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## BANKING OUTPUT & PRICE INDICATORS FROM QUARTERLY REPORTING DATA

Paolo Guarda and Abdelaziz Rouabah

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## Table des matières

1. Introduction ........ 6

2. Methods ........ 7

3. Data ........ 8

4. Empirical Results ........ 11

    4.1. Classifying financial products as outputs and inputs ........ 13

    4.2. Aggregating outputs and inputs ........ 14

    4.3. Price indices ........ 16

    4.4. Total Factor Productivity ........ 17

5. Conclusions and Directions for Further Research ........ 20

6. References ........ 22

7. Annex 1: structural model and system of share equations ........ 24

8. Annex 2: Statistics and correlations ........ 26



Address: 2, Boulevard Royal - L-2983 Luxembourg
Telephone: (+352) 4774 - 1
Fax: (+352) 4774 - 4910
Internet: http://www.bcl.lu
E-mail: sg@bcl.lu
Téléx: 2766 IML LU

# BANKING OUTPUT & PRICE INDICATORS FROM QUARTERLY REPORTING DATA*

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**Abstract**

The measurement of banking output (and therefore productivity) has long been controversial. This article applies the user cost approach in Fixler and Zieschang (1999) to quarterly reporting data from Luxembourg's banking sector. This requires associating the flows in the profit-and-loss account to different assets and liabilities in the balance sheet. The user cost of each asset/liability is then calculated as the difference between the rate at which it generates revenues/costs and a "reference rate" representing the opportunity cost of funds. A negative user cost then identifies an asset or liability as an output and a positive user cost identifies it as an input in the production process. In theory, this data-driven approach is capable of combining elements of both the two traditional approaches to measuring banking output (the production and intermediation approaches) since these classify inputs and outputs on an a priori basis. In practice, our results suggest that neither of these conventional approaches is wholly consistent with the data for Luxembourg. We then use multilateral Törnqvist indices to aggregate outputs and inputs separately and show that the resulting series are robust to alternative measures of the reference rate. The difference between the output and input index provides a measure of Total Factor Productivity (TFP) and an implicit price index is also derived. Results suggest that productivity growth in Luxembourg's banking sector has been high since the mid-1990s, displaying volatile but persistent dynamics and moving pro-cyclically. Productivity varies widely across banks but larger banks (in terms of total assets) tend to be more productive.

JEL classifications: G21, D24, C34

Keywords: banks, total factor productivity, Törnqvist index numbers

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Correspondence to paolo.guarda@bcl.lu or abdelaziz.rouabah@bcl.lu

## Non-technical summary

The measurement of banking output (and therefore productivity) has long been controversial. This article applies the user cost approach in Fixler and Zieschang (1999) to measure output and prices in Luxembourg's banking sector using quarterly reporting data. This requires associating the flows in the profit-and-loss account to different assets and liabilities in the balance sheet. The user cost of each asset/liability is then calculated as the difference between the rate at which it generates revenue/cost flows and a "reference rate" representing the opportunity cost of funds. A negative user cost then identifies an asset or liability as an output and a positive user cost identifies it as an input in the production process. In theory, this data-driven approach is capable of combining elements of both the two traditional approaches to measuring banking output (the production and intermediation approaches) since these classify inputs and outputs on an a priori basis. In practice, our results suggest that neither of the conventional approaches is wholly consistent with the data for Luxembourg. We then use our classification to aggregate outputs and inputs separately using multilateral Törnqvist indices. These indices use a representative firm as a benchmark to track the performance of the banking sector through time. Results suggest that gross output grew at an average annual rate of 11% over the period 1994Q1-2006Q4. This high rate of growth is largely attributable to the process of European financial integration, but also to financial innovation, with balance sheets benefiting from the introduction of new financial instruments and from changes in the regulatory framework. Results are robust to several alternative measures of the reference rate. Total Factor Productivity (TFP), measured by the difference between the output and input indices, averaged 4% annual growth on the same sample. Productivity growth has been volatile but persistent, moving pro-cyclically. Productivity varies widely across banks but seems to be higher in large banks. An implicit price index is also derived and suggests that the price of financial intermediation services fell over the sample period. This may be attributed to new information and communication technologies, whose impact on costs should be larger in the banking sector. Alternatively, it could be associated with the downwards trend in nominal interest rates over the last decade. By compressing the interest rate margin, lower interest rates have reduced the user cost of both assets and liabilities and therefore the implicit price of output. In this context, the user cost approach is particularly attractive because it provides a price measure for financial intermediation services that are indirectly charged, meaning that their price cannot be directly observed.

## Résumé non-technique

La mesure de la production bancaire (et donc de la productivité) a été longtemps sujette à des multiples controverses. Cet article adopte l'approche de Fixler and Zieschang (1999) afin de mesurer la production et les prix du secteur bancaire luxembourgeois à partir des reporting trimestriels transmis à la BCL. Cette méthode consiste en l'affectation des différents flux affichés dans le compte de pertes et profits aux différents postes de l'actif et du passif bilantaires. Ainsi, nous calculons un taux pour chaque poste d'actif ou du passif en fonction des flux de revenus ou de coûts associés. Ensuite, la différence entre ce taux et un taux de référence censé capturer le coût d'opportunité du capital financier fournit le coût d'usage du poste en question. Un coût d'usage négatif permet de catégoriser un poste de l'actif ou du passif comme étant un produit, tandis qu'un coût d'usage positif revient à l'identifier comme étant un entrant dans le processus de production. Or, les deux approches traditionnelles à la production bancaire, qui sont l'approche « intermédiation » et l'approche « production », se fondent sur une classification à priori. Par contre, l'approche par le coût d'usage opère une classification issue des données et peut, théoriquement, combiner des éléments afférents aux deux approches précitées. En pratique, nos résultats suggèrent qu'aucune des deux approches traditionnelles n'est tout-à-fait cohérente avec les données luxembourgeoises. Après l'identification des produits et des intrants, nous les agrégeons

séparément à l'aide d'indices Törnqvist multilatéraux. Ces derniers sont construits sur la base d'une entreprise représentative qui sert d'étalon pour suivre l'évolution temporelle du secteur bancaire. Il ressort que la production totale a progressé à un taux annuel moyen de 11% sur la période 1994T1-2006T4. Ce taux élevé trouve son origine à la fois dans la progression de l'intégration financière européenne et dans le processus d'innovation financière traduit par l'introduction de nouveaux instruments financiers ou de nouvelles formes de régulation. La forte progression de la production demeure robuste à l'adoption de multiples mesures du taux de référence. La Productivité Totale des Facteurs (PTF), mesurée par la différence entre les indices de la production et des intrants, a augmenté de 4% par an sur le même échantillon. La croissance de la productivité est volatile mais les fluctuations y afférentes sont persistantes et pro-cycliques. Le niveau de la productivité varie à travers les banques, mais il semble qu'il est plus élevé dans les établissements de grande taille. Nous avons calculé également un indice des prix implicites, dont l'évolution suggère que les prix des services d'intermédiation financière ont baissé sur la période considérée. Ceci pourrait refléter l'introduction des nouvelles technologies d'information et de communication, dont l'impact sur les coûts serait particulièrement favorable pour le secteur bancaire. Cette baisse de prix peut être attribuée également à la tendance baissière des taux d'intérêt nominaux au cours de la dernière décennie. La compression de la marge d'intérêt qui en résulte a réduit le coût d'usage des actifs comme des passifs et par voie de conséquence le prix implicite de la production. Dans ce contexte, l'approche par le coût d'usage est particulièrement attractive parce qu'elle fournit une mesure de prix pour les services d'intermédiation financière qui ne sont pas directement facturés, c'est-à-dire ceux dont le prix ne peut pas être observé directement.

## 1. Introduction

The measurement of banking output has long been a controversial issue (see Triplett, 1990, Fixler and Zieschang, 1991, or Berger and Humphrey, 1992). The conceptual and empirical difficulties common in other service industries are exacerbated in the banking sector by the lack of agreement on the definition of banking output. In large part, the problem is due to the two different types of bank revenues: net interest (the difference between interest collected on loans and interest payments made to deposits) and explicit service charges. The latter are largely insufficient to cover non-interest costs of operation (i.e. wages, rents, equipment, etc.), so at least some of the services provided by banks must be paid by net interest income rather than explicit service charges.

Fixler and Zieschang (1991) attribute the controversy on measuring banking output to long-held differences in how interest is viewed. Some prefer to view interest as a *transfer payment* from borrowers to lenders (depositors) for foregone consumption. On this view, the intermediary services provided by banks are not productive. In fact, national accounts used to exclude interest flows from value added arguing that they are contaminated by "pure interest" reflecting intermediation. As a result, most banking output used to be excluded from measured GDP. On the other view, banks' net interest income is considered a *payment for services* they provide to the community (payments services and money creation), to the depositor (recordkeeping, safekeeping, and interest payments on deposits) or to the borrower (funding, credit rating). On this view, it is easier to accept that banks use net interest income partly to pay for services that are not explicitly charged to customers.

However, this also means that banks purchase funds from depositors with bartered depositor services and not just interest payments. This casts doubts on the practice of automatically classifying liabilities as inputs and assets as outputs. Interest payments required by liabilities (i.e. deposits) are partially offset by explicit service charges paid by depositors, who may also face minimum deposit requirements or limits to the number of checks written per month. On the other hand, interest flows generated by assets (i.e. loans) are partially offset by the cost of attendant services (credit checks, recordkeeping, withdrawals).

Hancock (1985, 1986) developed a theory of production for the financial firm in which the input or output status of individual financial products can be determined empirically. This approach is based on the user cost of money as developed by Donovan (1978) and Barnett (1980). The user cost of each *asset* is calculated as the difference between its holding revenue rate and a reference rate representing the bank's opportunity cost of funds. The user cost of each *liability* is calculated as the difference between its holding cost rate and the bank's opportunity cost of funds. When a positive user cost is attached to an asset (because its holding revenue rate is insufficient to cover the opportunity cost of funds) this will contribute to the financial firm's costs and the asset is therefore classified as an input. When the opposite is true, the asset adds to the firms' revenue and is therefore classified as an output. The same is true of liabilities, which can also be classified endogenously as either inputs or outputs depending on the sign of the associated user cost. For example, if the holding cost on a deposit (interest paid plus the cost of services not explicitly charged) falls short of the opportunity cost of funds, then the deposit will add to revenue and thus be classified as an output.

Fixler and Zieschang (1992a) and Fixler (1993) applied the user cost approach to calculate an index of commercial banking output and prices. Using the exact index number results of Caves, Christensen and Diewert (1982a), they constructed a Törnqvist index with superlative properties and showed that it was robust to alternative choices of the reference rate measuring the opportunity cost of funds (i.e. the interest rate on 90-day Treasury Bills, 1-year and 2-year Treasury Notes). Fixler and Zieschang (1992*a*) used a distance function approach to estimate the opportunity cost of funds econometrically and confirmed that results are robust to use of several additional measures of the opportunity cost of funds (i.e. banks' rate of return on assets). Fixler and Zieschang (1992*b*) allowed for quality change by extending the index of banking output to incorporate additional information. Fixler and Zieschang (1993) studied banks' technical efficiency using an index number approach based on their user cost measure of output. Finally, Fixler and Zieschang (1999) further explored the impact of quality adjustment on measures of productivity in the banking sector.

These methods were first applied to quarterly data on banks in Luxembourg by Dimaria (2001) and by Guarda and Rouabah (2006). The present paper extends the analysis in several different directions. First, this paper uses a larger set of banks and covers a longer period (1994Q1 to 2006Q4). Second, it uses a slightly different breakdown of financial products, treating commission income as a financial product (included among directly charged services) and classifying commissions paid as an input on ex ante grounds. Third, the user costs constructed for certain assets take account of the associated write-downs and transfers to/from provisions. Fourth, aggregate measures of output and inputs for the banking sector are constructed as multilateral Törnqvist indices. These use a representative firm approach to guarantee transitivity in our panel data context. They also make it possible to analyse quarter-to-quarter productivity developments across individual firms.

Section 2 outlines the methods used in more detail. Section 3 describes the data and discusses some trend behaviour. Section 4 presents the resulting Törnqvist indices of output and prices in the banking sector and compares them to the corresponding series from the national accounts. The final section presents some conclusions.

## 2. Methods

The user cost for the *i*th asset is calculated as the difference between the reference rate measuring banks' opportunity cost of funds and the holding revenue rate for that class of assets:

$$u_{ai}^{t} = \rho - h_{ai}^{t} \tag{1}$$

where the user cost $u_{ai}^{t}$ may vary depending on the period $t$ as well as the asset class $ai$. The reference rate measuring the opportunity cost of funds is denoted $\rho$ and $h_{ai}^{t}$ denotes the holding revenue rate for asset $ai$ in period $t$. In theory, the holding revenue on an asset class accounts for both capital gains and provisions for loan losses on the given asset.

The user cost of liability $j$ is calculated as the difference between banks' holding cost rate and the reference rate measuring the opportunity cost of funds.

$$u_{lj}^{t} = h_{lj}^{t} - \rho \tag{2}$$

where again the user cost $u_{lj}^{t}$ may vary across periods $t$ and across liability classes $lj$. The holding cost rate for liability $lj$ in period $t$ is denoted $h_{lj}^{t}$ and includes not only interest payments net of explicit service charges but also the product of $\rho$ and the reserve requirement on the given class of liabilities[1].

Obviously, for equations (1) and (2) to be operational, we need some measure of $\rho$, the reference rate measuring the opportunity cost of funds. We consider several possible alternatives, including the econometric estimate proposed by Fixler and Zieschang (1992*a*). This last measure potentially allows $\rho$ to vary across banks, to reflect the heterogeneous nature of the sample in Luxembourg. In practice, the econometric estimate of the reference rate is based on Fixler and Zieschang's assumption that

[1] Following Fixler and Zieschang (1992), we drop discounting terms to simplify the analysis.

the opportunity cost of funds is a constant fraction of the total rate of return on assets (which varies across banks and across periods). This fraction is estimated by specifying an output distance function conditional on the level of liabilities. Adopting a translog functional form, the parameters of this function are recovered from an estimated system of share equations[2]. The system is estimated by iterated seemingly unrelated regression, imposing the cross-equation restrictions required for symmetry. The covariance matrix of the disturbances across equations is singular because of the restriction that they must add to zero. Therefore one of the share equations can be dropped from the system. Without loss of generality, we drop the equation for Y5, directly charged services (see table 1 below). Multiplying the estimated value of the intercept in the distance function by the total rate of return on assets produces a bank-specific series for $\rho$, the reference rate measuring the opportunity cost of funds.

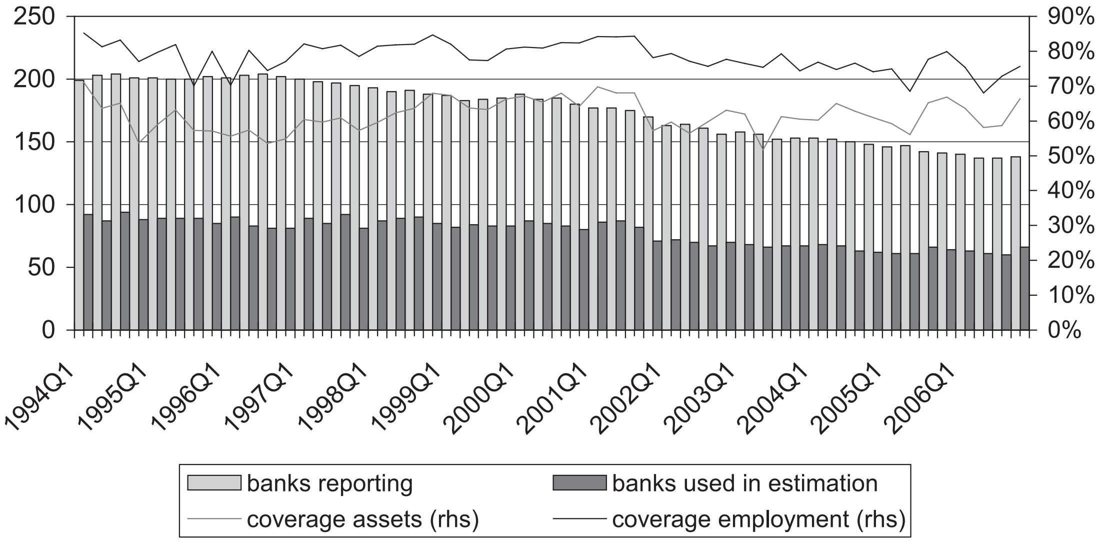
The results reported below are based on the estimation of the system of share equations over the whole sample. In principle, it would be preferable to estimate the parameters separately for each quarter in the sample, allowing for changing technology and the shifting composition of the sample of banks. This would also make it possible to test the assumption of fixed cross-term coefficients that is implicit when calculating Törnqvist productivity indices. Finally, quarter-specific estimates of the technology would also make it possible to decompose total factor productivity growth of individual banks into the separate effects of technical progress, changing efficiency and variable returns to scale. Unfortunately, quarter-by-quarter estimates proved rather volatile. In particular, this approach produced two outlying values (below 0.4) of the crucial parameter phi in 1996Q3 and 1998Q3. To avoid the implausible volatility that this would imply for the estimate of $\rho$, it was preferred to use the whole-sample estimate of phi, applying it to $r_{TA}$ to derive a time-varying value of $\rho$.

## 3. Data

The dataset includes observations from all banks reporting since 1994Q1. On average, 176 banks reported each quarter; however the exact number of banks per period varies as some banks enter, leave or merge each quarter. Unfortunately, many of the banks reporting did not provide all the data required for our analysis. In particular, subsidiaries of banks established in other EU member states are subject to lower reporting requirements. In addition, some banks are specialised in some very restricted line of business so they report zeros in many asset/liability positions, which is problematic for our assumed translog functional form. To ensure a more homogenous sample, we restricted our analysis to banks that reported all four of the asset categories and both of the liability categories presented in Table 1 below. This reduces the number of banks to only 78 per quarter on average. However, the banks that were removed from the sample were predominantly smaller banks, often both in terms of balance sheet and in terms of the number of employees. In fact, the sub-sample of banks considered covered 52% to 71% of the aggregate balance sheet for the sector, with the coverage rate for total assets averaging 62%. In terms of employment, the sub-sample of banks considered represented 68% to 85% of all jobs in the banking sector, averaging 79% over the whole sample period.

[2] See annex for details.

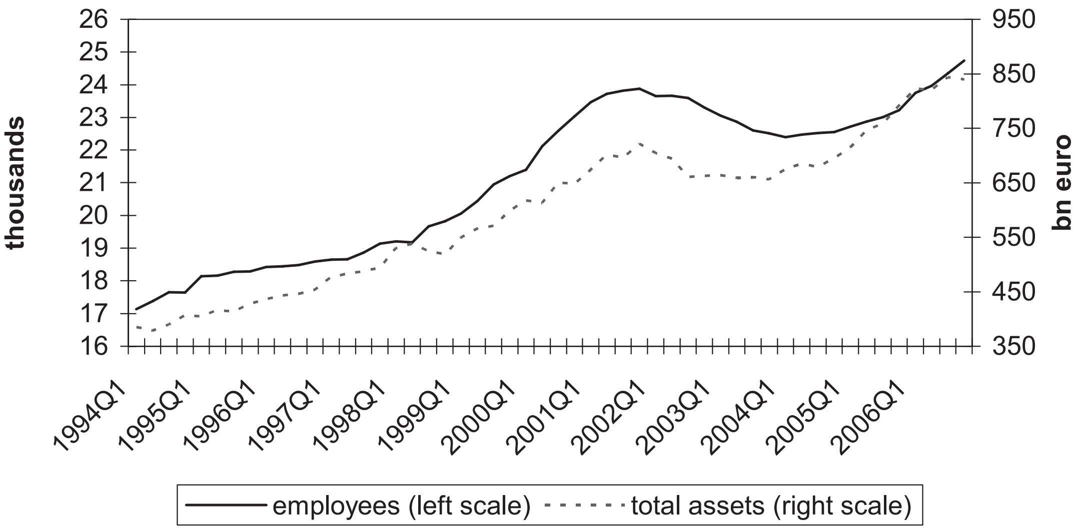
***Figure 1: Number of banks and coverage rates***




Source : BCL

The observable decline in the number of banks from over 200 to less than 140 reflects the move towards consolidation in the European banking sector, as mergers between parent banks in Germany, France, Belgium or other EU countries lead to mergers in their Luxembourg subsidiaries. In fact, despite this decline in the number of banks, the Luxembourg financial sector has continued to grow both in terms of total assets and in terms of employment. This is confirmed by the following figure plotting the evolution of total assets and employees summed over all banks for each individual quarter in the sample. Both employment and total assets grew strongly until the beginning 2001Q4, when the financial sector started to contract. Both series have recovered since, with the expansion beginning earlier in total assets than in employment.

***Figure 2: Employment and total assets in Luxembourg's banks***




Source : BCL

Following Fixler and Zieschang (1992*a*), we aggregate assets and liabilities drawn from the balance sheet into different product classes, with each class potentially either an input or an output. The definition of the different classes depends in part on the available detail regarding the corresponding holding revenues or costs for the given asset or liability under consideration. The following is our version of Table 6.1 in Fixler and Zieschang (1992*a*):

***Table 1: Components of Financial Product Aggregates***

| *Aggregate financial product* | *BCL code* | *Description* |
|---|---|---|
| *Loans & leases:* | | |
| Y1 | B1-04.000<br>B1-05.000 | Loans to customers<br>Leases |
| Y2 | B1-03.000 | Loans to depository institutions |
| *Securities:* | | |
| Y3 | B1-02.000<br>B1-06.000 | Government securities<br>Fixed income securities |
| Y4 | B1-07.000<br>B1-08.000<br>B1-09.000 | Shares<br>Participations<br>other variable income securities |
| *Directly charged services:* | | |
| Y5 | P4-04.000<br>P4-01.600<br>P4-01.700<br>P4-06.000<br>P4-07.000 | Commission income<br>Gains on foreign exchange trades<br>Gains from financial instruments<br>Gains from financial operations<br>Other non-interest income |
| *Deposits & other liabilities:* | | |
| Y6 | B2-01.000<br>B2-03.000<br>B2-07.000 | Deposits by banks<br>Securities issued<br>Subordinated debt |
| Y7 | B2-02.000 | Deposits by customers |

Source: BCL

In general, the input/output status of these different financial product classes was determined by the sign on their associated user cost. This was calculated as the difference between a reference rate (the "opportunity cost of funds" denoted $\rho$ below) and the holding revenue rate for assets or the holding cost rate for liabilities (see equations 1 and 2). For asset classes Y1, Y2, Y3 and Y4, the holding revenue rate was constructed from respective interest or other income but was also adjusted for any write-downs in value and any transfers to/from provisions to reflect foreign exchange or credit risk. The holding revenue rate for each asset was constructed for each bank in each quarter.

Financial product Y5, directly charged services, was classified as an output on an a priori basis. In fact, Y5 represents the sum of several series in the profit-and-loss account that cannot be associated with any particular asset or liability class in the balance sheet[3]. For liability classes Y6 and Y7, the holding cost rate was constructed from interest and other costs. Here also, the holding cost rate of each liability class was calculated for each bank in every quarter. In our data set, some implausible rates (in excess of 100%) were observed for the holding costs/revenue rates on some assets/liabilities. To avoid the influence of these outliers, we dropped the observations associated with the top 0.5% of the distribution (across all periods) of each holding cost/revenue rate for which implausible values appeared.

[3] Fixler and Zieschang had an equivalent product labelled Y7 in their case.

We follow Fixler and Zieschang (*1992a*) in identifying three additional inputs on an *a priori* basis: labour (x1), capital (x2, including both tangible and intangible fixed assets) and purchased materials and services (x3, including non-wage administrative costs and commissions paid). Along with those assets/liabilities that are classified as inputs, these will be aggregated into an input index that will then be used to calculate total factor productivity growth.

## 4. Empirical Results

To construct the user cost associated with each asset/liability, equations (1) and (2) require not just an estimate of holding revenues/costs but also a measure of the reference rate $\rho$ that is supposed to represent the opportunity cost of funds. We followed Fixler and Zieschang, experimenting with several alternative measures. These include (1) the 3-month EURIBOR interbank rate, $r_{EURIBOR}$; (2) the total return on assets (including revenue from directly charged services), $r_{TA}$; (3) the rate of return on assets, $r_A$ (excluding revenue from directly charged services); and (4) the required rate to cover the interest cost of liabilities, $r_{REQ}$. The latter three measures were calculated as asset-weighted sample means. Finally, (5) an econometric estimate using a system of share equations from a translogarithmic cost function (see Annex 1 for details).

Rate (2) the asset-weighted sample mean of the *total* return on assets is computed as:

$$r_{TA} = \frac{\sum_{n=1}^{N} \left[ \sum_{i=1}^{4} h_{in} y_{in} + R_{5n} \right]}{\sum_{n=1}^{N} \sum_{i=1}^{4} y_{in}}$$

where i=1…4 indexes the assets, n=1…N indexes the banks in the given quarter and $R_{5n}$ represents the income from directly charged services in the bank *n*. Rate (3) is the asset-weighted sample mean of the holding return on assets (excluding $R_{5n}$), computed as:

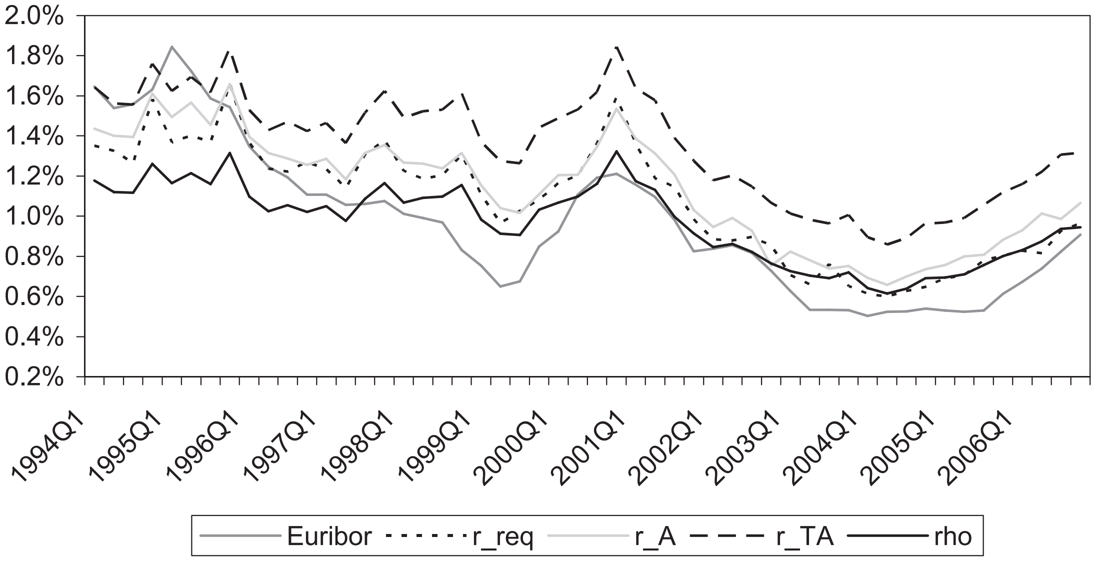
$$r_A = \frac{\sum_{n=1}^{N} \sum_{i=1}^{4} h_{in} y_{in}}{\sum_{n=1}^{N} \sum_{i=1}^{4} y_{in}}$$

Rate (4), the asset-weighted sample mean of the required rate of return on assets, is computed:

$$r_{REQ} = \sum_{n=1}^{N} \left[ \frac{\sum_{i=1}^{4} y_{in}}{\sum_{n=1}^{N} \sum_{i=1}^{4} y_{in}} \right] \cdot \left[ h_{6n} / (1 - k_n) \right]$$

where $k_n$ is the ratio of reserves (currency and coin and deposits with central banks) to deposit and other liabilities in the *n*th bank.

***Figure 3: Alternative measures of the opportunity cost of funds (quarterly rates)***




Source : BCL

Figure 3 plots these different reference rates measuring the opportunity cost of funds as quarterly rates since 1994Q1. The figure indicates that these rates display a substantial degree of co-movement. All series are characterised by a falling trend over the sample period, with a sharp but transitory upturn between 2000 and 2002 and a recovery starting in 2004/2005. The degree of co-movement could be considered surprising given that the series $r_{REQ}$, $r_A$ and $r_{TA}$ are calculated as asset-weighted means from our sample, while the Euribor 3-month interbank rate is downloaded from the ECB database (prior to 1999 it is constructed by aggregating across European countries). The co-movement of the econometrically estimated opportunity cost of funds, $\rho$, is less surprising, since it is calculated simply as the product of the total return on assets ($r_{TA}$) and an estimated constant (this was near 0.70 when estimated over the sample as a whole, corresponding closely to the values estimated by Fixler and Zieschang).

Table 2 reports the alternative reference rates measuring the opportunity cost of funds (asset-weighted means for $r_{REQ}$, $r_A$ and $r_{TA}$) in selected quarters of the sample. The results are converted from quarterly to annual rates and are similar in magnitude to those reported by Fixler and Zieschang (1992a, table 6.3) for US banks over the sample 1984 to 1988.

***Table 2: Reference Rates – Asset-Weighted Means (annual rates)***

| Variable | Description | 1994Q1 | 1996Q1 | 2000Q1 | 2004Q1 | 2006Q1 |
|---|---|---|---|---|---|---|
| $r_{euribor}$ | 3-month interbank rate | 0.0675 | 0.0550 | 0.0375 | 0.0203 | 0.0272 |
| $r_{REQ}$ | Required rate | 0.0551 | 0.0557 | 0.0474 | 0.0247 | 0.0335 |
| $r_A$ | Asset rate | 0.0587 | 0.0570 | 0.0491 | 0.0280 | 0.0377 |
| $r_{TA}$ | Total asset rate | 0.0673 | 0.0626 | 0.0608 | 0.0363 | 0.0472 |
| $\rho$ | Econometric opportunity cost | 0.0541 | 0.0580 | 0.0480 | 0.0213 | 0.0354 |

Source: BCL

### 4.1. Classifying financial products as outputs and inputs

The next table reports the output status of the different financial products (in the columns) according to the different reference rates used to measure the opportunity cost of funds (in the rows). The figures in the table represent the fraction of quarters in the sample for which the user cost of the given financial product took a negative sign (output status). The user cost is calculated by comparing the asset-weighted mean of the holding cost/revenue of a given financial product to the reference rate in the given row. Note that in three cases ($r_{REQ}$, $r_A$ and $r_{TA}$) the reference rate measuring the opportunity cost of funds is also an asset-weighted mean. In the fifth column, no results are reported for Y5 because directly charged services were classified as an output on a priori basis.

***Table 3: Output Status of financial products according to sign of user cost***

| | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 |
|---|---|---|---|---|---|---|---|
| $r_{euribor}$ | 85% | 81% | 90% | 69% | - | 12% | 87% |
| $r_{REQ}$ | 71% | 40% | 88% | 58% | - | 60% | 100% |
| $r_A$ | 62% | 15% | 81% | 52% | - | 77% | 100% |
| $r_{TA}$ | 10% | 0% | 38% | 40% | - | 100% | 100% |
| $\rho$ | 98% | 73% | 96% | 67% | - | 29% | 88% |

Source: BCL

The only consistent result is for Y7 (deposits from customers). This is considered an output (holding costs below the opportunity cost of funds) in an overwhelming majority of quarters for all the alternative measures of the reference rate. This result is inconsistent with the intermediation view of the banking firm, which classifies deposits a priori as an input. Instead, it seems more consistent with the production view of the banking firm, which concentrates on the capital and labour inputs required to produce the services that are associated with deposits but not explicitly charged. In any event, this evidence that deposits should be considered outputs underscores the importance of considering the user cost of a given financial product, meaning the gap between the reference rate and the rate at which cost and revenue flows are generated.

For the other financial products, the classification is not as straightforward. When the opportunity cost of funds is measured by the total return on assets ($r_{TA}$) assets Y1 to Y4 are generally classified as inputs, and liabilities Y6 and Y7 are always classified as outputs, in direct contradiction with the intermediation approach to measuring banking output. Apparently, this result is due to the fact that $r_{TA}$ is generally the highest measure of the opportunity cost of funds. We conclude that $r_{TA}$ could be a misleading guide and prefer to ignore this row of the table. On the basis of the other rows, it seems reasonable to classify the following assets as outputs: Y1 (Loans to customers), Y3 (Fixed income securities) and, less convincingly, Y4 (Variable income securities). A question mark remains over Y2 (Loans to banks) which is traditionally considered an output but tends to be classified as an input (except when the reference rate is the Euribor 3-month interbank rate). We believe this result reflects the low rate at which inter-bank deposits attract interest flows in a competitive market and therefore we choose to classify this asset also among the outputs. This is consistent with the findings by Fixler and Zieschang.

In summary, these results reveal that the sign of the user cost on a given asset or liability may be fairly constant or fairly volatile through time, depending both on the financial product in question and on the reference rate chosen to measure the opportunity cost of funds.

## 4.2. Aggregating outputs and inputs

Now that financial products are classified as inputs and outputs, one can construct Törnqvist indices measuring the level of outputs and inputs. The Törnqvist index does not require knowledge of the underlying technological parameters and Caves, Christensen and Diewert (1982a) showed that it is *exact* when the underlying functional form is translog. This is an appealing feature since the translogarithmic functional form provides an approximation to *any* arbitrary functional form. For this reason, the Törnqvist index is also called *superlative*. The Törnqvist *output* index for comparing periods *s* and *t* can be written in logarithmic form:

$$\ln Q_{st} = \sum_{i=1}^{N} \left( \frac{\omega_{is} + \omega_{it}}{2} \right) \left( \ln q_{it} - \ln q_{is} \right) \tag{3}$$

where the $q_{it}$ represent the i=1…N outputs in period *t* and *s* and $\omega_{it}$ indicates the share of output *i* in total revenues in period *t*. The index $Q_{st}$ indicates the growth rate between period *s* and *t* and can be used to calculate a chain-linked output quantity index. Analogously, the Törnqvist *input* index for comparing periods *s* and *t*, can be written in logarithmic form:

$$\ln X_{st} = \sum_{i=1}^{M} \left( \frac{\gamma_{is} + \gamma_{it}}{2} \right) \left( \ln x_{it} - \ln x_{is} \right) \tag{4}$$

where $x_{it}$ represents the i=1…M inputs in period *t* and the weights $\gamma_{it}$ measure the share of input *i* in total costs in period *t*. Total factor productivity (TFP) can be measured by the Törnqvist TFP index, obtained as the ratio of the Törnqvist output index and the Törnqvist input index. In logarithmic form, this can be written:

$$\ln(TFP_{st}) = \ln(Q_{st}) - \ln(X_{st}) \tag{5}$$

An increase in the TFP index represents that part of an increase in output that is not accounted for by a corresponding increase in inputs.

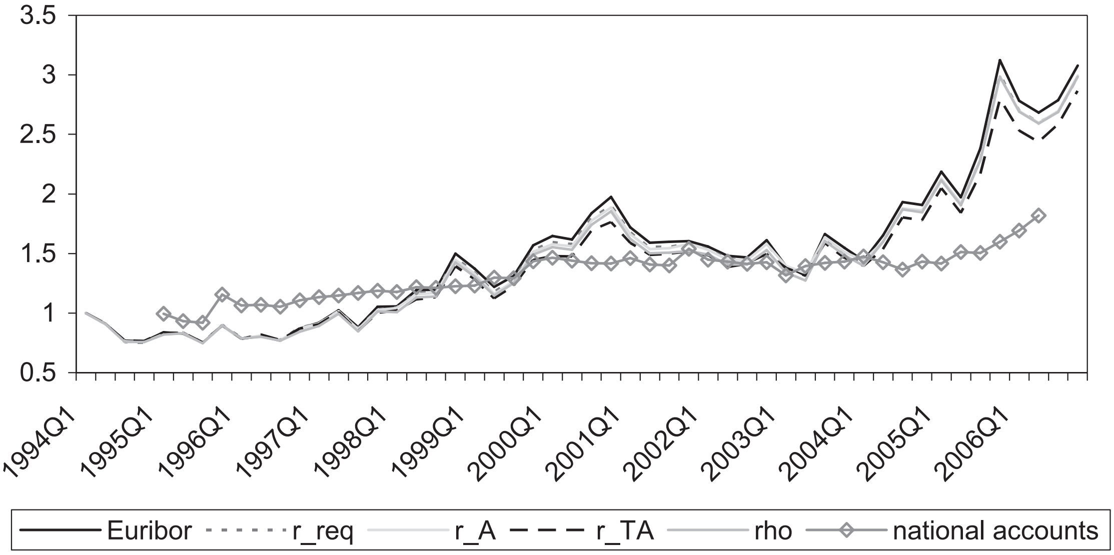
Following Fixler and Zieschang, output revenue shares are constructed by multiplying each financial product with the negative of its user cost summing across products (see their equation 10). However, in a panel data context if this operation is repeated for each bank then productivity comparisons will not be transitive (bank A may be more productive than bank B, and bank B more productive than bank C, but it will not follow that bank A will be more productive than bank C). Following Fixler and Zieschang (1993) we avoided this problem by constructing a multilateral output index[4] along the lines of Caves, Christensen and Diewert (1982b). Multilateral indexes ensure transitivity by relying on a hypothetical firm that provides an unambiguous basis for comparison when observations have no natural ordering (as in a cross-section of individual firms). A hypothetical firm was constructed for each quarter, with subcomponent expenditure shares equal to the arithmetic mean of expenditure shares across all firms in that period and subcomponent quantities equal to the geometric means of the subcomponent quantities across all firms in that period. Having constructed a hypothetical firm for each cross-section, these hypothetical firms were then chain-linked together over time starting with an arbitrary initial value of 100.

Figure 4 presents the resulting Törnqvist indices of gross output, each calculated with a different measure of the opportunity cost of funds. Our results confirm the finding by Fixler and Zieschang that the output indicator is robust to different choices of the reference rate, as all the Törnqvist indices are very close.

[4] See section 2.2 of Good et al. (1997) for an accessible discussion of multilateral indices.

The average year-on-year growth ranges from 10.8% to 12.0% depending on the choice of reference rate (opportunity cost of funds). After a period of contraction in 1995 and 1996 (around -2%), output expanded rapidly in 1997 to 2000. There was a new contraction from 2001 to 2003 (-1% to -7%). Since then average annual growth has returned to its previous very high rates, although preliminary estimates suggest slower growth in the latter half of 2006.

***Figure 4: Gross output Törnqvist volume indices and national accounts series***




Source : BCL

Figure 4 also plots the volume series (rebased to 1995=1) for gross output of the banking sector as provided by the national accounts department of STATEC. The upwards trend in the level of the series is not as pronounced as in the Törnqvist indices, with the national accounts series averaging only 5.2% year-on-year growth, which is about half the average growth in the Törnqvist output indices. However, there is some similarity in the short-run dynamics, especially over the years 2000-2003. In fact, year-on-year growth in the national accounts series has a linear correlation coefficient of about 28% with all five Törnqvist output indices. The national accounts series is less volatile, with a standard deviation of year-on-year growth around 7.3%, which is less than half the figure for the Törnqvist output indices. See annex 2 for more details.

The discrepancy between the Törnqvist output indices and the national accounts series could be attributed to a variety of factors. First, the two measures have different coverage: our indicators are based on only a sub-sample of reporting banks, while the national accounts series is based on aggregated data for the sector as a whole. Second, the Törnqvist output indices is constructed solely from the sample of data from individual banks whereas the national accounts series is part of a consistent description of the whole economy and therefore may be affected by the "balancing" exercise required to reconcile price and volume data from different sources. Third, the Törnqvist multilateral indices may overstate growth since banking consolidation through mergers and acquisitions reduces the number of banks while leaving the aggregate balance sheet largely unaffected. As a result, growth in the balance sheet of the representative bank may be overstated relative to that of the banking industry as a whole. Finally, the national account series is constrained to follow the methodology spelled out in the European System of Accounts (ESA 95)[5] as modified by European Council Regulation 448/98 and

[5] For details, see Michaux and Origer (2003).

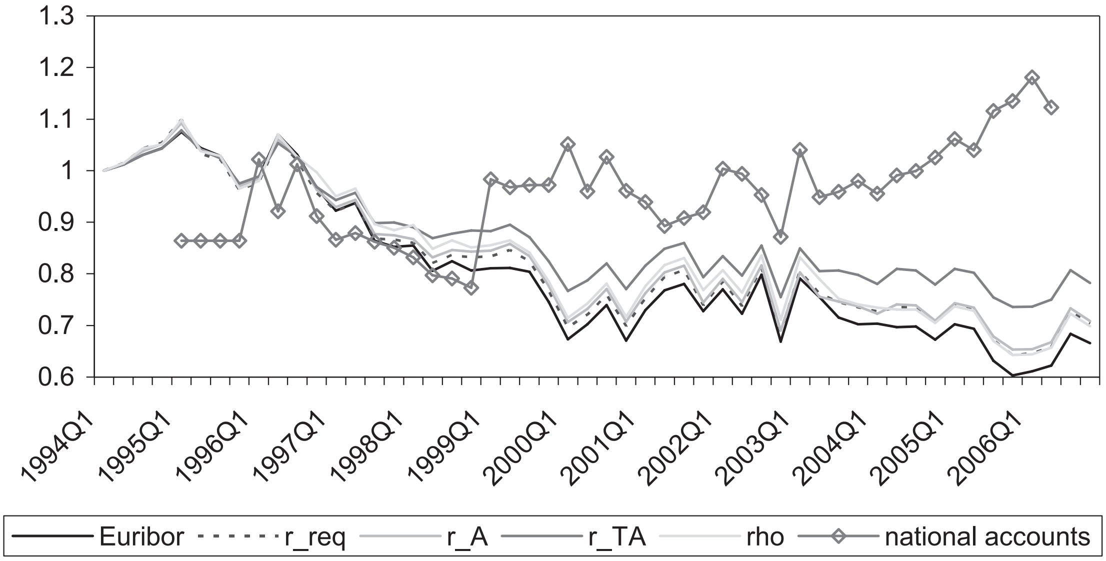
European Commission Regulation 1889/2002. Among other specific recommendations, these regulations require the separate treatment of all gains/losses on financial derivatives (forward contracts, swaps, etc.)

The high rate of output growth in Luxembourg's banking sector can plausibly be attributed to increasing financial integration in Europe but also to the effects of financial innovation. By innovation, we mean not only the introduction of new financial instruments and services but also changes to the framework of financial regulation. Both these factors can contribute to the increase in balance sheets.

### 4.3. Price indices

An implicit output price index can be obtained by dividing nominal output (the sum of all seven financial products in table 1) by real output (obtained by applying the quarter-on-quarter growth rates of the Törnqvist output index starting from the level of nominal output in 1994Q1). Figure 5 plots this implicit output price using each of the different reference rates considered above, along with the implicit price of gross output from the national accounts series. As could be inferred from the preceding figure, the results are pretty robust to the choice of reference rate used to measure the opportunity cost of funds. For all the reference rates considered, the implicit price of gross output actually falls over the sample. The average year-on-year growth rate for the implicit prices derived from the Törnqvist indices ranges from –2.18% to –3.48%. This contrasts with the national accounts data, where the implicit price of gross output averaged positive annual growth of 3.18% over the period 1995Q1 to 2006Q2. For the year-on-year growth rates, the linear correlation coefficient between the national accounts series and those calculated from the Törnqvist indices vary from -0.04 to +0.06. This lack of correlation may reflect an apparent level shift in the national accounts price series between 1998Q4 and 1999Q1. The national account series is much more volatile, with a standard deviation of 10.5% in the year-on-year growth rate, as opposed to a maximum standard deviation of 7.5% for the price series calculated from the Törnqvist indices. See annex 2 for more details

***Figure 5: Gross output implicit prices from Törnqvist indices and national accounts***




Source : BCL

An external measure of prices in the sector is provided by the HICP sub component "financial services" which refers mostly to directly charged services (i.e. service fees charged for standard operations). This price index averaged even higher growth (5.62% year-on-year over the same sample). Our results using the user cost method suggest that this increase in the price of directly charged services has been more than offset by the fall in the price of cost intermediation services indirectly measured. Such a fall in the prices of indirectly measured intermediation services may be attributed to the introduction of new information and communication technologies (ICT) that are supposed to help reduce costs, particularly in the banking sector. Alternatively, the fall in prices of financial intermediation services could be attributed to the downwards trend in nominal interest rates over the last decade. By compressing the interest rate margin, lower interest rates have reduced the user cost of both assets and liabilities and therefore the implicit price of output. A similar fall in the price of financial services was found by Fixler and Zieschang (1992a) and by Fixler (1993) using US data over the latter half of the 1980s, another period when nominal interest rates were declining.

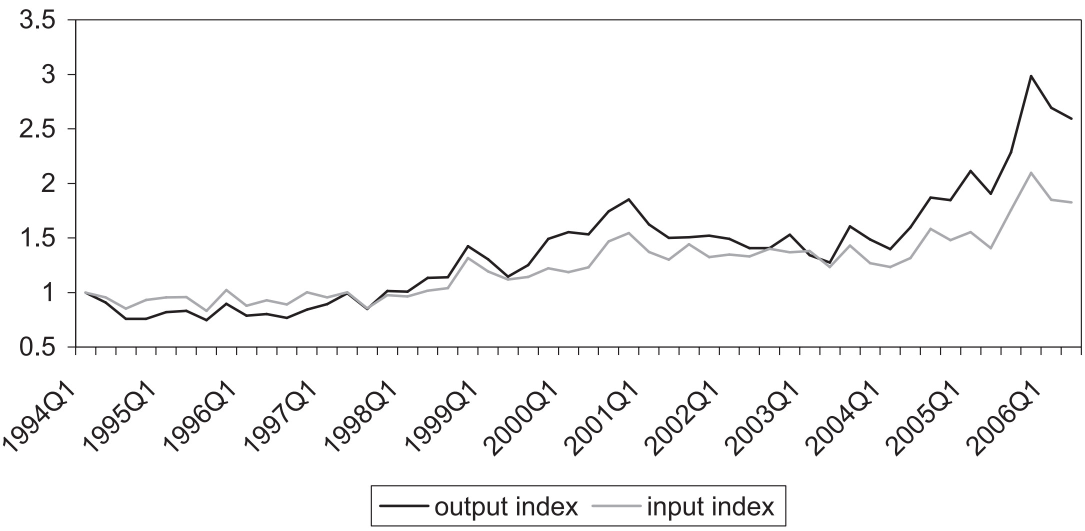
In this context, the user cost approach is particularly attractive because it provides a price measure for financial intermediation services that are indirectly charged, meaning that their price cannot be directly observed. Instead, national accounts price indicators are generally restricted to the limited set of price series that can be directly observed (such as commissions, share prices, foreign exchange rates, and final demand deflators).

## 4.4. Total Factor Productivity

Figure 6 plots the Törnqvist input index along with the output index for the econometrically estimated opportunity cost of funds $\rho$ (as indicated in Figure 4, results are similar for other reference rates). The Törnqvist input index aggregates growth in labour (x1), tangible and intangible fixed assets (x2), purchased materials and services[6] (x3) and interbank deposits (y6) with weights based on their individual cost shares.

After the contraction early in the sample, the output index increased faster than the input index until the end of 2000, suggesting gains in TFP. There followed a decline in output until the two series crossed again in 2003, after which the growth in output is consistently higher than the growth in inputs. The output index grew at an average rate of about 11% year-on-year, while the input index grew only about 7% on average. This suggests that total factor productivity grew at around 4% on average, as is confirmed below.

***Figure 6: Törnqvist output and input index using econometric opportunity cost*** $\rho$

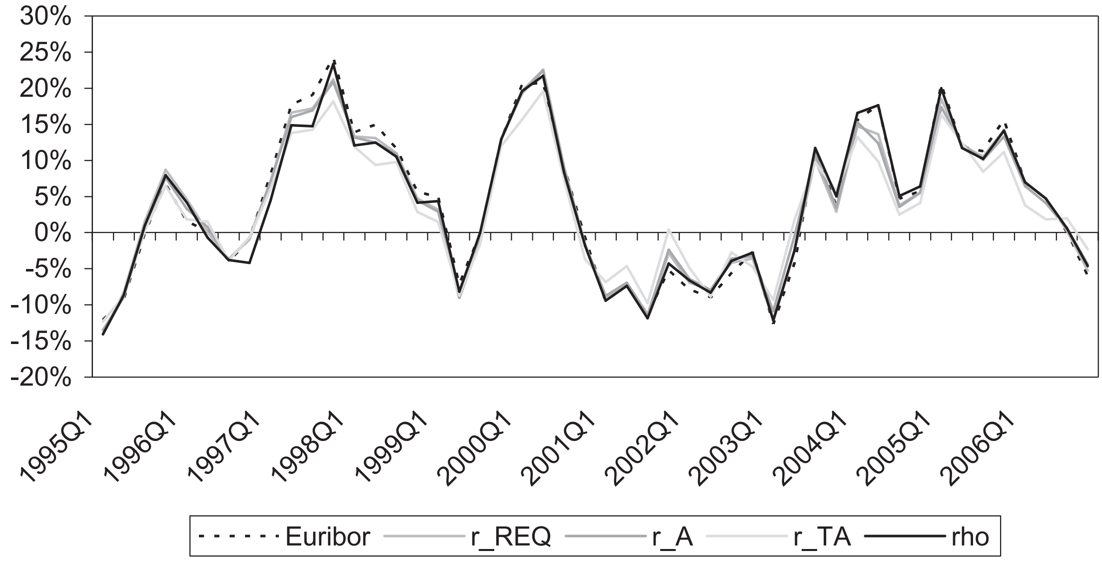



Source : BCL

[6] This includes commissions paid.

Figure 7 plots the year-on-year growth rates of the chain-linked index of TFP obtained from the logarithm of the ratio of the Törnqvist output and input indices. TFP growth is high, volatile but persistent. The average growth rate is between 3.4% and 4.3%; the standard deviation around 9% and the first-order autocorrelation coefficient ranges from 0.66 to 0.70 (see annex 2 for more details). Year-on-year TFP growth is negative in 18 of the 52 quarters available and displays no clear trend, apparently cycling around a fixed mean value.

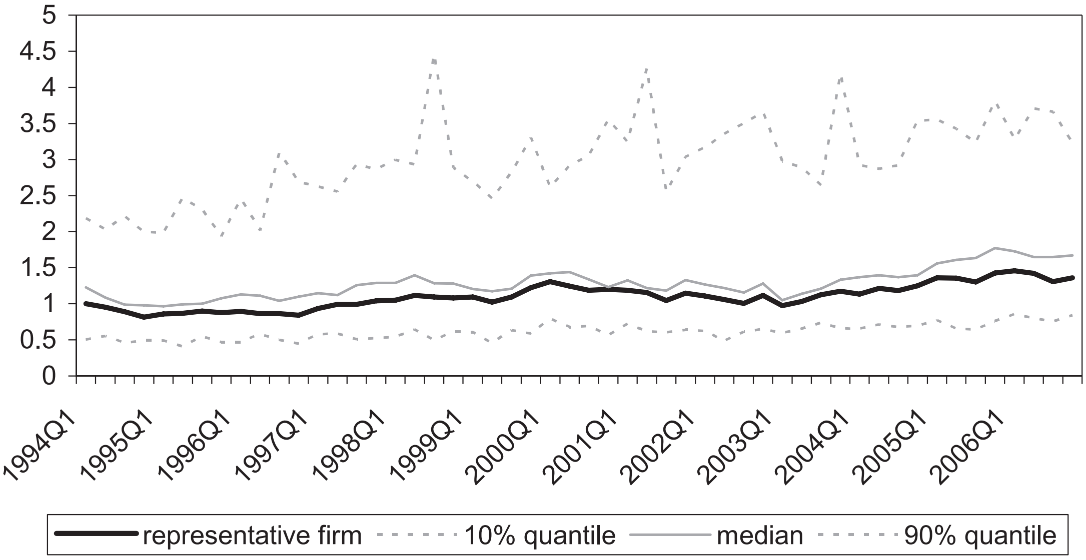
***Figure 7: Total Factor Productivity growth in Luxembourg's banking sector (y-o-y)***




Source : BCL

For each individual firm in each quarter, TFP was calculated relative to the representative firm in the given quarter. Figure 8 provides an indication of the distribution of TFP across firms by plotting the top and bottom deciles of the level of TFP, along with the level of TFP for the representative firm. The dark line in the middle corresponds to the level of TFP in the representative firm calculated from the geometric average of output and input quantities across firms in the given quarter. The dotted lines at the top and bottom correspond respectively to the 90% and 10% quantile of the distribution, meaning that 10% of banks in the given quarter have a level of TFP above the top line and 10% have a level of TFP below the bottom line. The lighter line in the middle represents the median of the distribution (50% quantile), meaning that half of all banks in the given quarter are above the line and half below. Note that the distribution of TFP is asymmetric, with the 10% quantile systematically closer to the median than the 90% quantile. This is to be expected as there is a zero bound implicit in the calculation of the TFP scores relative to the representative firm. However, note that the representative firm is relatively close to the median of the distribution, so it remains a valid reference point despite the asymmetry. In effect, Figure 8 is telling us that although there are some exceptional banks with TFP levels three or four times greater than the median, most of the distribution is concentrated around the level indicated by the representative firm. These high performers should therefore be considered anomalies or outliers compared to the rest of the population. This point is explored further in the following figure, which analyses TFP separately for large and small banks.

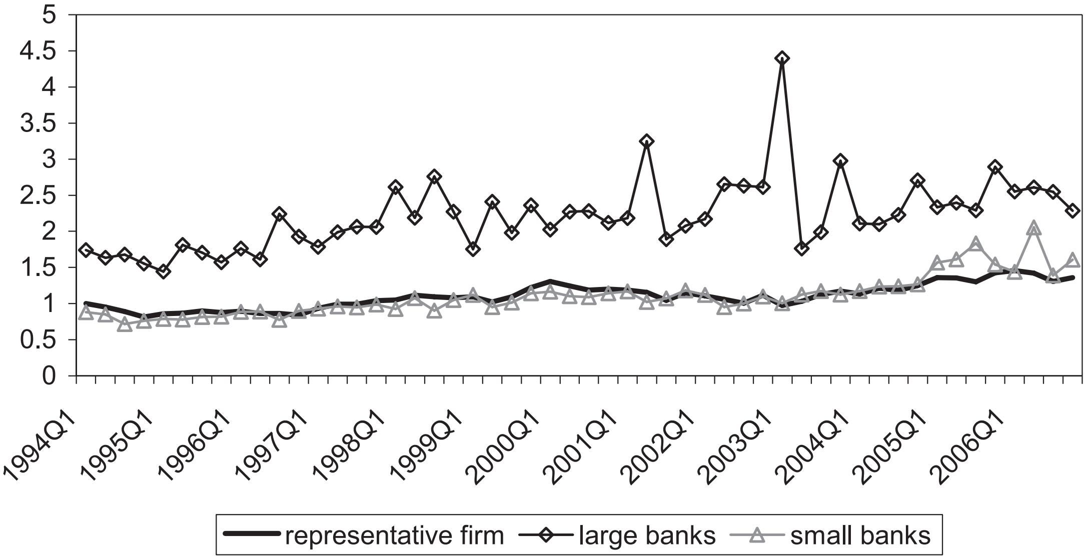
*Figure 8: Distribution of Total Factor Productivity across firms each quarter*




Source : BCL

The median value of total assets was calculated across all banks in each quarter. Banks with total assets above this median value were classified as “large” and banks with total assets below this median value were classified as “small”. Note that a bank with total assets near the median value may be classified as “large” in one quarter and as “small” in the following. Figure 9 plots the level of TFP (relative to the representative firm) averaged separately over “large” and “small” banks, along with the TFP of the representative firm (for the whole sample) that appeared in the previous figure. From this figure, it appears that the large banks tend to be more productive on average, while small banks tend to have productivity near that of the representative firm. Note that we dispose of only 77 observations per quarter on average, so splitting these into “large” and “small” banks severely reduces the number of banks on which each of the sub-sample measures is based. It may be that the level of TFP averaged across “large banks” is strongly influenced by one or two outliers. It may seem puzzling that towards the end of the sample there are quarters in which both the large and the small banks have TFP levels above that of the representative firm. Bear in mind that the representative firm is constructed as a geometric average of the output and input quantities across all firms. The productivity of individual banks is then calculated relative to this representative firm, and the lines labelled “large banks” and “small banks” are simple arithmetic averages of these relative efficiency scores for different sets of firms in each quarter. An alternative approach that was not attempted here would be to classify banks ex ante as “large” or “small” for all their duration and then construct representative firms separately for the two classes of bank. However, in this case it would only be possible to compare relative movements in TFP, as their initial relative position would be unknown.

*Figure 9: Total Factor Productivity in large and small banks*




Source : BCL

## 5. Conclusions and Directions for Further Research

This paper implemented the user cost approach to classify banks' balance sheet items as financial inputs or outputs in the intermediation process. The resulting data-driven classification combined elements from both the production and intermediation approach, which are often used to classify banking inputs and outputs on *a priori* grounds. Outputs and inputs were then aggregated separately using Törnqvist indices and a measure of TFP growth in Luxembourg's banking sector was derived. Results suggest that the volume of gross output in this sector grew at an average of 11% and that prices fell about 3% on average over the period. This strong growth reflects substantial gains in productivity, with TFP growth fluctuating cyclically around an annual average of 4%. Analysis within each quarter suggested that the distribution of TFP varies widely across banks and that "large" banks (measured in terms of total assets) tended to be more productive.

Compared to the national accounts series, the Törnqvist output index has the important advantage that it is available at a quarterly frequency. This is useful both for monitoring current developments and for developing forecasting methods. In Luxembourg a timely indicator of banking output would be particularly useful since banking represents about 25% of total value added in the economy. However, the Törnqvist indices presented are *gross* output measures, while the banking sector's contribution to GDP is determined by value added, which is gross output net of intermediate consumption. In fact, the value added concept assumes that production is separable with respect to inputs other than physical capital and labour. It is possible to test this separability assumption using the parameters of the estimated distance functions. Such an analysis could shed further light on the sources of growth in the financial sector.

The TFP measure presented here only provides an appropriate measure of technological progress under several restrictive assumptions. First, firms are assumed to be technically and allocatively efficient, otherwise changes in TFP may actually reflect variations in the average level of efficiency in production. Following Berger and Humphrey (1992), it would be possible to calculate the level of efficiency of individual firms in given quarters using the parameters of the translogarithmic distance function recovered from the system of share equations estimated in the current paper. Stochastic frontier methods could then be applied to evaluate changes in average efficiency from one quarter to the next. Second, if the assumption of constant returns to scale is violated, the Törnqvist measure of TFP may be contaminated by scale effects induced when firms change their volume of production. The estimated parameters of the distance function could also be used to test the hypothesis of constant returns to scale and to correct the TFP measure for scale effects where present. Again, using the estimated parameters it would be possible to calculate Malmqvist indices and their various decompositions for this purpose. Finally, the estimated parameters of the distance function can be used to quantify technical progress as measured by shifts in the average technology represented by the distance function.

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## 7. Annex 1: structural model and system of share equations

This annex describes the structural model used to derive the estimated system of share equations. This model provided an econometric estimate of the opportunity cost of funds $\rho$. The description that follows draws largely from Fixler and Zieschang (1992a) as well as McFadden (1978):

Assume that banks transform the non-financial inputs labour (x1), fixed capital (x2), and purchased materials and services[7] (x3) into financial products. The first of these inputs is measured in number of employees, while the others are in euros. The financial products include Y1 = loans to customers and leases; Y2 = loans to banks; Y3 = government and other fixed income securities; Y4 = equity and other variable income securities; Y5 = directly charged services; Y6 = deposits by banks, securities issued and subordinated debt; Y7 = deposits by customers. All financial products are measured in euros and can be classified either as inputs or as outputs depending on the sign of their user cost.

Let the bank's production technology be characterised by an output distance function D:

$$D(\mathbf{x}, \mathbf{y}) = [\max\{\theta : (\mathbf{x}, \theta\mathbf{y}) \in T\}]^{-1} \tag{6}$$

Where $\mathbf{x}$ is the vector of inputs, $\mathbf{y}$ is vector of outputs, and T is the technology set. Since D is the reciprocal of the factor $\theta$ such that $\theta\mathbf{y}$ is just producible with $\mathbf{x}$, the function D will be bounded above by unity. The classic duality result derived by Shephard demonstrates that if D is convex and increasing in $\mathbf{y}$ then it is dual to the revenue function $\pi(\mathbf{x}, \mathbf{p})$ that can be defined

$$\pi(\mathbf{x}, \mathbf{p}) = \max_y \{ \mathbf{p}'\mathbf{y} : (\mathbf{x}, \mathbf{y}) \in T\}$$

where $\mathbf{p}$ is a vector of known nonzero, nonnegative prices. If one of the outputs in the vector $\mathbf{y}$ is actually an input (meaning that the functions D and $\pi$ have negative derivatives with respect to this element of the vector $\mathbf{y}$) then the profit function $\pi$ can be re-interpreted as the general restricted profit function. McFadden (1978, p.66) proposed the latter as a unified approach encompassing the cost-minimising, revenue-maximising and profit-maximising solutions. Thus the general restricted profit function can represent (as special cases) the revenue, profit or cost functions (the latter with a negative sign). McFadden distinguished the *fixed* from the *variable* outputs and/or (negative) inputs. Hancock (1985, 1986) used the restricted profit function to focus on interest rate and substitution elasticities of financial products. Fixler and Zieschang (1992a) used the obverse of the well-known Shephard-Hotelling lemma that yields the vector of revenue-maximising outputs as $\mathbf{y}^* = \nabla_{\mathbf{p}}\pi(\mathbf{x}, \mathbf{p})$ in order to estimate the vector of shadow prices $\mathbf{p}^* = \nabla_{\mathbf{y}}D(\mathbf{x}, \mathbf{y})$. To do this, they estimated the conditional distance function $D_c$, which is conditioned on the level of deposits. In the present paper, the distance function is conditioned on two classes of deposits: Y6 = deposits by banks and securities issued; Y7 = deposits by customers.

$$D_c = (\mathbf{x}, Y6, Y7; \mathbf{y_c}) = [\max \{ \theta\ D( x, \theta\mathbf{y_c}, Y6, Y7) \leq 1\}]^{-1} \tag{7}$$

where $\mathbf{y_c}$ represents the vector (Y1, Y2, Y3, Y4, Y5) excluding the fixed outputs (deposits). This conditional function is appropriate in the present context as deposits and other liabilities represent accounting inputs (a use of funds) even though they may be a source of financial services output. Since the output distance function is conditional on deposits and liabilities, it generates a system of gross revenue share equations, relating asset receipts to total asset income. Instead, an unconditional distance function would generate a system of net revenue share equations, relating (positive) asset income and (negative) deposit payments to net asset income. From an econometric point of view, this would be problematic because net asset income shares would not be bounded between zero and one, so the results would be more sensitive to random variation in the interest rates that are effectively endogenous variables in the system.

7 Including non-wage administrative costs and commissions paid.

The gross revenue shares can be calculated as follows

$$\omega_i = \frac{(h_i - \rho)y_i}{R_A - \rho A} \qquad i = 1,\ldots,4$$

Where $h_i$ = the holding revenue rate on the ith asset; $\rho$ = the opportunity cost of funds (rate); $R_A = \Sigma_i h_i y_i + R5$ = total asset holding revenue (i=1,...,4) plus directly charged services; R5 = income from services produced other than those associated with asset/liability products; $A = \Sigma_\iota y_i$ = total assets; and $\omega_i = \nabla_{\ln yi} \ln D_c(\mathbf{x}, Y6, Y7; \mathbf{y_c})$. Fixler and Zieschang (1992a, p. 230) assume that the opportunity cost rate $\rho$ is a constant fraction $\phi$ of the total return on assets $r_{TA}$ such that $\phi = \rho/r_{TA}$ and restate these equations with a change of variables as

$$w_i = \phi \cdot s_i + (1 - \phi) \cdot \omega_I \qquad i = 1,\ldots,4 \tag{8}$$

Where $w_i = h_i y_i/R_A$ = the holding revenue share of the *i*th product in overall asset income; $s_i = y_i/A$ = the asset portfolio share of the *i*th product; $r_{TA} = R_A/A$ = the total rate of return on assets including service charge income.

Assuming the conditional distance function is translog, the economic shares $\omega_i$ can be written:

$$\omega_i = \alpha_i + \sum\nolimits_{j=1}^{7} \gamma_{yy,ij} \ln y_j + \sum\nolimits_{k=1}^{3} \gamma_{yx,ik} \ln x_k \qquad i = 1,\ldots,5 \tag{9}$$

The familiar restrictions $\sum\nolimits_{i=1}^{5} \alpha_i = 1$, $\sum\nolimits_{i=1}^{5} \gamma_{yy,ij} = 0$, j=1,…,7; and $\sum\nolimits_{i=1}^{5} \gamma_{yx,ik} = 0$, k=1,2,3 follow from homogeneity of the distance function. Substituting (9) into (8) and appending an error term:

$$w_i = \phi s_i + \mu_i + \sum\nolimits_{j=1}^{7} \psi_{yy,ij} \ln y_j + \sum\nolimits_{k=1}^{3} \psi_{yx,ik} \ln x_k + \varepsilon_i \qquad i = 1,\ldots,4 \tag{10}$$

$$w_5 = \mu_5 + \sum\nolimits_{j=1}^{7} \psi_{yy,5j} \ln y_j + \sum\nolimits_{k=1}^{3} \psi_{yx,5k} \ln x_k + \varepsilon_5 \tag{11}$$

where $\mu_i = (1-\phi)\alpha_i$; $\psi_{yy,ij} = (1-\phi)\gamma_{yy,ij}$; $\psi_{yx,ik} = (1-\phi)\gamma_{yx,ik}$ and the earlier homogeneity restrictions can be restated using this new parameterization. Note that since the shares $w_1$ to $w_5$ must add up to unity, there is a restriction on the stochastic error terms added to the five equations: $\Sigma_i \varepsilon_i = 0$. Equations (10) and (11) represent the system of equations estimated in this paper by iterated seemingly unrelated regression. The covariance matrix of the disturbance terms across the five equations is singular because they are restricted to add up to zero. Therefore one of the five share equations can be dropped arbitrarily and its parameters recovered via the homogeneity restrictions.

## 8. Annex 2: Statistics and correlations

This annex reports the sample statistics for gross output, its implicit price series and for total factor productivity (TFP). In all the following tables, the first five columns are based on the Törnqvist indices constructed using different measures of the reference rate. In the tables for gross output and its implicit price, an additional column refers to the national accounts series. The first row reports the number of observations. The following rows report the mean, median (50% quantile), maximum, minimum and standard deviation of the series. The excess skewness statistic measures the degree of asymmetry of the distribution around its mean and the kurtosis statistic measures peakedness or flatness relative to the normal distribution. Measures of kurtosis below the normal value of 3.0 suggest the possible presence of outliers or "fat tails" in the distribution. In the tables for gross output and its implicit price, the final row reports the linear correlation coefficient with the national accounts measure (over the common sample).

***Table 4: Year-on-Year growth in gross output***

| | $r_{euribor}$ | $r_{REQ}$ | $r_{REQ}$ | $r_{TA}$ | $\rho$ | **National accounts** |
|---|---|---|---|---|---|---|
| Observations | 48 | 48 | 48 | 48 | 48 | 42 |
| Mean | 11.98 | 11.65 | 11.58 | 10.87 | 11.65 | 5.17 |
| Median | 13.96 | 12.51 | 12.78 | 10.33 | 11.65 | 5.32 |
| Maximum | 63.68 | 61.74 | 60.39 | 57.33 | 61.57 | 20.11 |
| Minimum | -18.74 | -18.08 | -17.47 | -16.30 | -17.99 | -9.11 |
| Std. Dev. | 18.73 | 18.11 | 17.71 | 16.50 | 18.22 | 7.29 |
| Skewness | 0.52 | 0.54 | 0.54 | 0.63 | 0.53 | 0.27 |
| Kurtosis | 2.81 | 2.84 | 2.82 | 3.00 | 2.83 | 2.44 |
| Correlation with national accounts | 0.285 | 0.298 | 0.292 | 0.264 | 0.284 | 1.000 |

Source : BCL, STATEC

Table 4 reports the results for year-on-year growth in gross output. The Törnqvist output indices based on the five different reference rates are characterised by a higher mean than the national accounts series. Volatility is also higher as measured by the standard deviation. There is little evidence of asymmetry as the skewness statistic is near zero. However, the kurtosis coefficient is generally less than the normal value of 3, suggesting the possible presence of outliers in the tails of the distribution. The correlation of year-on-year growth in the Törnqvist output indices with the national accounts series varies between 26% and 29%.

**Table 5: Year-on-Year growth in implicit prices of gross output**

| | $r_{euribor}$ | $r_{REQ}$ | $r_{REQ}$ | $r_{TA}$ | $\rho$ | National accounts |
|---|---|---|---|---|---|---|
| Observations | 48 | 48 | 48 | 48 | 48 | 42 |
| Mean | -3.48 | -3.08 | -3.00 | -2.18 | -3.12 | 3.18 |
| Median | -4.87 | -3.56 | -3.07 | -1.37 | -3.64 | 4.53 |
| Maximum | 10.37 | 10.05 | 9.65 | 7.83 | 10.01 | 25.78 |
| Minimum | -16.98 | -16.60 | -16.35 | -13.16 | -16.44 | -15.24 |
| Std. Dev. | 7.50 | 7.24 | 6.94 | 5.61 | 7.17 | 10.53 |
| Skewness | 0.18 | 0.16 | 0.11 | 0.01 | 0.20 | 0.19 |
| Kurtosis | 1.95 | 2.04 | 2.07 | 2.11 | 1.96 | 2.26 |
| Correlation with national accounts | 0.035 | 0.017 | 0.042 | 0.061 | -0.040 | 1.000 |

Source : BCL, STATEC

Table 5 reports the same statistics for the implicit prices of gross output as calculated from the Törnqvist output index using the five different measures of the reference rate and from the national accounts series. Average year-on-year growth rate of the implicit price series is negative for all five measures based on the Törnqvist indices, while it is positive for the national accounts series. The median is below the mean for four of the five Törnqvist series, suggesting some asymmetry in the distribution. In terms of volatility, the situation is reversed compared to the output series, as the standard deviation of all five Törnqvist series is below that of the national accounts series. There is more evidence suggesting the presence of outliers than in the previous table, as the Kurtosis coefficient is closer to 2.0 than to the normal value of 3.0. Finally, the linear correlation coefficients with the national accounts series are much lower than for the gross output series and not significantly different from zero.

**Table 6: Year-on-Year growth in Total Factor Productivity (TFP)**

| | $r_{euribor}$ | $r_{REQ}$ | $r_{REQ}$ | $r_{TA}$ | $\rho$ |
|---|---|---|---|---|---|
| Observations | 48 | 48 | 48 | 48 | 48 |
| Mean | 4.28 | 4.01 | 3.98 | 3.38 | 4.01 |
| Median | 4.71 | 4.21 | 3.51 | 2.25 | 4.41 |
| Maximum | 24.18 | 22.57 | 22.49 | 19.61 | 23.36 |
| Minimum | -12.63 | -14.10 | -13.50 | -12.28 | -14.05 |
| Std. Dev. | 10.26 | 9.72 | 9.43 | 8.26 | 9.86 |
| Skewness | 0.12 | 0.04 | 0.06 | 0.04 | 0.08 |
| Kurtosis | 1.90 | 2.00 | 2.03 | 2.08 | 2.04 |

Source : BCL

Table 6 reports the same statistics for year-on-year growth in Total Factor Productivity calculated from the Törnqvist indices using the five different measures of the reference rate. In this case, we do not have a national accounts series for comparison purposes, since we have no indication of the level of the input of physical capital in the banking sector. Average year-on-year TFP growth is around 4%

for all five measures of the reference rate. It is somewhat lower for $r_{TA}$ (3.38%) and somewhat higher for $r_{EURIBOR}$ (4.28%). The median generally differs from the mean, suggesting some asymmetry in the distribution. The standard deviation varies from 8.26% ($r_{TA}$) to 10.26% ($r_{EURIBOR}$). Skewness is always positive (suggesting a long right tail) but does not seem to be significant. Again there seems to be evidence of platykurtosis, with a coefficient close to 2 suggesting the presence of outliers (fat tails) with respect to the normal distribution.