alpha^2\phi^2}{3c}}\right), \\ \mu_2 = \frac{1}{2}\left(\rho + \sqrt{(\rho+2\gamma)^2 - \frac{2\alpha^2\phi^2}{3c}}\right). \end{cases} \tag{26}$$

One checks that under condition (16), $\mu_1 < 0$, $\mu_2 > 0$.[15]

Under this condition the system converges to a steady state with positive investment for both countries,

$$\widehat{\lambda_i} > 0, \qquad \text{and} \qquad \widehat{\lambda_j} < 0.$$

[15] One can check that system (25) converges if $\rho > \frac{\alpha^2\phi^2}{3c\gamma} - \gamma > \frac{\alpha^2\phi^2}{6c\gamma} - \gamma$.

Therefore, since system (25) implies no investment for country j, but shows positive investment in the long run, it must be that country j starts investing at some point.

Identifying the switching time T

To identify when the lagging country j starts investing, there are two periods to be considered. Period I, in which only country i is investing, is described by the system (25), and Period II, in which both countries are investing is the one described in the main text (see system (14)). The transition from no investment in country j to positive investment is identified as follows: First, we solve the system in Period I, and then we impose continuity conditions on the trajectories of the state and costates in both systems at the switching time T. Superscripts I and II indicate solutions of period I and period II respectively.

The solution of the two coupled equations of system (25) is,

$$\begin{cases} y^I(t) = c_1 v_{11} e^{\mu_1 t} + c_2 v_{21} e^{\mu_2 t} + \widehat{y}, \\ \lambda_i^I(t) = c_1 v_{12} e^{\mu_1 t} + c_2 v_{22} e^{\mu_2 t} + \widehat{\lambda}_i, \end{cases} \tag{27}$$

where $\overrightarrow{v_l} = (v_{l1}, v_{l2})^T$ is the corresponding eigenvector of eigenvalue μ_l, and c_l are undetermined constants, with $l = 1, 2$.

From the definition of an eigenvector, and taking $v_{l1} = 1$, it follows,

$$\overrightarrow{v_l} = \begin{pmatrix} v_{l1} \\ v_{l2} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{\alpha^2}{6(\rho+\gamma-\mu_l)} \end{pmatrix}, \qquad l = 1, 2.$$

Substituting the eigenvectors above into (27), it follows that the general solution is

$$\begin{cases} y^I(t) = c_1 e^{\mu_1 t} + c_2 e^{\mu_2 t} + \widehat{y}, \\ \lambda_i^I(t) = \dfrac{c_1 e^{\mu_1 t}\ \alpha^2}{6(\rho+\gamma-\mu_1)} + \dfrac{c_2 e^{\mu_2 t}\ \alpha^2}{6(\rho+\gamma-\mu_2)} + \widehat{\lambda}_i. \end{cases} \tag{28}$$

Substituting $y^I(t)$ into the last equation in (25), it follows

$$\begin{aligned} \lambda_j^I(t) &= \left[C_0 + \tfrac{\alpha^2}{6(\rho+\gamma)}\left(\tfrac{3}{\alpha} - \widehat{y}\right) + \tfrac{c_1\alpha^2}{6(\mu_1-\rho-\gamma)} + \tfrac{c_2\alpha^2}{6(\mu_2-\rho-\gamma)}\right] e^{(\rho+\gamma)t} \\ &\quad - \frac{c_1 e^{\mu_1 t}\ \alpha^2}{6(\mu_1-\rho-\gamma)} - \frac{c_2 e^{\mu_2 t}\ \alpha^2}{6(\mu_2-\rho-\gamma)}, \end{aligned} \tag{29}$$

where C_0 is the undetermined constant of integration.

Now, the necessary continuity conditions at the switching point T are,

$$\lim_{t\to T^-} \lambda_i^I(t) = \lim_{t\to T^+} \lambda_i^{II}(t) = \lim_{t\to T^+} \frac{\alpha^2}{6(\rho+\gamma-\nu)} y_T e^{\nu(t-T)} + \overline{\lambda}_i,$$

$$\lim_{t\to T^-} \lambda_j^I(t) = \lim_{t\to T^+} \lambda_j^{II}(t) = \lim_{t\to T^+} \frac{\alpha^2}{6(\rho+\gamma-\nu)} y_T e^{\nu(t-T)} + \overline{\lambda}_j, \tag{30}$$

and

$$\lim_{t\to T^-} y^I(t) = \lim_{t\to T^+} y^{II}(t),$$

where $\lambda_i^{II}(t)$, $\lambda_j^{II}(t)$ and $y^{II}(t)$ are given by (19) with the dynamic system of period II starting at time T with gap y_T.

More specifically, when period II starts at $t = T$, $y^{II}(t) = y_T e^{\nu(t-T)}$, $\forall t \geq T$, which helps reduce the continuity condition for the state y to

$$y_T = y^I(T) = c_1 e^{\mu_1 T} + c_2 e^{\mu_2 T} + \widehat{y}, \tag{31}$$

Therefore, the continuity equation for λ_i and (28) imply that for country i, at the switching time T, it must be that,

$$\frac{\alpha^2}{6}\left[\frac{c_1 e^{\mu_1 T}}{\rho+\gamma-\mu_1} + \frac{c_2 e^{\mu_2 T}}{\rho+\gamma-\mu_2}\right] + \widehat{\lambda}_i = \lambda_i^{II}(T) = \frac{\alpha^2 y_T}{6(\rho+\gamma-\nu)} + \overline{\lambda}_i. \tag{32}$$

Moreover, for country j, the condition for continuity of λ_j at T can be derived as follows. From the Kuhn-Tucker condition (Proposition 1), if at $t = T$ country j starts investing, it must be that

$$\lambda_j^{II}(T) = 0 \qquad \rightarrow \qquad \frac{\alpha^2 y_T}{6(\rho+\gamma-\nu)} = -\overline{\lambda}_j = \frac{\alpha}{2(\rho+\gamma)}.$$

Therefore,

$$y_T = \frac{3(\rho+\gamma-\nu)}{\alpha(\rho+\gamma)} \equiv \overline{y_0}. \tag{33}$$

Furthermore, considering that from (30), $\lambda_j^I(T) = \lambda_j^{II}(T) = 0$, then from (29) we get

$$\begin{aligned} &\frac{\alpha^2}{6}\left[\frac{c_1 e^{\mu_1 T}}{\mu_1-\rho-\gamma} + \frac{c_2 e^{\mu_2 T}}{\mu_2-\rho-\gamma}\right] \\ &\quad = \left[C_0 + \frac{\alpha^2}{6}\left[\frac{1}{\rho+\gamma}\left(\frac{3}{\alpha} - \widehat{y}\right) + \frac{c_1}{\mu_1-\rho-\gamma} + \frac{c_2}{\mu_2-\rho-\gamma}\right]\right] e^{(\rho+\gamma)T} \end{aligned} \tag{34}$$

Next, one combines the initial condition,

$$y_0 = c_1 + c_2 + \widehat{y} \ , \tag{35}$$

with the conditions (31), (32) and (34) obtained from the continuity equations to infer the constants C_0, c_1, c_2 and identify T.

From (31), one gets,

$$c_2 e^{\mu_2 T} = \overline{y_0} - \widehat{y} - c_1 e^{\mu_1 T}.$$

Substituting into (32) and rearranging terms, it follows

$$\begin{aligned} c_1 e^{\mu_1 T} &= \frac{(\rho+\gamma-\mu_1)(\rho+\gamma-\mu_2)}{\mu_1-\mu_2}\left[\frac{\overline{y_0}(\nu-\mu_2)}{(\rho+\gamma-\nu)(\rho+\gamma-\mu_2)}\right. \\ &\quad \left. + \frac{\widehat{y}}{\rho+\gamma-\mu_2} + \frac{6(\overline{\lambda}_i-\widehat{\lambda}_i)}{\alpha^2}\right] \equiv \widehat{F}. \end{aligned} \tag{36}$$

Similarly,

$$\begin{aligned} c_2 e^{\mu_2 T} &= \frac{(\rho+\gamma-\mu_1)(\rho+\gamma-\mu_2)}{\mu_2-\mu_1}\left[\frac{\overline{y_0}(\nu-\mu_1)}{(\rho+\gamma-\nu)(\rho+\gamma-\mu_1)}\right. \\ &\quad \left. + \frac{\widehat{y}}{\rho+\gamma-\mu_1} + \frac{6(\overline{\lambda}_i-\widehat{\lambda}_i)}{\alpha^2}\right] \equiv \widehat{G}. \end{aligned} \tag{37}$$

Thus,

$$c_1 = \widehat{F} e^{-\mu_1 T} \qquad \text{and} \qquad c_2 = \widehat{G} e^{-\mu_2 T}. \tag{38}$$

Combining with (35), it follows

$$c_1 + c_2 = \widehat{F} e^{-\mu_1 T} + \widehat{G} e^{-\mu_2 T} = y_0 - \widehat{y}. \tag{39}$$

It can be shown that there exists a unique solution $T > 0$ to (39). That completes the proof.

A.3 Proof that at least one country always invests

Optimal investment (20) is obtained by assuming that both players invest in infrastructure and Proposition 3 presents a situation in which initially only country i invests. It turns out that the case in which no country invests, i.e., $I_i = 0, I_j = 0$, cannot occur in absence of acquisition costs, $b = 0$.

To show that, assume that at least for some time, $I_i = 0, I_j = 0$, so the dynamic system becomes:

$$\begin{cases} \dot{y} = -\gamma y, \\ \dot{\lambda}_i = (\rho+\gamma)\lambda_i - \frac{\alpha^2}{6} y - \frac{\alpha}{2}, \\ \dot{\lambda}_j = (\rho+\gamma)\lambda_j - \frac{\alpha^2}{6} y + \frac{\alpha}{2}, \end{cases} \tag{40}$$

From the the first equation, it is easy to see the state solution is

$$y(t) = y_0 e^{-\gamma t}$$

and the the steady state of linear system (40) is the same as the one in Proposition 2. In other words, in the long-run, $\overline{\lambda_i} > 0$ and $\overline{\lambda_j} < 0$. Thus, $I_i = I_j = 0$ can not last forever.

To determine when players start to invest, we rely in the explicit solution:

$$\lambda_i(t) = \left[a_i(0) - \frac{\alpha}{2(\rho+\gamma)} - \frac{\alpha^2 y_0}{6(\rho+2\gamma)}\right] e^{(\rho+\gamma)t} + \frac{\alpha}{2(\rho+\gamma)} + \frac{\alpha^2 y_0}{6(\rho+2\gamma)}\, e^{-\gamma t}$$

and

$$\lambda_j(t) = \left[a_j(0) + \frac{\alpha}{2(\rho+\gamma)} - \frac{\alpha^2 y_0}{6(\rho+2\gamma)}\right] e^{(\rho+\gamma)t} - \frac{\alpha}{2(\rho+\gamma)} + \frac{\alpha^2 y_0}{6(\rho+2\gamma)}\, e^{-\gamma t},$$

where $a_i(0)$ and $a_j(0)$ are integral constants. Converging to the steady state can happen, if and only if the first terms in the above two equations vanish. Thus, the explicit solution can be reduced to

$$\begin{cases} \lambda_i(t) = \dfrac{\alpha}{2(\rho+\gamma)} + \dfrac{\alpha^2\, y_0\, e^{-\gamma t}}{6(\rho+2\gamma)}, \\ \lambda_j(t) = -\dfrac{\alpha}{2(\rho+\gamma)} + \dfrac{\alpha^2\, y_0\, e^{-\gamma t}}{6(\rho+2\gamma)} \end{cases} \tag{41}$$

Obviously, $\lambda_i(t) > 0$ for any $t \geq 0$. In other words, the initially advanced country i always invests in infrastructure. As to the lagging country j, investing is conditional on the size of the initial gap y_0, thus the problem returns to the one studied in the last subsection.

Corollary 1 *For the game described above, $I_i^*(t) > 0$, $\forall t > 0$.*

A.4 Proof of Proposition 4

Preliminaries

With acquisition costs $b > 0$, the Kuhn-Tucker conditions link investment to shadow values (see (22)),

$$I_i = \begin{cases} 0, & \lambda_i < \frac{b}{\phi}, \\ \dfrac{\phi\lambda_i - b}{c}, & \lambda_i \geq \frac{b}{\phi}, \end{cases} \qquad I_j = \begin{cases} 0, & \lambda_j > -\frac{b}{\phi}, \\ \dfrac{-\phi\lambda_j + b}{c}, & \lambda_j \leq -\frac{b}{\phi}. \end{cases}$$

In other word, countries invest iff the discounted marginal product of public capital, measured by the co-state $|\lambda|$, exceeds the effective acquisition cost b/ϕ. Otherwise, $I = 0$.

Recall the steady state of the linear system when both countries invest

$$\bar{y} = 0, \qquad \bar{\lambda}_i = \frac{\alpha}{2(\rho+\gamma)}, \quad \bar{\lambda}_j = -\frac{\alpha}{2(\rho+\gamma)}.$$

(a) Low effective cost: both countries always invest

According to (19), solution of system (14), when both countries invest, we see that $\lambda_i(t)$ and $\lambda_j(t)$ decrease continuously over time. Therefore, if $\lim_{t\to\infty} \lambda_l(t) = \overline{\lambda_l} = \frac{\alpha}{2(\rho+\gamma)} > \frac{b}{\phi}$, then $\lambda_l(t) > \frac{b}{\phi}$ for all $t \geq 0$. It follows that according to the Kuhn-Tucker conditions (22), both countries always invest in infrastructure.

(b) High effective cost: neither country ever invests

Consider the no-investment dynamics (both at the corner), the dynamic system coincides with system (40), and its explicit solution is

$$\begin{cases} \lambda_i^{b,0}(t) = \dfrac{\alpha}{2(\rho+\gamma)} + \dfrac{\alpha^2\, y_0\, e^{-\gamma t}}{6(\rho+2\gamma)}, \\ \lambda_j^{b,0}(t) = -\dfrac{\alpha}{2(\rho+\gamma)} + \dfrac{\alpha^2\, y_0\, e^{-\gamma t}}{6(\rho+2\gamma)}. \end{cases} \tag{42}$$

If at $t = 0$,

$$\lambda_i^{b,0}(0) < \frac{b}{\phi} \qquad \text{and} \qquad \lambda_j^{b,0}(0) > -\frac{b}{\phi}, \tag{43}$$

then,

$$\lambda_i^{b,0}(t) < \frac{b}{\phi} \qquad \text{and} \qquad \lambda_j^{b,0}(t) > -\frac{b}{\phi} \qquad \text{for all} \qquad t \geq 0$$

because $\lambda_i^{b,0}(t)$ is decreasing with respect to time t, and $\lambda_j^{b,0}(t) \to \bar{\lambda}_j > -\frac{b}{\phi}$ when $t \to \infty$. According to the Kuhn-Tucker conditions (22), none of the two countries invests in infrastructure. The first inequality in (43) yields $\frac{b}{\phi} > \bar{b} \equiv \frac{\alpha}{2(\rho+\gamma)} + \frac{\alpha^2 y_0}{6(\rho+2\gamma)}$ and once this holds, the second inequality in (43) holds automatically.

(c) Intermediate effective cost: two stopping times, $0 < T_j < T_i < \infty$

Assume
$$\frac{\alpha}{2(\rho+\gamma)} \;<\; \frac{b}{\phi} \;<\; \bar{b},$$
and $y_0 \le \bar{y}_0$. Then the unique Nash equilibrium exhibits three periods (Fig. 2): Period I, both countries invest; Period II, only country i invests; Period III, no country invests. We show the existence and uniqueness of the two switching times T_j and T_i, and that the following piecewise investment rule holds,

$$I_i^b(t) = \begin{cases} \dfrac{\phi\lambda_i^{b,I}(t) - b}{c}, & 0 \le t < T_j, \\ \dfrac{\phi\lambda_i^{b,II}(t) - b}{c}, & T_j \le t < T_i, \\ 0, & t \ge T_i; \end{cases} \qquad I_j^b(t) = \begin{cases} -\dfrac{\phi\lambda_j^{b,I}(t) + b}{c}, & 0 \le t < T_j, \\ 0, & t \ge T_j. \end{cases} \tag{44}$$

with $\lambda_i^{b,I}, \lambda_j^{b,I}$ the interior (both-invest) costates on $[0, T_j]$, and $\lambda_i^{b,II}$ (as well as $\lambda_j^{b,II}$) the mixed (only i invests) costates on $[T_j, T_i]$.

We build the proof in three steps, each analyzing one period separately starting from the last. The strategy is to construct three period solutions and link them with continuity at the switching points T_j, T_i:

- Period III (none invests): the dynamics is described by system (40), and its solution denoted by $(y^{b,III}, \lambda_i^{b,III}, \lambda_j^{b,III})$ on $[T_i, \infty)$.
- Period II (only i invests): the dynamics is described by system (25) with $I_j = 0$, and its solution denoted by $(y^{b,II}, \lambda_i^{b,II}, \lambda_j^{b,II})$ on $[T_j, T_i]$.
- Period I (both invest): the dynamics is described by system (14), and its solution denoted by $(y^{b,I}, \lambda_i^{b,I}, \lambda_j^{b,I})$ on $[0, T_j]$.

In the following, we will see that the switching conditions are

$$\lambda_j^{b,I}(T_j) = \lambda_j^{b,II}(T_j) = -\tfrac{b}{\phi}, \qquad \lambda_i^{b,II}(T_i) = \lambda_i^{b,III}(T_i) = \tfrac{b}{\phi}.$$

Therefore, country j hits its lower threshold first, while i reaches its upper threshold last.

Step 1 — Last period (Period III): no country invests, and switching order. Assume there is a last time $T_b = \max\{T_i, T_j\}$ after which $I_i = I_j = 0$. Then the no-investment system, which coincides with (40), admits the closed-form solution (for $t \ge T_b$)
$$y^{b,III}(t) = y_b e^{-\gamma(t-T_b)},$$

$$\lambda_i^{b,III}(t) = \frac{\alpha}{2(\rho+\gamma)} + \frac{\alpha^2 y_b}{6(\rho+2\gamma)} e^{-\gamma(t-T_b)}, \qquad \lambda_j^{b,III}(t) = -\frac{\alpha}{2(\rho+\gamma)} + \frac{\alpha^2 y_b}{6(\rho+2\gamma)} e^{-\gamma(t-T_b)}.$$

Both $\lambda_i^{b,III}, \lambda_j^{b,III}$ are strictly decreasing in t. Hence the last switch must be characterized by equality at the threshold $|\frac{b}{\phi}|$. If the last country to switch were i, then $\lambda_i^{b,III}(T_i) = b/\phi$ pinpoints

$$y_b = y_i^b := \frac{6(\rho+2\gamma)}{\alpha^2}\left(\frac{b}{\phi} - \frac{\alpha}{2(\rho+\gamma)}\right) > 0.$$

If instead j were last, we would obtain $y_b < 0$, which rules out j as the last switch.

Therefore, $T_b = T_i$ and $y(T_i) = y_i^b$. We can conclude that the advanced country invests longer.

Step 2 — Middle period (Period II): only i invests, unique backward bridge to $(y_i^b, \lambda_i^{b,II}, \lambda_j^{b,II})$. On $[T_j, T_i]$, the dynamics coincides with (25) but with terminal conditions at T_i coming from Step 1:

$$y^{b,II}(T_i) = y_i^b, \quad \lambda_i^{b,II}(T_i) = \frac{b}{\phi}, \quad \lambda_j^{b,II}(T_i) = \frac{b}{\phi} - \frac{\alpha}{\rho+\gamma}.$$

This linear three-dimensional system has a unique solution when shot backward from T_i. In closed form it is a two-mode (one stable, one unstable) bridge, and the terminal conditions pin down the integration constants; see below. Along this path, $\lambda_i^{b,II}(t) \searrow b/\phi$ while $\lambda_j(t) \nearrow \big(b/\phi - \alpha/(\rho+\gamma)\big)$ as $t \to T_i$. Crucially, since $\lambda_j^{b,II}(T_j) = -b/\phi$ (next step), $\lambda_j^{b,II}(t)$ is strictly increasing on $[T_j, T_i]$, so j indeed remains at the corner $I_j = 0$ throughout Period II.

The definition of $\lambda^{b,II}$ is: the unique solution to this linear three-equation system with the three terminal values above. In closed form (same mode structure as (25)):

$$\begin{aligned}
y^{b,II}(t) &= c_1^b e^{\mu_1 t} + c_2^b e^{\mu_2 t} + \hat{y}, \\
\lambda_i^{b,II}(t) &= c_1^b \frac{\alpha^2}{6(\rho+\gamma-\mu_1)} e^{\mu_1 t} + c_2^b \frac{\alpha^2}{6(\rho+\gamma-\mu_2)} e^{\mu_2 t} + \hat{\lambda}_i, \\
\lambda_j^{b,II}(t) &= \left[k_b + \frac{\alpha^2}{6(\rho+\gamma)}\left(\frac{3}{\alpha} - \hat{y}\right) + \frac{\alpha^2 c_1^b}{6(\mu_1-\rho-\gamma)} + \frac{\alpha^2 c_2^b}{6(\mu_2-\rho-\gamma)}\right] e^{(\rho+\gamma)t} \\
&\quad - \frac{\alpha^2 c_1^b}{6(\mu_1-\rho-\gamma)} e^{\mu_1 t} - \frac{\alpha^2 c_2^b}{6(\mu_2-\rho-\gamma)} e^{\mu_2 t},
\end{aligned}$$

where $\mu_1 < 0 < \mu_2$ are the eigenvalues of the (y, λ_i) sub-block, $\hat{y}, \hat{\lambda}_i$ are the steady-state values for (25), and c_1^b, c_2^b, k_b are chosen to satisfy the three terminal equations above. Solving those yields the explicit expressions

$$C_1^b := c_1^b e^{\mu_1 T_i} = \frac{6(\rho+\gamma-\mu_1)(\rho+\gamma-\mu_2)}{\alpha^2(\mu_1-\mu_2)}\left(\frac{b}{\phi}-\hat{\lambda}_i\right) - \frac{\rho+\gamma-\mu_1}{\mu_1-\mu_2}\,(y_i^b-\hat{y}),$$
$$C_2^b := c_2^b e^{\mu_2 T_i} = \frac{6(\rho+\gamma-\mu_1)(\rho+\gamma-\mu_2)}{\alpha^2(\mu_2-\mu_1)}\left(\frac{b}{\phi}-\hat{\lambda}_i\right) - \frac{\rho+\gamma-\mu_2}{\mu_2-\mu_1}\,(y_i^b-\hat{y}),$$

and then $k_b = k_b(T_i)$ from $\lambda_j^{b,II}(T_i) = \frac{b}{\phi} - \frac{\alpha}{\rho+\gamma}$. This completely defines $(y^{b,II}, \lambda_i^{b,II}, \lambda_j^{b,II})$.

Finally, what is relevant for Period I regarding Period II is the pair $\big(y^{b,II}(T_j), \lambda_\ell^{b,II}(T_j)\big)$: from the first line above

$$y^{b,II}(T_j) = c_1^b e^{\mu_1 T_j} + c_2^b e^{\mu_2 T_j} + \hat{y},$$

which will be used to match $y^{b,I}(T_j)$. Moreover $\lambda_j^{b,II}(t)$ is increasing on $[T_j, T_i]$ and ends strictly above $-b/\phi$ at T_i, validating the corner $I_j = 0$ throughout Period II.

Step 3 — First period (Period I): both invest until j hits $-b/\phi$ once. On $[0, T_j]$ both invest, so the ODEs are exactly system (14) with initial state $y(0) = y_0$ and terminal costate values supplied by Step 2 at $t = T_j$:

$$\lambda_i^{b,I}(T_j) = \lambda_i^{b,II}(T_j), \qquad \lambda_j^{b,I}(T_j) = \lambda_j^{b,II}(T_j) = -\tfrac{b}{\phi}.$$

The definition of $\lambda^{b,I}$ is: the interior solution to (14) on $[0, T_j]$ satisfying those three boundary conditions. In closed form, the general solution is a linear combination of three eigenmodes,

$$y^{b,I}(t) = k_1 e^{\nu_1 t} + k_2 e^{\nu_2 t} + k_3 e^{\nu_3 t},$$
$$\lambda_i^{b,I}(t) = \sum_{m=1}^{3} \frac{\alpha^2}{6(\rho+\gamma-\nu_m)} k_m e^{\nu_m t} + \frac{\alpha}{2(\rho+\gamma)},$$
$$\lambda_j^{b,I}(t) = \sum_{m=1}^{3} \frac{\alpha^2}{6(\rho+\gamma-\nu_m)} k_m e^{\nu_m t} - \frac{\alpha}{2(\rho+\gamma)},$$

with $\nu_1 = \rho+\gamma > 0$ and $\nu_2 > 0 > \nu_3$ the eigenvalues of (14). Unlike the $b = 0$ global solution (which kills the growing modes by transversality), here we stop at a finite T_j, so the growing modes are kept to hit the boundary. The four unknowns (k_1, k_2, k_3, T_j) are determined by the conditions

$$y^{b,I}(0) = y_0, \quad \lambda_i^{b,I}(T_j) = \lambda_i^{b,II}(T_j), \quad \lambda_j^{b,I}(T_j) = -\tfrac{b}{\phi},$$

together with the state continuity at T_j: $y^{b,I}(T_j) = y^{b,II}(T_j)$. Because $\lambda_i^{b,I}$ and $\lambda_j^{b,I}$ are linearly dependent in (k_1, k_2, k_3), you can solve k_2, k_3 as functions of k_1, T_j, and then determine k_1 from $\lambda_i^{b,I}(T_j) = \lambda_i^{b,II}(T_j)$, yielding $k_m = k_m(T_j)$. Finally, the continuity of y

$$y^{b,II}(T_j) = c_1^b e^{\mu_1 T_j} + c_2^b e^{\mu_2 T_j} + \hat{y} \;=\; k_1(T_j)e^{\nu_1 T_j} + k_2(T_j)e^{\nu_2 T_j} + k_3(T_j)e^{\nu_3 T_j}$$

determines T_i (and thus T_j) uniquely once you substitute the Period II coefficients $c_\ell^b(T_i)$. Monotonicity of λ_i, λ_j in each period guarantees a single crossing.

This completes the construction of the proof and fixes all unknowns.

B Markov-perfect (feedback) approach via HJB

Consider the model with $b = 0$. In a Markov Perfect Equilibrium (MPE), strategies are functions of the current state y: $\tau_\ell = \tau_\ell(y)$, $I_\ell = I_\ell(y)$ with $\ell = i, j$.

Let $V_i(y)$ be country i's value. The HJB for i (given j's Markov strategy) is[16]

$$\rho V_i(y) = \max_{\tau_i, I_i \geq 0} \Big\{ \tau_i k_i(y, \tau_i - \tau_j) \;-\; \frac{c}{2} I_i^2 \;+\; V_i'(y)\left[\phi(I_i - I_j) - \gamma y\right] \Big\}.$$

By symmetry $V_j(y) = V_i(-y)$ (and hence $V_j'(y) = -V_i'(-y)$).

Because taxes τ_i, τ_j affect only the flow payoff and not the state dynamics, the tax subgame at each y is static. The HJB FOCs wrt τ_i and τ_j yield the same instantaneous Nash pair as in the main text (see eq. (13)):

$$\tau_i(y) = 1 + \frac{\alpha}{3} y, \qquad \tau_j(y) = 1 - \frac{\alpha}{3} y,$$

so that $k_i(y) = \frac{1}{2} + \frac{\alpha}{6} y$ and the implied revenue is

$$\tau_i(y) k_i(y) = \frac{1}{2} + \frac{\alpha}{3} y + \frac{\alpha^2}{18} y^2.$$

It follows that the HJB for country i can be written as,

$$\rho V_i(y) = \left[\tfrac{1}{2} + \tfrac{\alpha}{3} y + \tfrac{\alpha^2}{18} y^2\right] \;+\; \max_{I_i \geq 0} \Big\{ -\frac{c}{2} I_i^2 + V_i'(y)\left(\phi(I_i - I_j(y)) - \gamma y\right) \Big\}, \tag{45}$$

and symmetric for country j.

For interior investment ($I_i, I_j > 0$) the HJB FOC yields

$$I_i(y) = \frac{\phi}{c} V_i'(y), \qquad I_j(y) = -\frac{\phi}{c} V_j'(y). \tag{46}$$

[16] We intentionally use a stationary HJB. Given that nothing in the primitives of our model depends explicitly on t, the Bellman value for player ℓ can be written as a function of the current state only, $V_\ell(y)$, and the HJB is time-independent.

These are the feedback counterparts of the PMP rules found in the main text: $I_i = \frac{\phi}{c}\lambda_i$, $I_j = -\frac{\phi}{c}\lambda_j$, once we note the identity $\lambda_i = V_i'(y)$, $\lambda_j = V_j'(y)$.

Quadratic ansatz for the value function

Looking for symmetric quadratic value functions, we make the following guess:

$$V_i(y) = Ay^2 + By + D, \quad \text{and} \quad V_j(y) = Ay^2 - By + D.$$

Therefore,

$$V_i'(y) = 2Ay + B, \qquad V_j'(y) = 2Ay - B.$$

Then, plugging the interior rules above into the HJB equation (45) and matching the coefficients (y^2, y, constant) yields the system[17]

$$\begin{aligned} &\text{(i)} \quad (6\phi^2/c)\, A^2 - (2\gamma + \rho)A + \alpha^2/18 = 0, \\ &\text{(ii)} \quad \big(\rho + \gamma - \tfrac{2\phi^2}{c}A\big)B = \alpha/3, \\ &\text{(iii)} \quad \rho D = \tfrac{1}{2} - \tfrac{\phi^2}{2c}B^2. \end{aligned}$$

We solve (i) picking the stabilizing root (the "–" sign):[18]

$$A = \frac{c}{12\phi^2}\left((\rho + 2\gamma) - \sqrt{(\rho + 2\gamma)^2 - \tfrac{4}{3}\tfrac{\alpha^2\phi^2}{c}}\,\right) > 0 \ ,$$

and then from (ii) we get B,

$$B = \frac{2\alpha}{5\rho + 4\gamma + \sqrt{(\rho + 2\gamma)^2 - \tfrac{4}{3}\tfrac{\alpha^2\phi^2}{c}}} > 0 \ ,$$

while D follows from (iii), but it is not needed for policies.

Here, we adopt an affine-quadratic value function to characterize the dynamic optimization problem faced by each jurisdiction. While the resulting value function is convex in the state variable y, this does not contradict optimality in our setting. The standard requirement of concavity for value functions in maximization problems typically ensures global optimality and uniqueness. However, this condition presumes joint concavity of the flow payoff in both state and control variables, which does not hold in our setting due to the bilinear term ($y\tau$) in the objective. Consequently, the usual sufficiency theorems based on concavity do not apply. Instead, optimality is verified through: (1) Satisfaction

[17]Doing the same with country j's HJB would lead to the same three equations.

[18]This is required for V_i to satisfy the shadow-price TVC: $e^{-\rho t}V_i'\big(y(t)\big)\, y(t) \xrightarrow[t\to\infty]{} 0$.

of the Hamilton-Jacobi-Bellman (HJB) equation, (2) strict concavity of the Hamiltonian in the control variable I, ensuring unique optimal investment, and (3) compliance with transversality conditions under the stabilizing root.

The convexity of the value function reflects strategic incentives: jurisdictions with a larger infrastructure advantage derive increasing marginal value from further widening the gap. This feature is consistent with the model's prediction of delayed catch-up and strategic escalation, especially under low depreciation and high effectiveness of public capital.

Markov-perfect investment policies and closed-loop dynamics

Finally, the optimal feedback investment rules $I_i = \frac{\phi}{c}(B+2A)$ and $I_j = \frac{\phi}{c}(B-2A)$ are,[19]

$$\begin{cases} I_i^M(y) = \dfrac{\phi}{c}\left[\dfrac{2\alpha}{(5\rho+4\gamma+\sqrt{\Delta})} + \dfrac{2\,\alpha^2}{9\,(\,\rho+2\gamma+\sqrt{\Delta}\,)}y\right], \\ I_j^M(y) = \dfrac{\phi}{c}\left[\dfrac{2\alpha}{(5\rho+4\gamma+\sqrt{\Delta})} - \dfrac{2\,\alpha^2}{9\,(\,\rho+2\gamma+\sqrt{\Delta}\,)}y\right]. \end{cases} \tag{47}$$

where $\Delta = (\rho+2\gamma)^2 - \frac{4}{3}\frac{\alpha^2\phi^2}{c} > 0$ follows from the convergence condition.

From (47) one can see that when there is no gap $y=0$, both countries make positive investment, since $B>0$. Moreover, similar to the results of Proposition 3 in the main text, when the gap is too large, i.e., $y \geq \frac{B}{2A} \equiv \bar{y}^M$, the laggard country j does not invest. Nevertheless, when the gap is small enough both countries invest.[20]

The closed-loop law of motion (interior region), since $V_i' + V_j' = 4Ay$, is:

$$\dot{y} = \frac{\phi^2}{c}(V_i'+V_j') - \gamma y = \left(\frac{4\phi^2 A}{c} - \gamma\right)y =: \kappa\, y, \quad \text{with} \quad \kappa = \frac{1}{3}\left(\rho-\gamma-\sqrt{\Delta}\,\right) < 0 \quad \text{(stabilizing root)}.$$

It follows that $y(t) = y_0 e^{\kappa t}$, so the initial gap decays at rate κ. This implies faster convergence than open-loop, since the state decays at a more negative rate $\kappa < \nu$.

[19]Note the use of the difference-of-squares equivalence: $(\rho+2\gamma) - \sqrt{\Delta} = \frac{(\rho+2\gamma)^2-\Delta}{\rho+2\gamma+\sqrt{\Delta}} = \frac{\frac{4}{3}\frac{\alpha^2\phi^2}{c}}{\rho+2\gamma+\sqrt{\Delta}}$.

[20]Note that for country j not to invest, a gap larger than that under open-loop is required: $\bar{y}^M = \frac{9\,[\,\rho+2\gamma+\sqrt{\Delta}\,]}{\alpha[\,5\rho+4\gamma+\sqrt{\Delta}\,]} > \bar{y}$.

Comparison to the open-loop/PMP solution

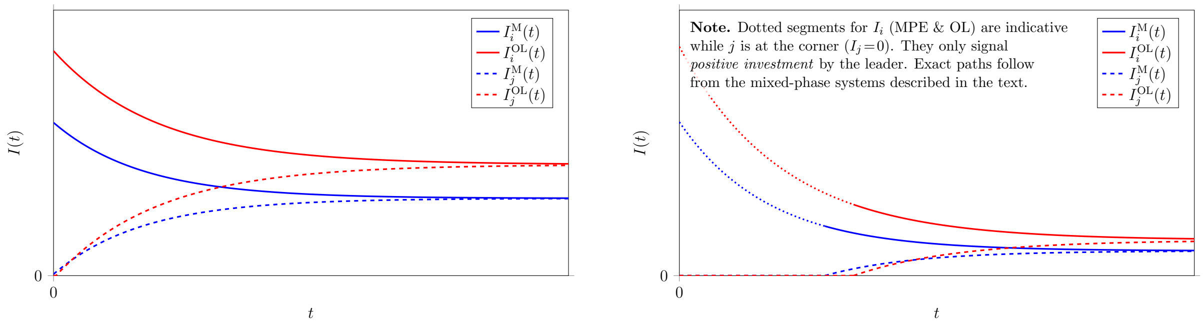
The rules in equation (47) are the Markov-perfect counterparts of the PMP optimal investment policies in equation (20), which for easier comparison can be written as,[21]

$$\begin{cases} I_i^{OL}(y(t)) = \dfrac{\phi}{c}\left[\dfrac{\alpha}{2(\rho+\gamma)} + \dfrac{\alpha^2}{3\,(\,\rho+2\gamma+\sqrt{\Delta}\,)}y(t)\right], \\[2ex] I_j^{OL}(y(t)) = \dfrac{\phi}{c}\left[\dfrac{\alpha}{2(\rho+\gamma)} - \dfrac{\alpha^2}{3\,(\,\rho+2\gamma+\sqrt{\Delta}\,)}y(t)\right]. \end{cases} \tag{48}$$

Therefore, both open-loop and markov-perfect solutions have the same form $a_0 \pm a_1 y$, but they only look the same. In fact, both equilibria deliver linear and symmetric rules, but with different coefficients. Taxes coincide, while investments do not. In particular, one can show that:

$$a_1^{\mathrm{M}} = \frac{2}{3}\, a_1^{OL}, \qquad a_0^{M} < a_0^{\mathrm{OL}},$$

so the Markov rule is flatter (weaker reaction to the gap y) and has a smaller intercept (lower baseline investment). Because j does adjust with the state, the current push on the state today triggers a strategic reaction tomorrow that leads i to invest less and smooth the response to the gap y compared with open-loop, ie., $I_i^M(y) < I_i^{OL}(y)$. However, the laggard j invests more in MPE only when the gap is sufficiently large $y^* < y < \bar{y}^M$. This contributes to faster convergence than open-loop towards the steady state (see Figure 3).



(a) Small gap $y_0 < \bar{y} < \bar{y}^M$ (b) Large gap $y_0 > \bar{y}^M > \bar{y}$

Figure 3: Investment Dynamics: Open-Loop vs. Markov-Perfect (b=0)

Bottom line: the optimal investment strategies in a MPE are qualitatively the same to those in an OL equilibrium, but show lower steady state investment levels and flatter

[21] The open-loop solution gives a time path $(I_i(t))_{t\geq 0}$. Since in our case $y(t)$ is monotone, we can parametrize that path as $I_i(y(t))$. However, note that a Markov-perfect policy must be a best reply at every state y, not just along the OL trajectory. OL co-states satisfy adjoint ODEs that effectively omit the rival-feedback term $\frac{\phi^2}{c}V_i' V_j'$ (they treat the rival's control as open-loop, i.e., not reacting to y). Rewriting the OL controls as $I_i(y(t))$ does not change that strategic content. OL and MPE coincide only under extra structure that kills the rival-feedback term, which is not the case in our model.

transition. This leads to faster convergence to the steady state gap, which is also the same. Tax policies are exactly the same.

C Illustrative mapping of policy "races" to the model

Figure 4 provides an illustrative (data-based) vignette for two policy narratives: competition between the EU and the U.S., and between the U.S. and China. Placements are shown as intervals to reflect proxy and mapping uncertainty. The purpose is to illustrate how "persistent gaps" versus "catch-up" narratives translate into the model's regimes, rather than to provide quantitative validation. Details are given below.

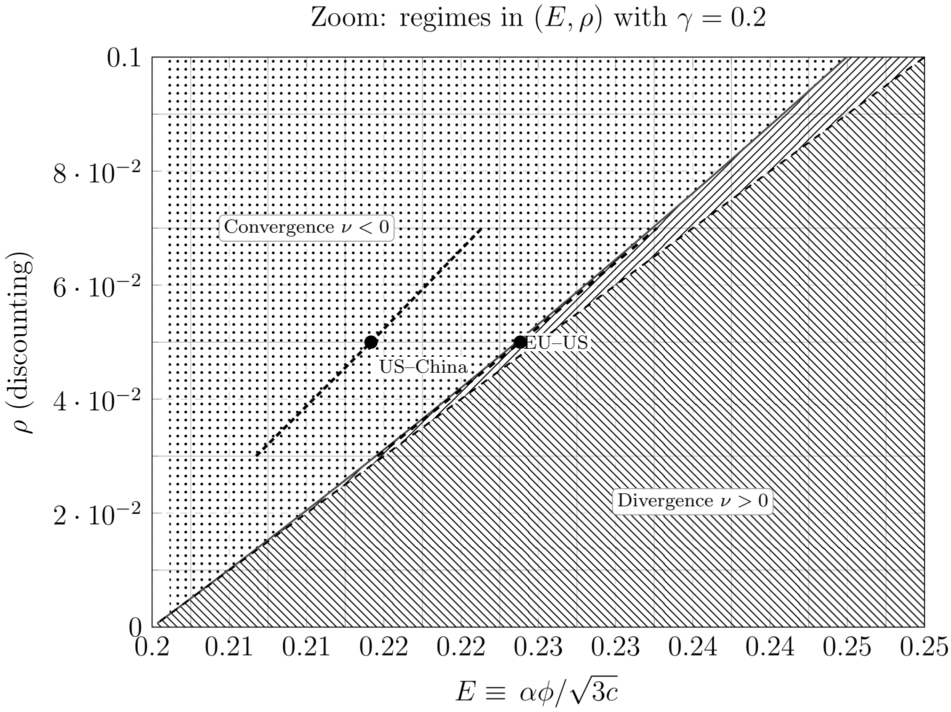


Figure 4: Zoomed regime map in the (ρ, E) plane at $\gamma = 0.2$, with illustrative interval placements for two policy "races" constructed as described in this appendix.

Consistent with the underlying narratives, the EU–U.S. baseline falls on the non-convergent side of the boundary, suggesting a "risk of persistent gaps" when E, the potency of infrastructure, is strong relative to effective patience and effective depreciation. Instead, the U.S.–China baseline lies on the convergent side, so the model predicts catch-up (shrinking gaps) in the relevant stock.

This vignette is deliberately *illustrative* (not a structural calibration). We map two real-world policy "races" onto the regime diagram in Figure 4 by: (i) choosing a transparent gap proxy y_t, (ii) summarizing its evolution by an average exponential trend, (iii) adopting conservative ranges for the discount rate ρ and diffusion/erosion parameter γ, and (iv) inverting the model-implied relationship between the trend and the composite index E.

Gap proxies and trends. For each race, define a scalar gap proxy y_t and compute the average log trend over a window of length T:

$$\hat{\nu} \equiv \frac{1}{T} \ln\left(\frac{y_T}{y_0}\right). \tag{49}$$

A negative (positive) $\hat{\nu}$ indicates catch-up (widening gaps).

For EU–U.S. we use the GDP-per-capita (PPP) shortfall reported in Draghi (2024). If the EU level is x_t below the U.S. level, then

$$y_t \equiv \frac{GDPpc_t^{US}}{GDPpc_t^{EU}} = \frac{1}{1-x_t},$$

and the report states that the per-capita gap rises from $x_{2002} = 0.31$ to $x_{2023} = 0.34$. For U.S.–China we proxy the gap by the ratio of gross domestic expenditures on R&D (GERD, current PPP dollars),

$$y_t \equiv \frac{GERD_t^{US}}{GERD_t^{CHN}},$$

using National Science Board and National Science Foundation (2025) (Table DISC-10).[22]

Parameter ranges. We take $\rho \in [0.03, 0.07]$ as a conservative real discount-rate range used in policy evaluation, and $\gamma \in [0.15, 0.25]$ as an illustrative reduced-form range capturing background erosion/diffusion forces. These intervals are used only to construct a placement range in (ρ, E)-space.

Implied effectiveness index. Using (15), the composite index $E \equiv \alpha\phi/\sqrt{3c}$ can be recovered from $(\hat{\nu}, \rho, \gamma)$ as

$$E = \frac{1}{2}\sqrt{(\rho + 2\gamma)^2 - (\rho - 2\hat{\nu})^2}. \tag{50}$$

We report a baseline placement at $(\rho, \gamma) = (0.05, 0.2)$ and an interval obtained from $\rho \in [0.03, 0.07]$ and $\gamma \in [0.15, 0.25]$. Importantly, the reported range for E is induced by varying (ρ, γ) through (50); it should not be combined with the endpoints of ρ as if E and ρ were independent. In Figure 4, the dashed intervals are computed by fixing $\gamma = 0.2$ (as in the regime map) and evaluating the inversion (50) at $\rho \in \{0.03, 0.05, 0.07\}$, using the corresponding $\hat{\nu}$ for each race, and connecting the three resulting points in (E, ρ)-space.

[22] NSB (2025) Table DISC-10: https://ncses.nsf.gov/pubs/nsb20257/assets/global-r-d-and-international-comparisons-2/tables/nsb20257-tabdisc-010.pdf, accessed December 30, 2025.

Table 1: Illustrative placement of two policy races in the (E, ρ) regime map (data-based vignette).

Race	Window	y_0	y_T	$\hat{\nu}$	E (base)	Implied E range
EU–US	2002–2023	1.449	1.515	0.00214	0.224	[0.164, 0.283]
US–China	2010–2022	1.925	1.137	-0.04387	0.214	[0.154, 0.274]

Notes. EU–US proxy: if EU is x_t below US in GDPpc (PPP), then $y_t = GDPpc^{US}/GDPpc^{EU} = 1/(1 - x_t)$ with $x_{2002} = 0.31$ and $x_{2023} = 0.34$ (Draghi). US–China proxy: $y_t = GERD_t^{US}/GERD_t^{CHN}$ (current PPP dollars), NSB (2025) Table DISC-10. Trend: $\hat{\nu} = \frac{1}{T}\ln(y_T/y_0)$. Implied index: $E = \frac{1}{2}\sqrt{(\rho + 2\gamma)^2 - (\rho - 2\hat{\nu})^2}$. Baseline uses $(\rho, \gamma) = (0.05, 0.2)$; intervals use $\rho \in [0.03, 0.07]$, $\gamma \in [0.15, 0.25]$. Implied E range over (ρ, γ) : $\{E(\rho, \gamma; \hat{\nu}) : \rho \in [\underline{\rho}, \bar{\rho}],\ \gamma \in [\underline{\gamma}, \bar{\gamma}]\}$. This is not an independent confidence interval for E; it is the sensitivity range induced by varying (ρ, γ) in the stated rectangle while holding $\hat{\nu}$ fixed.

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