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THE MISCONCEPTION OF THE OPTION VALUE OF DEPOSIT INSURANCE AND THE EFFICACY OF NON-RISK-BASED CAPITAL REQUIREMENTS IN THE LITERATURE ON BANK CAPITAL REGULATION

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THE MISCONCEPTION OF THE OPTION VALUE OF DEPOSIT INSURANCE AND THE EFFICACY OF NON-RISK-BASED CAPITAL REQUIREMENTS IN THE LITERATURE ON BANK CAPITAL REGULATION [*]

Paolo Fegatelli [†]

Abstract: This study shows how the misconception of the *option value of deposit insurance* by Merton (1977) and its later misuse by Keeley and Furlong (1990), among others, have led some literature supporting the adoption of binding non-risk-based capital requirements to derive incorrect conclusions about their efficacy. This study further shows that what Merton defines as the *option value of deposit insurance* is actually a component of a bank's limited liability option under a third-party deposit guarantee. As such, it is already included in the value of the bank's equity capital, and the flawed definition makes the Keeley-Furlong model internally incoherent.

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Résumé non-technique

Au cours des 20 dernières années, le monde académique a adopté plusieurs approches différentes dans l'analyse de l'efficacité des ratios de solvabilité bancaire forfaitaires. Plusieurs voies ont été explorées : l'information asymétrique (aléa moral et antisélection), l'hétérogénéité des banques et les implications systémiques (par exemple, la procyclicité). Il y a lieu de noter que, parmi ces voies, l'approche de la gestion de portefeuille (qui envisage les intermédiaires financiers comme gérant un portefeuille) a été abandonnée, l'attention se concentrant avant tout sur les rapports de solvabilité sensibles au risque, dans le cadre du débat sur la mise en place de Bâle II. Un tel changement de perspective a néanmoins eu lieu sans une déclaration définitive sur l'efficacité des ratios forfaitaires. En conséquence, bien avant l'explosion de la crise des subprimes en 2007, de nombreux universitaires et praticiens s'étaient déjà interrogés sur le bien-fondé des critères de Bâle II et sur l'opportunité de remplacer les règles complexes de Bâle II par d'autres principes "naturels" ou *de bon sens* basés sur des critères plus simples. Dans ce contexte, ce travail prouve l'incohérence interne de Keeley et Furlong (1990), et revalide implicitement les conclusions de Koehn et Santomero (1980) : un ratio de solvabilité plus élevé non basé sur le risque, considéré isolément, pourrait se révéler contreproductif pour la prévention du risque systémique et de solvabilité des banques. Une implication majeure est qu'indépendamment de l'efficacité des réformes en cours, l'adoption de règles basées dans une moindre mesure sur le risque des actifs ferait fausse route.

D'un point de vue théorique, cette analyse aspire à clarifier la définition de *valeur d'option de l'assurance-dépôt*, introduite par Merton (1977), ainsi que sa relation avec l'option de responsabilité limitée des banques. Cet exercice de clarification s'avère nécessaire puisque la littérature concernant la réglementation des capitaux a largement - et souvent incorrectement - utilisé le concept de "valeur d'option de l'assurance-dépôt". Ainsi, des économistes tels que Keeley, Furlong et d'autres, ayant construit leur approche sur celle de Merton, ont été d'une certaine façon induits en erreur par le caractère flou de la définition. Pourtant, ce travail montre que la définition de la *valeur d'option d'assurance-dépôt* donnée par Merton est doublement ambiguë. Premièrement, ce que Merton définit comme "la valeur d'option de l'assurance-dépôt" est en réalité une composante de "l'option de

responsabilité limitée" des actionnaires sous une garantie de dépôt par un tiers. Ces deux concepts sont distincts quoiqu'étroitement liés, comme l'explique pleinement cet article à la lumière du travail original de Merton. Deuxièmement, Merton n'est pas tout à fait clair sur les bénéficiaires ultimes d'une garantie d'assurance-dépôt par un tiers : ces bénéficiaires sont-ils les actionnaires de la banque ou les déposants/créanciers? Ainsi, il importe de considérer explicitement la prime de risque demandée par les déposants/créanciers, pour pouvoir conclure avec Merton que "le résultat de la garantie est de créer un afflux supplémentaire de liquidité à la firme [bancaire]" (donc, aux actionnaires de la banque).

1. Introduction

One of the most prominent debates within the existing literature on capital regulation relates to how financial intermediaries act as portfolio managers. In that vein, this paper tackles the question of how effective *"flat," non-risk-based* capital requirements are for insured banks that are von Neumann-Morgenstern utility maximizers in the context of complete, contingent claims markets without possibilities of arbitrage.

Over the past 20 years, scholars have adopted several different approaches to answer this question,[1] hinging upon a variety of issues ranging from asymmetric information (moral hazard and adverse selection) to problems of bank heterogeneities and systemic implications (e.g., procyclicality).[2] Interestingly, scholars abandoned the portfolio management approach as the focus shifted from "flat" to risk-based capital ratios, which was certainly induced by the overheating debate on the complexities and potential drawbacks of Basel II[3] at the turning of this century. Such a change of perspective took place without a conclusive statement on the efficacy of those simpler, non-risk-based capital requirements. As a consequence, well before the outburst of the 2007's subprime crisis many scholars and practitioners had put in question whether the full implementation of Basel II would have been a real achievement[4], or even if it would have been more opportune to step back to some kind of "natural" or

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[1] See for instance the classification outlined in VanHoose (2006) as well as the earlier literature review in Santos (2001).

[2] See—in chronological order—Flannery (1989), Besanko and Kanatas (1993), Boot and Greenbaum (1993), Thakor (1996), Blum (1999), Santos (1999), Milne (2002), Kopecky and VanHoose (2004a, 2004b), Jeitschko and Jeung (2005), and Kopecky and VanHoose (2006), among others. About the moral hazard problem related to deposit insurance and government safety nets, see the works of Demirgüç-Kunt and Detragiache (2002), Demirgüç-Kunt and Huizinga (2004), Nier and Baumann (2006), Hoggart *et al*. (2005), Pennacchi (2006), Huizinga and Nicodème (2006) as well as the earlier study of Avery and Berger (1991)

[3] Basel Committee on Banking Supervision (2006).

[4] See the interesting paper of Decamps et al. (2004) for a suggestion on how to rebalance the three pillars (capital ratios, supervisory review, and market discipline) of the 2006 proposal. See also the ultimate implication of the extensive literature review in VanHoose (2006), which affirms that "the intellectual foundation for bank capital regulation in general and for the proposed Basel II system specifically is not particularly strong" (p. 51). Another proposal to reform Basel II – with regard to the value-at-risk approach – comes out of the analysis of Alexander and Baptista (2006).

common-sense rule based on simple criteria and non-risk-based capital requirements.[5] In this context, this paper proves the internal inconsistency of Keeley and Furlong (1990; "KF" hereafter), thus implicitly revalidating the conclusions of Koehn and Santomero (1980; "KS" hereafter): *higher non-risk-based capital requirements when taken alone might even be counterproductive in preventing bank credit and systemic risk*. A major implication is that regardless of how effective the ongoing reforms are, less risk-based capital regulation would probably be a step back into the wrong direction.[6]

From a theoretical point of view, this study also aims to clarify the definition of *option value of deposit insurance*, first introduced by Merton (1977; "M" henceforth), and its relation with banks' limited liability option. In particular, the literature on capital regulation has widely—and often improperly—used the concept of "option value of deposit insurance" so that not only KF, but others[7] who also built on M's approach were somehow misled by its fuzzy definition. The resulting paradox is that although today the M model is amended and more coherent from a credit-risk-modeling point of view[8], it is still accepted and "used" ambiguously in the debate on banks' microeconomic behavior after the imposition of higher non-risk-based-capital requirements.

Hence, this study shows that M's definition of *option value of deposit insurance* is substantially unclear in two respects. First, what M defines as the 'option value of deposit insurance' is actually a component of the shareholders' "limited liability option" under a third-party deposit guarantee. The two concepts are different though strictly related, and this study fully explains their relationship by reassessing M's original work. Second, M is unclear about who really benefits from a third-party deposit insurance guarantee: the bank shareholders or the depositors (bondholders). Thus, only when we explicitly consider the risk premium required by the depositors/bondholders we can agree with M

[5] Lannoo (2001). Given these premises, it should not come as a surprise that the new rules enclosed in the proposals being finalized by the Basel Committee later in 2010 (known as 'Basel III') have already been defined as potentially "catastrophic" by bankers and non-bank business leaders, who claim that such rules might substantially reduce the ability of banks to lend to industry.

[6] Indeed, for KF non-risk-based regulation taken alone is sufficient to decrease overall risk, whereas for KS it is not. Thus, risk-based regulation becomes a possible corrective action in KS's framework, but it is useless in KF's view.

[7] See for instance the works of Sharpe (1978), Kareken and Wallace (1978), Dothan and Williams (1980), and Furlong and Keeley (1989).

that “the result of the guarantee is to create an additional cash inflow to the [banking] firm” (namely, the bank's shareholders).

This paper is organized as follows. Section 2 recalls the works of KS and KF in order to focus on one critical, unclear point of KF’s framework. Section 3 proves a link to an inconsistency in M's work. A solution to M’s contradiction is in Section 4. Finally, sections 5 and 6 demonstrate unequivocally that the KF model is incoherent because the option value of deposit insurance is already included in the value of the bank’s equity capital—directly contradicting KF’s definition.

2. The controversy on the effectiveness of non-risk-based capital regulation

The model of KS is based on a classical (i.e., Markowitz-Tobin) portfolio analysis with the further constraint of a regulatory capital requirement. The goal is to analyze a bank’s portfolio allocation after the imposition of more binding non-risk-based capital-adequacy rules. Such a portfolio allocation determines the probability of bank failure, as measured by an indicator[9] considered a precursor of value at risk. The results undermine the efficacy of a supervisory policy centered on the (exclusive) use of non-risk-based capital requirements. In fact, KS find that as the capital constraint increases, the bank's probability of failure might increase or decrease depending on the bank’s degree of risk-aversion (specifically, on the coefficient of relative risk aversion of that bank’s utility function).[10] At a systemic level, the final distribution of failure risk for the banking industry is more dispersed after the imposition of a higher capital-asset ratio, though the industry mean depends upon the underlying distribution of risk aversion.

[8] See Altman et al. (2002) for a review of the related literature.

[9] Derived from Roy (1952).

[10] This happens because for any bank the degree to which the portfolio reshuffling occurs is dependent upon the relative risk-aversion coefficient of the respective utility function. For highly risk-averse institutions, the elasticity value of high-risk assets is less than the elasticity for other institutions possessing less risk aversion.

In general, the literature has used a mean-variance framework to analyze the effects of non-risk-based capital regulation on the asset and bankruptcy risk of insured, *utility-maximizing* banks.[11] Other authors have reached opposite conclusions by considering insured, *value-maximizing* banks.[12] The work of KF is perhaps the most notable example of this second strand of literature, though in reality KF uses all the assumptions KS adopted plus an extra one (i.e. the existence of an option value of deposit insurance, priced à la Black-Scholes-Merton).[13] KF show that when deposit insurance underprices risk, banks seeking to maximize the value of their stockholders' equity will attempt to maximize the value of the insurance subsidy by increasing their asset risk and leverage. Formally, KF starts by expressing the expected gross return on capital in two equivalent forms:

$$E(Z) = \int_{P*}^{\infty} \left[\frac{A_0 P - L_0 R}{K_0} \right] f(P) dP \ , \tag{1a}$$

$$E(Z) = \left\{ \left[\frac{A_0}{K_0} \right] \left[E(P) - R \right] + R \right\} + \int_{-\infty}^{P*} \left[\frac{L_0 R - A_0 P}{K_0} \right] f(P) dP \, , \tag{1b}$$

where

$E(Z)$: gross expected return on capital, which equals $1+E_p$, where E_p is the expected rate of return per unit of capital in the bank portfolio of assets and liabilities;

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[11] Kahane (1977), Blair and Heggestad (1978), Koehn and Santomero (1980), Kim and Santomero (1988), and Rochet (1992), all use utility maximization models to show that an increase in the required equity-to-asset ratio might either increase or decrease the portfolio risk chosen by a bank.

[12] Apart from those already mentioned in note 6, see the earlier works of Mingo (1975) and Mingo and Wolkowitz (1977), both of which use a deterministic framework. Differently from the conclusions of other authors using a value-maximizing approach, Gennotte and Pyle (1991) find that also value-maximizing banks may actually increase portfolio risk and the probability of failure, if bank investments are subject to decreasing returns to investment.

[13] Indeed, KF describe a framework that is common also to other authors adopting a utility-maximizing approach, such as Kahane (1977) and Kim and Santomero (1988), including the two basic assumptions that a minimal form of capital regulation is in place (a bank owner must invest his or her entire net worth in the bank), and the deposit insurance premium is zero.

K_0 , L_0 , A_0=K_0+L_0 : initial capital endowment / liabilities (deposits) / assets at time zero[14];

P : gross return on the bank's optimal portfolio of assets (P=1+$\tilde{r}$, where $\tilde{r}$ is the stochastic rate of return on assets);

R : gross return on liabilities (R=1+r, where r is the riskless rate of return);

$f(P)$: the probability density function of P, supposed to be normal;

$P^* = [L_0/A_0]R$: the lowest asset return for which depositors are repaid in full.

Equation (1a) indicates that the expected gross return on capital is the expected value of gross asset returns minus liability obligations, *conditional on nonbankruptcy*. Under the equivalent (1b), the first term represents the formulation of KS, and the second term represents the expected value—conditional on bankruptcy—of the obligations to depositors in excess of returns on assets, per dollar of invested capital (by definition, such obligations are positive in each bankruptcy state, because if $P < P^*$, then $L_0R - A_0P > 0$). According to KF, this second term in equation (1b) corresponds to the option value of deposit insurance as described by M.[15] *The KS analysis neglects this term.*

Analogously to M, KF also use the Black-Scholes option pricing model to analyze the effects of the option value of deposit insurance on banks' portfolio leverage and bankruptcy risk. KF conclude that when a bank takes into account the option value of deposit insurance, its unconstrained risk-return frontier is not linear anymore; it becomes infinitely convex because the expected return on capital is the sum of the expected return posited by KS plus the expected return of the option. In turn, this implies

[14] Notice that A_0 indicates just the initial level of the *total amount* of risky assets, without regard to the internal composition of the asset portfolio, because this problem has been already solved previously (portfolio separation). Thus, the internal structure of the bank's asset portfolio is supposed to be optimal, and the question now reduces to how much levering the optimal asset portfolio involves.

[15] According to M, the option value of deposit insurance corresponds to the expected cost borne by the insurance agency (the guarantor) to protect the depositors' funds. Notice that M does not really clarify whether a deposit insurance guarantee benefits the bank's shareholders and/or the depositors (we will make this point clear in Section 3). In general, all the value-maximizing literature on capital regulation has commonly assumed the first hypothesis, namely that the bank's owners achieve the financial advantage of the guarantee, in order to demonstrate that binding capital rules prevent banks' preference for an infinite degree of leverage.

that banks with an absolute risk-aversion index below a critical threshold[16] will prefer an infinitely leveraged portfolio,[17] making some form of overall capital-adequacy rules absolutely necessary to limit those banks' failure risks. KF claim that the utility-maximization literature does not support their conclusions regarding the effects of bank capital regulation, because neglecting the option value of deposit insurance mischaracterizes a bank's investment-opportunity set.[18]

To summarize the conclusions of KF, a bank can attract deposits at a fixed, promised deposit rate unrelated to the bank's underlying risk thanks to the deposit insurance guarantee, which is ultimately responsible for a twofold nonlinearity between expected return and leverage, and between risk and leverage. In reality, equation (1a) says something different: that the expected gross return on capital is equal to the expected value of gross asset returns minus liability obligations, *conditional on nonbankruptcy*. In effect, if bankruptcy occurs ($P < P^*$), KF simply assume that the gross return on capital is zero.

Thus*, KF mix the effects of the shareholders' limited liability with the effects of the deposit insurance clause*. As a matter of fact, it is not necessary to assume deposit insurance (as KF do) in order to have a censored distribution of gross returns on capital. The event "zero return on capital" is conditional on bank failure, not on deposit insurance.[19] Banks can fail even when their deposits are insured, and bankruptcy has the same drastic consequences for managers and shareholders in any case, regardless of deposit insurance.[20]

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[16] Such a critical threshold corresponds to the value by which the convexity of the bank risk-return frontier becomes greater than the convexity of the bank utility function. Notice that KF does not explicitly make this point; rather, they just assert that "risk-aversion alone will not necessarily be sufficient to limit leverage and asset risk" (p.82).

[17] This is equivalent to saying that with an infinitely convex risk-return frontier the "market capital requirements" (Berger et al., 1995) for the bank are equal to zero.

[18] Contrary to KS, KF also assert that the expected net marginal cost (expected interest cost plus an assumed fixed-rate premium) of deposits to the bank declines as the quantity of deposits increases, because the option value of the deposit guarantee increases as leverage increases. Thus, KS would confuse the expected cost of deposits with the promised return to depositors under situations where the probability of default is zero.

[19] We implicitly assume that the bank's owners have a limited liability in case of failure.

[20] ...or the implementation of a "too-big-to-fail" policy, for example. It is true that a too-big-to-fail policy may permit the bank's "good" assets, premises, operating franchises, and the bank as a corporation to survive the failure. However, even in this case the bank's owners would lose any claim to future profitability, because the capital they invested in the bank's shares would be wiped out anyway and managers would probably lose their jobs.

In sum, from this point of view, the deposit insurance guarantee is a contingent claim that changes the depositors' expected return, but it does not directly benefit the managers or the owners, except for the constant riskless rate that the bank pays for its funding under such a provision.[21] Because apparently the second term in (1b) stems primarily from the possibility of bankruptcy rather than from deposit insurance, an analogous quantity could be found also in a framework in which deposit insurance is absent but the possibility of a bank failure is allowed.[22] This makes it necessary to specify three things:

1. The conceptual difference between the definitions of *option value of deposit insurance* and *limited liability option value*;
2. The economic relationship linking the two option values;
3. The net impact of the two options on the bank's risk-return frontier.

3. Who benefits from a deposit insurance guarantee?

The overlap of the option value of deposit insurance and the limited liability option traces back to M's well-known work on the cost of deposit insurance and loan guarantees. Although the cost of a deposit insurance guarantee for a third-party guarantor is correctly estimated by applying the option pricing theory, what is not clear in M's description[23] of the working of a loan guarantee (analogous to a deposit insurance guarantee for a bank) is who really benefits from the same guarantee: the shareholders—as finally asserted by the author—or the creditors—as the model itself describes. Cutting the knots of this question is extremely important to identify the ambiguity of the analysis in KF.

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[21]In the absence of deposit insurance, the marginal funding costs are usually assumed to be nonlinearly increasing. Note that in this analysis we do not consider any possible implications related to the managers' "moral hazard" that may arise as a consequence of introducing a deposit insurance guarantee.

[22]The only difference in this case would be that the return to depositors, R, would be an increasing function of the level of leverage (assuming that markets are efficient and depositors are risk-sensitive). See Gollier *et al*. (1995) for an analysis of the consequences of the limited liability option on the risk-taking behavior of a rational decision-maker with a concave utility function.

[23] See M, pp. 6-7.

M considers a "simple model of a firm that borrows money by issuing a single homogeneous debt issue"[24] that does not require any interim or coupon payments (term discount issue). The firm agrees to pay the bondholders the amount $L_0 \cdot R$ at maturity, and in the event of default, the bondholders take over the firm's residual assets, which are now worth $A_0 \cdot P < L_0 \cdot R$. Thus, at maturity, the value of the debt is Min[($A_0 \cdot P$), ($L_0 \cdot R$)], and the value of the firm's equity is equal to Max[0, ($A_0 \cdot P$) – ($L_0 \cdot R$)].

However, under a third-party guarantee of the payment to the bondholders, the maturity value of debt changes. "The terms of the guarantee are that in the event the management does not make the promised payment to the bondholders, the guarantor will meet these payments"[25] without any uncertainty, so that now the bondholders will always receive $L_0 \cdot R$ at maturity. The cost for eliminating the risk is entirely borne by the guarantor, who has a net payout or loss of [($L_0 \cdot R$) – ($A_0 \cdot P$)] in case of insolvency by the firm. In other words, the guarantor issues a contingent claim with a maturity value of –Min[0, ($A_0 \cdot P$) – ($L_0 \cdot R$)] *in favor of the bondholders*. Table 1 illustrates an *ex-ante* evaluation of the scheme of payoffs at maturity with and without a deposit insurance guarantee.

Table 1. Bank shareholders' and bondholders' payoffs, according to Merton (1977).

Maturity value:	*For the shareholders*	*For the bondholders*	*For the guarantor*
Without a third-party guarantee:	Max[0, ($A_0 \cdot P$)–($L_0 \cdot R$)]	Min[($A_0 \cdot P$), ($L_0 \cdot R$)]	0
With a third-party guarantee:	Max[0, ($A_0 \cdot P$)–($L_0 \cdot R$)]	$L_0 \cdot R$	Min[0, ($A_0 \cdot P$)–($L_0 \cdot R$)]
Direction of change for the expected value at maturity:	=	↑	↓

The situation presented in Table 1 shows that only the bondholders profit from a third-party guarantee. Nonetheless, M concludes that "(i)n effect, the result of the guarantee is to create an

[24] *Ibid*., p. 6. Note that in this analysis nothing would change if we substituted the words *firm* and *debt* with the words *bank* and *deposit* respectively. To keep things clear, in our exposition we retained the same notation adopted by KF.

[25] *Ibid*., p. 7.

additional cash inflow to the firm" of $-\text{Min}[0, (A_0 \cdot P) - (L_0 \cdot R)]$ dollars,[26] where "the firm" clearly means "the shareholders."[27]

We can follow two different paths now. First, we can suppose that the guarantor's contract does not benefit bondholders, but only the shareholders, who simply add the option value of deposit insurance to the value of their equity capital. As we show in the Appendix, without a theoretical justification, this assumption is merely an academic hypothesis whose consequences just evidence the contradictions it generates when placed in the KF framework.

On the other hand, Section 4 illustrates a scenario that rationalizes M's assertion that "the result of the guarantee is to create an additional cash inflow to the firm." We will then derive the implications for correct definitions of *option value of deposit insurance* and *limited liability option*, as well as how they really affect the expected gross return on bank capital. This theoretical setting, however, confutes irremediably the analysis of KF.

4. Deposit insurance as a subsidy to banks

Some "ideal" conditions underlie M's analysis. Two major assumptions shared with the literature relating to the option pricing theory are market completeness and the absence of arbitrage.[28] They in turn assume the validity of the efficient-market hypothesis, because a "perfect" market (implied by market completeness and absence of arbitrage) is also "fully efficient." Notice that the absence of informational asymmetries between bank managers and depositors/bondholders is an assumption (again, not made explicit by the author) that contradicts the *raison d'être* of a deposit guarantee: protecting depositors against

[26] *Ibid.*

[27] M's subsequent financial evaluation of the cost of this guarantee to the guarantor does not help to solve our dilemma. As M does, we purposely leave out any consideration concerning the managers' "moral hazard" that could spring from the introduction of a third-party guarantee, associated with a relaxation of the activity addressed to monitor the risks incurred by the same managers.

[28] See Black and Scholes (1973), Cox and Rubinstein (1985), and Dothan (1990) among others. In turn, such assumptions imply that the market is perfect (i.e., there are no taxes, no transactions costs, and no short-selling restrictions; assets are infinitely divisible; and all market participants have perfect information about the state dependent cash flows of all assets.

incomplete information concerning the financial health of the bank.[29] Nevertheless, when we accept this framework as a first approximation to reality, that is, in a quasi-fully efficient world,[30] then we can derive a solution to the issue of who really benefits from a deposit insurance guarantee. In effect, *under these conditions, the deposit insurance guarantee turns out to be a subsidy to the banks* (according to M's conclusive assertion), whereas depositors are left indifferent in terms of expected utility.

This statement can be proved as follows. Differently from the scheme described by M, suppose that in the absence of a loan guarantee, at maturity the bondholders require the risk premium, RP, in addition to the certainty equivalent, $L_0 \cdot R$, of the amount lent at time zero, L_0. Given the efficient-market hypothesis, RP has to be such that at time zero the bondholders' expected utility of $\mathrm{Min}[(A_0 \cdot P), (L_0 \cdot R)+RP]$ equals the utility of $L_0 \cdot R$ under the condition of certainty:

$$E[U(X)] = U(L_0 \cdot R)\,, \tag{2a}$$

with

$$X = \begin{cases} (L_0 \cdot R) + RP\,, & for\,(L_0 \cdot R) + RP < A_0 \cdot P\,, \quad with\,\Pr[(L_0 \cdot R) + RP] = q\,, \\ \\ A_0 \cdot P \quad , & for\,(L_0 \cdot R) + RP > A_0 \cdot P\,, \quad with \quad \Pr[A_0 \cdot P] = 1 - q \quad . \end{cases}$$

Or assuming, for example, a binomial distribution for P, with $A_0 \cdot p_u \geq (L_0 \cdot R)+RP$, and $A_0 \cdot p_d < (L_0 \cdot R)+RP$:

$$qU(L_0 \cdot R + RP) + (1-q)U(A_0 \cdot p_d) = U(L_0 \cdot R)\,, \tag{2b}$$

where $U(\cdot)$ indicates the utility function. Table 2 shows the *ex-ante* evaluation of the scheme of payments at maturity with and without the guarantee if the bondholders are risk-averse.

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[29] The current scarcity of liquidity in many financial markets following the subprime crisis makes this observation (already highlighted by Rochet, 1992) even more relevant, as markets have never been so far from being frictionless and efficient.

Table 2. Bank shareholders' and bondholders' payoffs, with bondholders' risk aversion.

Maturity value:	*For the shareholders*	*For the bondholders*	*For the guarantor*
Without a third-party guarantee:	Max[0, $A_0 \cdot P-(L_0 \cdot R+RP)$]	Min[$A_0 \cdot P$, $(L_0 \cdot R)+RP$]	0
With a third-party guarantee:	Max[0, $(A_0 \cdot P)-(L_0 \cdot R)$]	$L_0 \cdot R$	Min[0, $(A_0 \cdot P)-(L_0 \cdot R)$]
Direction of change for the expected value at maturity:	↑	↓	↓

In a perfectly efficient market, the risk premium *RP* requested by the bondholders/depositors fully reflects the level of risk incurred, so that *a priori* the payoffs to the bondholders in both cases lie on the same indifference curve (equations [2a]-[2b]). However, in the presence of the guarantee the return of the same "quantity" of bonds is lower, because we assume the bondholders are risk-averse and cannot "beat the market." This means that with the guarantee, the bondholders require only the riskless rate, giving up for free the part of the risk premium *RP* related to their risk-aversion.[31] Given the concavity of the bondholders' utility function, the expected value of their "uninsured" payoff Min[$(A_0 \cdot P)$, $(L_0 \cdot R)+RP$] is expressed by the following relation, where ε is the positive default-adjusted risk premium:

$$E[X] = (L_0 \cdot R) + \varepsilon \ , \qquad (3a)$$

with

[30] It can be proved that the classical option pricing is "on average" true, even given liquidity risk in the framework of imperfect markets. See, for instance, Jarrow (2007).

$$X = \begin{cases} (L_0 \cdot R) + RP\ , & for\,(L_0 \cdot R) + RP < (A_0 \cdot P)\ , \quad with\,\Pr[(L_0 \cdot R) + RP] = q\ , \\ (A_0 \cdot P) \quad , & for\,(L_0 \cdot R) + RP > (A_0 \cdot P)\ , \quad with\ \Pr[(A_0 \cdot P)] = 1 - q\ \ . \end{cases}$$

For a binomial distribution of P, we have:

$$q(L_0 \cdot R + RP) + (1-q)(A_0 \cdot p_d) = L_0 \cdot R + \varepsilon\text{, with } A_0 \cdot p_d < L_0 \cdot R. \tag{3b}$$

This explains the reduction in the bondholders' payoff in Table 2. Analogously, under full market efficiency and depositors' risk-aversion, a guarantee on deposits *a priori* decreases the expected value of deposits, because in this case they are not assumed to earn any risk premia. Thus, the shareholders' advantage from a third-party (deposit insurance) guarantee is twofold, as they receive:

1. The *direct* subsidy –Min[0, ($A_0 \cdot P$)–($L_0 \cdot R$)]≡Max[0, ($L_0 \cdot R$)–($A_0 \cdot P$)] from the guarantor and
2. The *indirect* subsidy ε from the bondholders/depositors.

The sum of the two subsidies forms the limited liability option *for the bank's shareholders under a third-party guarantee of deposits*:

$$Max[0, A_0P - L_0R] - Max[0, A_0P - (L_0R + RP)] = Max[0, L_0R - A_0P] + \varepsilon\,. \tag{4}$$

Notice that the guarantor's subsidy –Min[0, ($A_0 \cdot P$)–($L_0 \cdot R$)]≡Max[0, ($L_0 \cdot R$)–($A_0 \cdot P$)] corresponds to the final payoff of the put option KF defined as "the option value of deposit insurance" (equation 1b): under a third-party deposit guarantee, this value characterizes the opposite of the cost of the deposit insurance guarantee to the guarantor, but not the total benefits of the limited liability option to the bank's shareholders. In fact, $Max[0, L_0R - A_0P]$ is equal to the limited liability option for the shareholders only when the default-adjusted risk premium ε is zero—namely, when the

[31] It is as if the same bondholders paid an insurance premium to be 100% sure to receive the amount (L_0•R) at maturity.

bondholders/depositors are risk-neutral. With the exception of this special case, the option value of deposit insurance and the limited liability option value diverge, as shown in Table 3.[32]

Table 3. Relationship between option value of deposit insurance and limited liability option.

Default-adjusted risk premium	*Depositors' attitude*	*Relationship between option value of deposit insurance and limited liability option*
$\varepsilon > 0$	Risk-averse	Limited liability option > Option value of deposit insurance
$\varepsilon = 0$	Risk-neutral	Limited liability option = Option value of deposit insurance
$\varepsilon < 0$	Risk-lover	Limited liability option < Option value of deposit insurance

Because $Max[0, L_0R - A_0P] \geq 0$ by definition, we have now delineated a setting in which M's assertion that "the result of the guarantee is to create an additional cash inflow to the [banking] firm" can be justified in spite of his previous analysis and independently from any consideration on the managers' moral hazard and the level of risk-aversion of the bondholders/depositors.

5. A reformulation of the model of Keeley and Furlong (1990)

In the model proposed in Section 4, we can rewrite equation (4) to isolate the payoff to the shareholders under a third-party guarantee:

$$Max[0, A_0P - L_0R] = Max[0, A_0P - (L_0R + RP)] + Max[0, L_0R - A_0P] + \varepsilon . \qquad (5)$$

The same relationship can be rewritten as:

[32] This model could be easily adapted to the case of a market with informational asymmetries between bank

$$Max[0, A_0P - (L_0R + RP)] = A_0P - L_0R - \varepsilon\,. \tag{6}$$

Plugging (6) in (5), we obtain:

$$Max[0, A_0P - L_0R] = A_0P - L_0R + Max[0, L_0R - A_0P]\,. \tag{7}$$

It is easy to verify that equation (7) is just another formulation of the model proposed by KF (see equations [1a]-[1b]). The only difference is that equation (7) shows a firm's payoff to its shareholders under a third-party guarantee *at maturity*, whereas in equation (1b) KF consider the *expected value* of the same payoff per unit of capital *ex-ante*. To see this, observe that equation (7) corresponds to the final payoff of a call option whose value can be expressed in terms of the put-call parity:

$$C_1^E[A_1, L_1] = A_0 - L_0 + P_1^E[A_1, L_1], \tag{8}$$

where

A_t , L_t : bank assets / liabilities at time t , with $t = 0{,}1$;

$C_1^E[A_1, L_1]$, $P_1^E[A_1, L_1]$: European call / put option on $A_1 = A_0P$ (with $p \in P$, $0 \le p < +\infty$), at a strike price equal to $L_1 = L_0R$, and maturity $t = 1$;

Ex-ante, at time zero, the expected value of (8) at maturity $t = 1$ is:

$$\int_{P^*}^{\infty} [A_0P - L_0R] f(P)dP = A_0E(\widetilde{P}) - L_0R + \int_{-\infty}^{P^*} (L_0R - A_0P) f(P)dP\,. \tag{9}$$

depositors and shareholders if we assume that ε is a function of the degree of market (in)efficiency.

Dividing the two members of (9) by the amount of bank's own capital, $K_0 = A_0 - L_0$, produces the expected gross return on capital of equations [1a]-[1b]:

$$\int_{P*}^{\infty}\left[\frac{A_0 P - L_0 R}{K_0}\right] f(P)dP = E(Z) = \left\{\left[\frac{A_0}{K_0}\right]\left[E(\widetilde{P}) - R\right] + R\right\} + \int_{-\infty}^{P*}\left[\frac{L_0 R - A_0 P}{K_0}\right] f(P)dP \quad (10)$$

Thus, equations (7), (8), and (10) are just three alternative ways to formulate the same relationship. Indeed, this confirms that *the KF model implies the assumptions of the absence of arbitrage and market completeness,* which characterize the classical option pricing theory.[33] Although KF do not make such assumptions explicitly, it is clear that their work can be criticized upon this ground.

6. Conclusions

Based on the results in Section 5, we can now prove our initial statements.

First, when the bank's capital is modeled as a call option on the bank's assets at a strike price equal to the value of the bank's liabilities ($C_1^E[A_1, L_1]$, see equations [7]-[8]-[10]), we observe that *the option value of deposit insurance is already included in the value of the call option (the equity capital) and does not have to be added to the latter*.

Second, in equilibrium the call option $C_1^E[A_1, L_1]$ equals the value of the equity capital at time zero.[34] KF define this value as $K_0 = A_0 - L_0$, but such a definition contradicts equation (8)—the put-call parity—which shows that the value of the call option $C_1^E[A_1, L_1]$ is equal to $K_0 + P_1^E[A_1, L_1]$. In effect, in complete markets without arbitrage, bank shareholders should not be able to trade their call

[33] In fact, KF make use of the Black-Scholes formula when they estimate the option value of deposit insurance.

[34] This assertion is proved in Black and Scholes (1973). Many others have followed the path-breaking approach of Black and Scholes to model the claims of a firm's stockholders: see for example Jensen and Meckling (1976).

option at a different price from the cost of the portfolio that replicates the payoffs from the same call option. *This contradiction undermines the whole analysis of KF*.

An open issue concerns the solution of such a contradiction. In fact, proving the inconsistency of KF's theoretical setup does not reduce the problem to the original form proposed by KS. KF have at least one important merit: the recognition of the skewness of bank return distributions, because a bank is ultimately worth something different from the mere sum of its assets. In other words, the value of a bank to its shareholders does not simply equal the value of its portfolio of assets and liabilities. Rather, the probability that a bank will go bankrupt "on their watch" is also of crucial importance.[35]

[35] In another work (Fegatelli, 2010), we show that one obvious way to solve this puzzle is by adopting an intertemporal approach in which the possibility of bankruptcy emerges any moment during the life of the bank, not just at some arbitrary "maturity date." This involves redefining the risk-return frontier of a bank's asset portfolio in each period within the shareholders' time horizon. Also in this context, however, the efficacy of non-risk-based capital adequacy rules is rather questionable.

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Appendix

Adding KF's option value of deposit insurance to the option value of the bank's equity capital

Following Merton's statement that "the result of the [deposit insurance] guarantee is to create an additional cash inflow to the firm", in this appendix we assume that the bank's asset portfolio includes the option value of deposit insurance for free, *à la* KF (the other basic hypotheses being the same as in KF, i.e. market completeness and absence of arbitrage). Differently from KF, however, in this case we introduce the possibility of a bank default *before* the expiration date of the option (corresponding to the time horizon of the shareholders).

We will prove that:

1. in equilibrium, the option value of deposit insurance is more than offset by the related cost of default;
2. in particular, the intrinsic value of the deposit insurance put option is always worth half of its cost to the bank (namely, the cost of default);
3. consequently, *in this setting*, while the analysis of KF is incomplete because does not take account of such a bankruptcy cost, *ultimately banks should not exist*, as there would be no incentive to leverage any portfolio of assets (that is, only mutual funds – with no liabilities – should exist).

We can prove the existence of the bankruptcy cost as follows. Similar to Black and Scholes (1973) for a generic firm in a single-period horizon, we model the equity of the bank as one non-standard American call option on the bank's assets at a strike price equal to the promised maturity value of the bank's liabilities (an analogous proof – available from the author upon request – can be offered by modeling the equity of the bank as a portfolio of European call options on the bank's assets at a striking price equal to the promised maturity value of the bank's liabilities). Then, the market value of bank equity must be equal to the value of this call option *plus* (according to KF) the value of the put option related to the possibility for the bank – in case of default – "to put the bank's assets to the insuring agency at a striking price equal to the promised maturity value of its liabilities" (KF, p. 78). In our case, this put option can be modeled as one non-standard American put option on the bank's assets at a strike price equal to the promised maturity value of the bank's liabilities. In principle, if we assume that the life of the bank can be potentially infinite, the two American options should have an *infinite* maturity. In practice, however, arbitrageurs and the bank's shareholders might want to consider only American options characterized by a reasonably long but *finite* maturity (qualitatively, the results do not change). If we assume, for sake of simplicity, that the underlying assets of the two options do not pay dividends, then we can write:

$$K_0 = A_0 - L_0 = C_u^A[A_t - P_u^A(A_t, L_t), L_t] + P_u^A(A_t, L_t) \quad , \qquad \text{(A.1)}$$

where

K_0 , L_0 , A_0 : market value of bank equity / bank liabilities (deposits) / bank assets at time zero;

$P_u^A(A_t, L_t)$: non-standard American put option on $A_t = A_0 \widetilde{P}^t$, at a striking price equal to $L_t = L_0 R^t \ (t = 1, \ldots, u)$, and maturity $u \to \infty$;

$C_u^A[A_t - P_u^A(A_t, L_t), L_t]$: non-standard American call option on $A_t - P_u^A(A_t, L_t)$, at a striking price equal to $L_t = L_0 R^t \ (t = 1, \ldots, u)$, and maturity $u \to \infty$;

$\widetilde{P}$: gross return on the bank's portfolio of assets, assumed to be random, which equals one plus the rate of return (we assume that $0 \leq p < +\infty$, with $p \in \widetilde{P}$);

R : gross return on liabilities (R=1+r, where r is the riskless rate of return).

Given the hypotheses of market completeness and absence of arbitrage, the put-call parity retains its validity, so that

$$K_0 = A_0 - L_0 =$$

$$= C_u^E[A_u - P_u^A(A_t, L_t), L_u] - P_u^E[A_u - P_u^A(A_t, L_t), L_u] + P_u^A(A_t, L_t) \quad , \qquad \text{(A.2)}$$

with

$C_u^E[A_u - P_u^A(A_t, L_t), L_u]$: European call option on $A_u - P_u^A(A_t, L_t)$, with $A_u = A_0\widetilde{P}^u$ and $P_u^A(A_t, L_t)$ defined above, at a striking price equal to $L_u = L_0R^u$, and maturity $u \to \infty$;

$P_u^E[A_u - P_u^A(A_t, L_t), L_u]$: European put option on $A_u - P_u^A(A_t, L_t)$, with $A_u = A_0\widetilde{P}^u$ and $P_u^A(A_t, L_t)$ defined above, at a striking price equal to $L_u = L_0R^u$, and maturity $u \to \infty$.

Moreover, we know from the option theory that the following relationship is valid:

$$C_u^A[A_t - P_u^A(A_t, L_t), L_t] = C_u^E[A_u - P_u^A(A_t, L_t), L_u] \ , \qquad \text{with} \quad t < u \ , \quad u \to \infty \ ,$$

so that eq. (A.1) can be re-written as:

$$K_0 = A_0 - L_0 = C_u^E[A_u - P_u^A(A_t, L_t), L_u] + P_u^A(A_t, L_t) \ . \tag{A.3}$$

After equating (A.2) and (A.3), we obtain

$$P_u^E[A_u - P_u^A(A_t, L_t), L_u] = 0 \ , \tag{A.4}$$

that is clearly absurd, given the definitions above. The explanation of this contradiction refers to a missing term in the definition of the market value of bank equity, as described by eq. (A.1). Indeed, the only put option $P_u^A(A_t, L_t)$ is not sufficient to characterize the state in which $L_t > A_t$ at time *t*. When the promised maturity value of the bank's liabilities is greater than the value of the bank's assets at time *t*, the bank can certainly exercise the put option $P_u^A(A_t, L_t)$, but – at the same time – *the bank loses all its claims relative to any successive period*. Such claims, namely the call option

$C_u^A[A_{t+k} - P_u^A(A_{t+k}, L_{t+k}), L_{t+k}]$, with k = 1, 2, ..., ∞, pertain to the proprietary rights of the bank's owners, and therefore should be part of the market value of equity, K_0.

The possible loss of these claims can be formalized in our analysis by introducing the American call option $C_u^A(\psi_t, 0)$, sold by the bank's shareholders at a striking price of zero. The value of the underlying asset ψ_t can be described by the following scheme:

$$\psi_t = \begin{cases} C_u^A[A_t - P_u^A(A_t, L_t), L_t], & for \quad A_t - L_t < 0 \\ \\ 0 & , \; for \quad A_t - L_t \geq 0 \end{cases} \quad , \qquad t = 1, \ldots, u, \quad u \to \infty.$$

Technically, the scheme of payments relative to $C_u^A(\psi_t, 0)$ could be appropriately represented by an exotic option combining the characteristics of a *binary* ('cash-or-nothing'), and a *compound* call option. For any future period t the call option $C_u^A(\psi_t, 0)$ represents the *cost* of the bankruptcy put option $P_u^A(A_t, L_t)$. *This cost is always twice the corresponding option value of bankruptcy*. In fact, eq. (A.1) now becomes

$$K_0 = A_0 - L_0 = C_u^A[A_t - P_u^A(A_t, L_t), L_t] + P_u^A(A_t, L_t) - C_u^A(\psi_t, 0) \quad . \tag{A.5}$$

Using the put-call parity again, and following the same steps as earlier, we obtain:

$$C_u^A(\psi_t, 0) = P_u^E[A_u - P_u^A(A_t, L_t), L_u] \quad . \tag{A.6}$$

Since

$$K_0 = A_0 - L_0 = C_u^E[A_u, L_u] - P_u^E[A_u, L_u] \; ,$$

eq. (A.5) can be rewritten as

$$P_u^E[A_u - P_u^A(A_t, L_t), L_u] = P_u^A(A_t, L_t) + C_u^A[A_t - P_u^A(A_t, L_t), L_t] -$$
$$- C_u^E[A_u, L_u] + P_u^E[A_u, L_u] \ . \tag{A.7}$$

By no arbitrage arguments, it can be proved that:

$$P_u^E[A_u - P_u^A(A_t, L_t), L_u] = 2P_u^E[A_u, L_u] \ , \tag{A.8a}$$

$$P_u^E[A_u, L_u] = P_u^A(A_t, L_t) \ , \tag{A.8b}$$

$$C_u^A[A_t - P_u^A(A_t, L_t), L_t] = C_u^E[A_u, L_u] \ . \tag{A.8c}$$

Finally,

$$P_u^E[A_u - P_u^A(A_t, L_t), L_u] = 2P_u^A[A_t, L_t] \ . \tag{A.9}$$

Thus, the bankruptcy put option $P_u^A[A_t, L_t]$ is always worth half of its cost to the bank.

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