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MARKET- AND BOOK-BASED MODELS OF PROBABILITY OF DEFAULT FOR DEVELOPING MACROPRUDENTIAL POLICY TOOLS

XISONG JIN and FRANCISCO NADAL DE SIMONE

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ABSTRACT

The recent financial crisis raised awareness of the need for a framework for conducting macroprudential policy. Identifying as early as possible and addressing the buildup of endogenous imbalances, exogenous shocks, and contagion from financial markets, market infrastructures, and financial institutions are key elements of a sound macroprudential framework. This paper contributes to this literature by estimating several models of default probability, two of which relax two key assumptions of the Merton model: the assumption of constant asset volatility and the assumption of a single debt maturity. The study uses market and banks' balance sheet data. It finds that systemic risk in Luxembourg banks, while mildly correlated with that of European banking groups, did not increase as dramatically as it did for the European banking groups during the heights of the financial crisis. In addition, it finds that systemic risk has declined during the second half of 2010, both for the banking groups as well as for the Luxembourg banks. Finally, this study illustrates how models of default probability can be used for event-study purposes, for simulation exercises, and for ranking default probabilities during a period of distress according to banks' business lines. As such, this study is a stepping stone toward developing an operational framework to produce quantitative judgments on systemic risk and financial stability in Luxembourg.

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The authors thank the FNR for its financial support. Correspondence can be sent to: Xisong Jin, Luxembourg School of Finance, 4 Rue Albert Borschette L-1246 Luxembourg, Tel: (+352) 46 66 44 5626; E-mail: xisong.jin@uni.lu; Francisco Nadal De Simone, Banque centrale du Luxembourg, 2, boulevard royal L-2983 Luxembourg, Tel: (+352) 4774-4518; E-mail: francisco.nadaldesimone@bcl.lu.

Résumé non-technique

L'objectif de cette étude est de proposer la construction d'un indicateur de fragilité financière, en l'occurrence la probabilité de défaut, pour le secteur bancaire luxembourgeois en s'appuyant sur la théorie des options développée par Black et Scholes (1973) et Merton (1974). Dans cette étude, le modèle de Merton est estimé en calculant la volatilité des actifs selon la méthode itérative KMV de Moody's. Néanmoins, le maintien de la stabilité financière demeure un objectif d'une nature prospective ; par conséquent, l'estimation de la volatilité sur des données historiques est sujette à de multiples critiques.

Partant, cette étude débute par l'estimation du modèle GARCH-MIDAS [1] d'Engle, Ghysels et Sohn (2008) qui, en décomposant la volatilité en ces deux composantes (court et long-terme), permet d'examiner les variations temporelles du risque de crédit. Dans une seconde étape, le modèle de Heston et Nandi (2000) a été adopté afin de neutraliser la contrainte de la constante de la volatilité. L'innovation de cette approche est la combinaison d'évaluation des options avec l'estimation d'un modèle GARCH à volatilité stochastique. Il semble que les résultats issus de cette approche permettent d'identifier les changements des PD relativement plus tôt.

Lorsque les domaines d'activité des banques sont caractérisés par la prédominance de la dette à long terme, l'utilité des modèles décrits précédemment est amoindrie dans la mesure où les probabilités estimées sont des probabilités à un an. Afin de remédier à cette contrainte, les auteurs de cette étude estiment le modèle de Delianedis et Geske (2003), lequel est caractérisé par la capacité d'estimer simultanément la PD à un an, la PD à long terme conditionnelle sur l'absence de défaut au cours de la première année, et de la probabilité totale de défaut.

Les données exploitées comprennent un échantillon de 20 groupes bancaires européens et de 35 banques luxembourgeoises. L'analyse construit trois indicateurs de risque de défaut. Les principaux résultats obtenus dans cette étude sont les suivants :
- l'évolution des indicateurs construits est synchrone avec les événements clés observés pendant la période sous revue ;
- le modèle GARCH-MIDAS et le modèle de Heston et Nandi identifient les changements du risque de crédit plus tôt que les autres modèles ;
- l'augmentation de la probabilité de défaut des banques luxembourgeoises suite à la crise a été moindre et moins variable que pour les groupes bancaires européens ;

[1] GARCH-MIDAS signifie modèles généralisés autorégressifs hétéroscedastiques conditionnels qui emploient des échantillons de données mixtes.

- la somme des actifs présentant une PD supérieure à 10%, soit à un an, soit à long terme, a régulièrement diminué entre décembre 2008 et décembre 2010 ;- il s'avère que les corrélations des PDs entre les banques ont augmenté durant la période précédant la crise.

L'application du modèle de Delianedis et Geske à des banques luxembourgeoises a permis tout d'abord d'illustrer dans quelle mesure l'évolution de la PD conditionnelle est cohérente avec les données historiques. Il a facilité, par ailleurs, l'examen de l'impact des changements de la structure de la dette d'une banque sur sa propre probabilité de défaut. Enfin, ce modèle révèle l'importance de la prise en compte de la structure de la dette des établissements financiers. En effet, la négligence de cette composante se traduirait par des biais importants des résultats.

De ce qui précède, il nous semble que les résultats de cette étude sont susceptibles d'améliorer notre compréhension de l'évolution du risque de crédit au Luxembourg permettant ainsi l'enrichissement de la batterie d'indicateurs macro-prudentiels développés au sein de la Banque centrale du Luxembourg.

I. Motivation

The crisis raised awareness of the need for a framework for conducting macroprudential policy. However, there is yet no widely accepted definition of macroprudential policy, or its objective or its instruments (Galati and Moessner, 2011). In this paper, the objective of macroprudential policy, in agreement with the ECB broad characterization of it, will be to limit systemic risk so as to minimize the costs of financial instability on the economy (ECB, June 2010). In this vein, macroprudential policy will seek to limit systemic risk viewed in its three main forms (ECB, December 2009): (1) as contagion risk[2], (2) as the risk of macro shocks that cause simultaneous problems to the economy, and (3) the risk of the unravelling of imbalances that have built up over time. As a result, models and instruments for achieving these objectives should aim at identifying as early as possible and addressing the build up of endogenous imbalances, exogenous shocks, and contagion from financial markets, market infrastructures, and financial institutions. Instruments to enhance the strength of the financial system deal both with the cross-sectional dimension of systemic risk, such as requests for more and better quality capital via the Revision of the Capital Requirements Initiative (CRD4), and with the time-dimension of systemic risk, such as the proposal for forward-looking provisioning.[3] Instruments to address imbalances include, for example, time-varying Loan-to-Value ratios (Goodhart and Hofmann, 2007) and time-varying margins or haircuts (CGFS, 2010).

The area where most models and instruments have been proposed or are currently under development, is the area of financial institutions. In particular, there is a fast growing analysis and research geared toward modelling financial institutions' interconnectedness. Whether by using a mixture of distributions to model dependence or by using copula or network analysis, models require the estimation of default probabilities as a first step. This paper, part of a large research project aimed at building and tracking financial stability in Luxembourg, uses the insights of contingent claims analysis (CCA), a generalization of the option pricing theory pioneered by Black and Scholes (1973) and Merton (1974). As thoroughly and convincingly shown in Gray and Malone (2008), CCA can be applied not only to the financial sector, but also to the private non-financial sector and to the sovereign. Given the goal of applying CCA not only to Luxembourg banks, but also to other sectors of the country's economy, this study has to use a methodology that is consistent with the absence of liquid markets for options on banks' debt, on non-financial firms' debt and on the country's sovereign debt.

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[2] The concept refers to the occurrence of an idiosyncratic shock affecting an important financial institution which gets in turn transmitted through the financial system and ends up affecting the real economy.

[3] See Borio and Drehmann (2009) for a useful characterization of the dimensions of macroprudential policy.

This study contributes to this literature by estimating several models of default probability, using market and banks' balance sheet data: the Merton model (1974), a combination of the Merton model with Engle *et al*'s GARCH-MIDAS model (2008), the Heston and Nandi model (2000) (with fixed price of risk) and the Delianedis and Geske model (2000). All but two banks in Luxembourg belong to large European banking groups. For the large European banking groups that have subsidiaries and branches in Luxembourg, market capitalization data are available, and thus, they are used. For the 33 Luxembourg subsidiaries or branches of those large banking groups, as their stock is not quoted in stock exchanges nor they issue options or bonds (at least not sufficiently to constitute a liquid market), balance sheet data is used. Balance sheet data is also used for the two 100% Luxembourg banks. This paper also proposes a few proxies for banking systemic risk, albeit without exploiting banks' interdependence at this stage.[4] Thus, those proxies are conceptually closer to the second form of systemic risk mentioned above, i.e., the risk of macro shocks that cause simultaneous problems to the economy. A main result is that systemic risk in Luxembourg, while mildly correlated with that of the European banking groups, did not increase as dramatically as it did for the European banking groups during the heights of the financial crisis. It also finds that systemic risk has declined during the second half of 2010, both for the banking groups studied as well as for the Luxembourg banks. In addition, this study illustrates how models of default probability can be used for event-study purposes, for simulation exercises, and for allowing the ranking of default probabilities during a period of distress according to banks' business lines. As such, this study is a stepping stone toward developing an operational framework to produce quantitative judgments on systemic risk and financial stability.

The paper is organized as follows. Next section provides a brief discussion of the set of models used to estimate default probabilities. Section III discusses the data. Section IV examines the empirical results. It presents first a few selected indicators of systemic banking risk and then illustrates a series of applications of the models to macroprudential objectives. Section V concludes.

II. Selected models to estimate default probabilities

In order to develop tools to measure and assess financial stability it is necessary to characterize instability. Approaches to deal with instability include, for instance, modelling financial institutions' default, analysing the financial system using extreme value theory, and allowing for episodes of market illiquidity. The approach taken in this

[4] Separate ongoing research explores different approaches to modelling dependent defaults.

study instead is to apply CCA to the analysis and measurement of credit risk, or as it is commonly referred to, structural credit risk modeling. Structural credit risk models attempt to assess the creditworthiness of a firm by modeling the evolution of the firm's asset values as a stochastic process, and by viewing bankruptcy as an endogenous random event linked to the value of the firm's assets. This section briefly discusses the set of models used to compute probabilities of default (PDs) for the banks used in this study.

1. The Merton Model

In the Merton model, equity owners are viewed as holding a call option on the firm's value after outstanding liabilities have been paid off. They also have the option to default if their firm's asset value falls below the present value of the notional amount—or book value—of outstanding debt ("strike price") owed to bondholders at maturity. In other words, when the market value of the firm's assets is less than the strike price, the value of equity is zero. Similarly, bond holders are viewed as writing a European put option to equity owners, who hold a residual claim on the firm's asset value if the firm does not default. Bond holders receive a put option premium in the form of a credit spread above the risk-free rate in return for holding risky corporate debt (and bearing the potential loss) due to equity owners' limited liability.

According to the Merton model, the market value of a firm's underlying assets follows a geometric Brownian motion (GBM) of the form:

$$dV_A = \mu V_A dt + \sigma_A V_A dW$$

where V_A is the firm's assets value, with an instantaneous drift μ (the expected rate of return of the firm), and an instantaneous asset return volatility σ_A. W is a standard Wiener process. If X is the book value of the debt which is due at time *T*, Black and Scholes' formula provides the market value of equity, V_E:

$$V_E = V_A N(d_1) - Xe^{-r(T-t)} N(d_2)$$

where $d_1 = \dfrac{\ln(\frac{V_A}{X}) + (r + \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A \sqrt{T-t}}$, $d_2 = d_1 - \sigma_A \sqrt{T-t}$, *r* is the risk-free rate, and $N()$ is the cumulative density function of the standard normal distribution. The PD of the firm is the probability that its assets' value will be less than the book value of its liabilities. The corresponding implied neutral PD is $\pi_N = N(-d_2)$. The "actual" PD is $\pi_A = N(-DD)$, where the distance-to-default, DD, is simply the number of standard deviations that the firm is away from default:

$$DD = \frac{\ln(\frac{V_A}{X}) + (\mu - \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A \sqrt{T-t}}$$

To get to "actual" PDs from neutral PDs, the latter must be adjusted by the market price of risk, which is estimated using a capital-asset pricing model in this study: $u = \rho_{A,M} \frac{\sigma_A}{\sigma_M}(u_M - r) + r$, where σ_M is market asset return volatility, $\rho_{A,M}$ is the correlation between firm's asset return and market asset return. Alternatively, historical recovery rates can be used to move from risk neutral to "actual" PDs. The derived "actual" PDs, however, could be still much higher than the observed PDs, the so-called "credit spread puzzle" (Huang and Huang, 2003). Moody's KMV deals with this issue by mapping distance to default into historical default probabilities. Chen, Collin-Dufresne and Goldstein (2009), instead, try to adjust PDs' levels by calibrating the pricing kernel to equity returns and aggregate consumption. Fortunately, the rankings are more meaningful than the levels, given the objective of this study.

A complication of CCA to calculate PDs is that the volatility of the underlying asset value is not directly observable. To calculate σ_A, Moody's KMV iterative procedure is used in this paper for the European banking groups.[5] For quoted financial institutions, the KMV approach implies taking daily equity data using a 12-month window to calculate historical assets volatility.[6] Regarding the value of debt, the KMV approach takes all obligations due in one year, plus half of the long-term debt. The KMV method is a simple two-step iterative algorithm to solve for assets volatility. The procedure uses an initial guess for volatility to determine the asset value and to de-lever the equity returns. The volatility of the resulting asset returns is used as the input to the next iteration of the procedure which, in turn, determines a new set of asset values and hence a new series of asset returns. The procedure continues in this manner until it converges.

2. The combined Merton and GARCH-MIDAS Model

The KMV approach of using a 12-month window of daily equity data to estimate σ_A, while practical, may not be appropriate for a central bank interested in assessing financial stability over time as the resulting PDs may not track risk timely. To cope with this handicap, at least partially, this study uses a straightforward result from the Merton

[4] Duan et al, (2004) show that the KMV estimates are identical to maximum likelihood estimates (MLE).

[6] See Section 2.5 for the approach followed to estimate σ_A in the case of Luxembourg banks for which quoted stock prices or options on stock are not available.

model, i.e., that the volatility of the firm's asset and its equity are related by the following optimal hedge equation:

$$\sigma_E = (\frac{V_A}{V_E})\frac{\partial V_E}{\partial V_A}\sigma_A .$$

As a result, asset values and asset volatilities can be directly estimated by simultaneously solving this optimal hedge equation and the call option equation. In this manner, it is feasible to get timely and steadily PDs by choosing the short- and long-run components of equity volatility. For this purpose, Engle et al's (2008) class of GARCH-MIDAS[7] models is preferred. GARCH-MIDAS distinguishes short- and long-run sources of volatility and *can* also link them directly to economic variables. In this study, the GARCH-MIDAS model with rolling window realized volatility (RV) is applied (the option with the macro variable is not explored here).[8]

In the GARCH-MIDAS model with rolling window RV, the return (*r*) on day *i* is written as: $r_i = \mu + \sqrt{\tau_i g_i}\varepsilon_i$, where volatility has two components, namely g_i , which accounts for short-lived daily fluctuations, and a secular component τ_i for all *i = 1, . . . ,N*, and $\varepsilon_i \mid \Phi_{i-1}$ ~ *N(0, 1)* with Φ_{i-1} the information set up to day (*i - 1*). The volatility of the short-term component g_i is a GARCH(1,1) process modeled by Engle *et al* (2008) as:

$$g_i = (1-\alpha-\beta) + \alpha\frac{(r_{i-1}-\mu)^2}{\tau_i} + \beta g_{i-1} .$$

Similarly, the long-run τ_i component is estimated by smoothing the rolling window realized volatility:

$$\tau_i = m + \theta\sum_{k=1}^{K}\varphi_k RV_{i-k}$$

where φ_k is a smoothing function and $RV_i = \sum_{j=1}^{N'} r_{i-j}^2$. The time span N' can be monthly, quarterly, or biannual at $N' = 22$, $N' = 65$ or $N' = 125$ respectively. K is determined by 'MIDAS lag years,' spanned in each MIDAS polynomial specification for τ_i . Since the PDs are estimated monthly in this study, a monthly time span and one MIDAS lag year

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[7] GARCH-MIDAS accounts for generalized autoregressive conditional heteroskedastic models using mixed data sampling.
[8] Engle *et* al (2008) model the long term component as driven by inflation and industrial production growth. They find that their model is at par in terms of out-of-sample prediction for horizons of one quarter and out-perform more traditional time series volatility models at longer horizons. They also find that at a daily level, inflation and industrial production growth account for between 10 % and 35 % of one-day ahead volatility prediction. Hence, macroeconomic fundamentals play a significant role even at short horizons. Ongoing separate research includes using the Merton and the GARCH-MIDAS models together not only to capture changes in PDs more timely, but also to model the macro financial factors underlying credit risk.

are used not only to catch credit events timely, but also to compare with Moody's KMV iterative procedure which typically takes daily equity data from the past year to calculate asset return volatility.

The GARCH-MIDAS model is estimated by Quasi-maximum likelihood first, and then the conditional short-run or long-run variance forecasts on day *i+1* are used to calibrate the PDs by the above-mentioned two equations on day *i.*[9] Therefore, the component dynamic volatilities will ideally make the estimated PDs less backward looking than the PDs that result from the estimation of volatility using the KMV approach. So the Merton GARCH-MIDAS model may catch credit risk more timely.

3. The Heston & Nandi GARCH Structure Model

Even when the Merton model is combined with the Engle *et al.'s* GARCH-MIDAS model in a two-step approach, instantaneous asset volatility is still constant by modeling. However, similar to equity returns, the actual distribution of the underlying asset value often has a fatter left-hand tail and a thinner right-hand tail than the lognormal. This will happen, e.g., if volatility is stochastic and negatively correlated with the price of the underlying asset. Many existing empirical studies have shown the importance of time-varying volatility, the leverage effect, negative skewness, and the existence of a "volatility smile". The approaches followed in the literature to deal with that modeling pitfall have been to either model volatility stochastically or to assume ex-ante a price probability distribution that is compatible with the alluded observed regularities (e.g., a mixed-log normal distribution).[10] Alternatively, a GARCH structure option model with a closed-form solution has been proposed by Heston & Nandi (2000). Heston (1993) model is a popular model in the continuous-time option valuation literature, while the NGARCH model of Duan (1995) is a standard model in the discrete-time literature. Both models contain stochastic volatility and a leverage effect. Heston and Nandi (2000) are 'bridging the gap' between the two streams in the literature, showing that Heston's (1993) model is the continuous-time limit of the Heston and Nandi (2000) model. To the authors' knowledge, this is the first model to provide a readily computed option formula for a random volatility model that can be estimated and implemented solely on the basis

[9] As in Engle *et al* (2008), standard errors are HAC.

[10] Recently, Arondel and Rouabah (2008) estimated risk-neutral densities subject to the constraint of minimizing the distance between the theoretical and the observed price of a given option for the Eurostoxx 50 index and the DJE Eurostoxx 300 index. Unfortunately, as stated above, as there are no option prices on Luxembourg banks' stock (nor on non-financial firms or the sovereign), this approach cannot be followed.

of observables. In addition, endogenizing GARCH-like time-varying volatility directly into a structural credit risk model could enable to identify changes in PDs earlier than by using the Merton model, a feature that would command a premium in the task of designing financial stability measures.[11]

According to Heston & Nandi (2000), the log return *R* of the firm's asset value follows a particular GARCH process:

$$R_{t+1} = \ln(V_{t+1}/V_t) = r + \lambda h_{t+1} + \sqrt{h_{t+1}}\, z_{t+1}$$
$$h_{t+1} = \omega + b h_t + a(z_t - c\sqrt{h_t})^2$$

where V_{t+1} denotes the underlying asset price on day *t +1*, *r* the risk free rate, *λ* the price of risk and h_{t+1} the daily variance on day *t +1* which is known at the end of day *t*. The z_{t+1} shock is assumed to be i.i.d. *N*(*0, 1*). The Heston & Nandi model captures time variation in the conditional variance, and the parameter *c* captures the so-called "leverage". The leverage effect refers to the observed negative relationship between shocks to returns and volatility which results in a negatively skewed distribution of returns. The market value of equity, V_E , has a formula similar to the Black and Scholes' formula for call options:

$$V_E = \frac{1}{2}V_A \frac{e^{-r(T-t)}}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{X^{-i\phi} f^*(i\phi+1)}{i\phi}\right]d\phi - Xe^{-r(T-t)}\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{X^{-i\phi} f^*(i\phi)}{i\phi}\right]d\phi\right),$$

where *Re* denotes the real part of a complex number and $f^*(\phi)$ denotes the generating function for the risk-neutral process for V_A . The difference with Black and Scholes' formula is that Heston & Nandi's depends on the current asset price and on the conditional variance $h(t+\Delta)$, itself a function of the observed path of the asset price. Therefore, volatility becomes a readily observable function of historical asset prices. The implied neutral probability of default is:

$$1-\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{X^{-i\phi} f^*(i\phi)}{i\phi}\right]d\phi\right).$$

Since the underlying asset price is not directly observed, the GARCH structural credit risk model is estimated by the Expectation Maximization (EM) algorithm proposed by

[11] Using S&P500 index options, Heston and Nandi (2000) show that the out-of-sample valuation errors from the single lag version of their GARCH model are substantially lower than those from the ad-hoc Black and Scholes model of Dumas, Fleming and Whaley (1998) that uses a separate implied volatility for each option to fit to the smirk in implied volatilities. The GARCH model remains superior even though the parameters of the GARCH model are held constant and volatility is filtered from the history of asset prices while the ad-hoc Black and Scholes model is updated every period. They attribute the improvement largely to the ability of the GARCH model to simultaneously capture the correlation of volatility with spot returns and the path dependence in volatility.

Malone, Rodriguez and Horst (2008). Similar to the KMV method, the EM procedure first uses an initial guess for the GARCH parameter vector to determine the asset values and to de-lever the equity returns by inverting the call option equation; it computes the series of log returns from the extracted series of asset values, and then it applies the Quasi-maximum likelihood method to estimate the new parameter vector of the GARCH process. The procedure continues in this manner until the researcher's chosen convergence criterion is reached. To compare with the Merton model, a one-year rolling window of past daily equity returns is used to calculate the GARCH parameter vector, and all obligations due in one year, plus half of the long-term debt are considered as the strike price.

In the Merton model, once daily asset values asset are estimated, the drift μ can be computed by calculating the mean of log returns. However, in many cases, the actual return on assets can be negative. Similarly, the one-year rolling window might not be good enough to estimate the price of risk λ, which is a crucial input to estimate the actual PD in the Heston & Nandi model. In order to compare with the Merton model, and using the expected rate of return formula $\mu = r + \left(\rho_{A,M}(u_M - r)/\sigma_A\sigma_M\right)\sigma_A^2$, the price of risk λ can be fixed by $\lambda = \rho_{A,M}(u_M - r)/\sigma_A\sigma_M$. This study estimates the Heston & Nandi model with a fixed price of risk.

4. The Delianedis and Geske Compound Option-based Structural Credit Risk Model

Previous models consider only a single debt maturity. However, debt maturity influences liquidity and the probability of default. This is an important drawback for a central bank or a supervisor interested in assessing and tracking banks' solvency. Geske (1977) and Delianedis and Geske (2003) consider a multi-period debt payment framework to which they apply compound option theory. This enables to account for the influence of the time structure of debt on the estimated PD.

Assume that a bank has long term debt, M_2, which matures at date T_2, and short term debt, M_1, which matures at date T_1. Between T_1 and T_2, the Merton model is valid as the bank's equity equals a call option giving the shareholder the right to buy the bank at the second payment date, T_2, by paying the strike price M_2. If at date T_1, the call option with the bank's value $\overline{V}$ equals at least the face value of the short term debt, M_1:

$$M_1 = \overline{V}N(k_2 + \sigma_A\sqrt{T_2 - T_1}) - M_2 e^{-r_{F_1}(T_2 - T_1)}N(k_2)$$

then the bank can roll over its debt. So, the refinancing problem, the right to buy the simple call option of the second period by paying the strike price at the first payment date, is exactly a compound option as follows:

$$V_E = V_A N_2(k_1 + \sigma_A\sqrt{T_1 - t}, k_2 + \sigma_A\sqrt{T_2 - t}; \rho) - M_2 e^{-r_{F_2}(T_2 - t)} N_2(k_1, k_2; \rho) - M_1 e^{-r_{F_1}(T_1 - t)} N(k_1)$$

where $\rho = \sqrt{\frac{T_2 - t}{T_1 - t}}$, $N_2()$ is a bivariate cumulative normal distribution, and,

$$k_1 = \frac{\ln(\frac{V_A}{\bar{V}}) + (r_{F_1} - \frac{1}{2}\sigma_A^2)(T_1 - t)}{\sigma_A\sqrt{T_1 - t}}, \quad k_2 = \frac{\ln(\frac{V_A}{M_2}) + (r_{F_2} - \frac{1}{2}\sigma_A^2)(T_2 - t)}{\sigma_A\sqrt{T_2 - t}}.$$

The richness of the model allows to calculate the following risk neutral PDs: (1) the total or joint probability of defaulting at either date T_1 or date T_2, i.e., $1 - N_2(k_1, k_2; \rho)$; (2) the short-run probability of only defaulting on the short-term debt at date T_1 , i.e., $1 - N(k_1)$ and; (3) the forward probability held today of defaulting on the long-term debt at date T_2 , conditional on not defaulting on the short-term debt at date T_1 , i.e., $1 - \frac{N_2(k_1, k_2; \rho)}{N(k_1)}$. Similar to the Moody's KMV iterative procedure, the Delianedis and Geske model is estimated by the two-step iterative algorithm. Regarding the maturity of value of debt, this study takes all short term obligations due in one year as a one-year maturity debt, and all long-term debt as a ten-year maturity debt.

5. The Book Value Based Merton and Delianedis and Geske Models

As Luxembourg bank subsidiaries and branches are not publicly quoted, an alternative approach has to be followed to calculate PDs.[12] Hillegeist *et al.* (2004) demonstrate that the market-based Merton's probability of default provides significantly more information about the probability of bankruptcy than do the popular accounting-based measures. However, Bharath and Shumway (2008) also examine the accuracy and PDs forecasting performance of the Merton model and find that most of its predictive power comes from its functional form rather than from the estimation method: the firm's asset value, its asset risk, and its leverage. In an application to Brazilian and Mexican banks, Souto *et al.* (2009) and Blavy and Souto (2009), respectively, show that the book-based Merton's credit risk measures are highly correlated with market-based Merton's credit risk measures.[13] This suggests that banks' financial statements are a crucial piece of

[12] This is even more the case because, as stated above, the methodology followed for estimating the models in this study should be also applicable to the non-financial sector and to the sovereign.
[13] See also Gray and Jones, 2006, for an early application of this idea.

information when forming market expectations about the probability of banks' default. This approach is followed here. The book value asset volatility is calculated by a rolling window (RW) as follows:[14]

$$\sigma_B = \sqrt{\sum_{t=1}^{N} (\ln(V_t^B / V_{t-1}^B))^2}$$

where V_t^B denotes the book value of total assets at time t , N represents a rolling window of four consecutive quarters. In addition, and using the same rolling window, the "negative" risk volatility (NRW) which places greater weight on negative shocks than on positive shocks, is also considered:

$$\sigma_B^{downside} = \sqrt{\sum_{t=1}^{N} \min(\ln(V_t^B / V_{t-1}^B),0)^2}$$

where *Min(. . .)* is the minimum function. The intuition for this choice for volatility modeling relies on the fact that negative shocks, rather than positive ones, are often a source of concern.

True, in empirical work, a dynamic volatility model is often preferred in order to track risks more timely. However, most dynamic volatility models require many more data points than are available for Luxembourg banks. Therefore, in this paper a third, alternative, approach to modeling volatility is used, the RiskMetrics (RM) filter/model. This model assumes a very tight parametric specification. The book value asset RM variance is defined as:

$$h^B{}_{t+1} = (1-\zeta)(\ln(V_t^B / V_{t-1}^B))^2 + \zeta h^B{}_t$$

where the variance forecast h_{t+1}^B for period *t+1* is constructed at the end of period *t* using the square of the return observed at the end of period *t* as well as the variance on period *t*. Although the smoothing parameter ζ may be calibrated to best fit the specific historical returns, RiskMetrics often simply fixes it at 0.94. To avoid the calibration difficulties associated to the limited data set used in this study, ζ is assumed to be same for all banks, and estimated by numerically optimizing the composite likelihoods (Varin et al, 2011), i.e., the sum of quasi maximum likelihood functions of the estimation sample over all banks simultaneously:

$$QMLE(\zeta) = -\frac{1}{2}\sum_{i=1}^{N} \sum_{t=1}^{T} (\ln(h_{t,i}) + (V_{t,i}{}^B / V_{t-1,i}^B)^2 / h_{t,i})$$

where N is number of banks, and there is a time series of T observations for each banks. In order to compare with the book value asset volatility estimated by the rolling window

[14] Following usual practice, quarterly volatility is annualized.

or the downside rolling window, the recursion is initialized by setting the initial σ_0^B equal to the first year book value asset volatility. Clearly, and similar to RW models, the means of quarterly assets returns in a large sample are assumed to be zeros to avoid the noises brought by the sample means to the RM variance process.

In order to have a more forward-looking measure, the variance forecast $\sigma^B{}_{t+1}$ can be used to calibrate PDs at time t. The book-value risk neutral PD (π_B for book-value PD) of the Merton model can be directly estimated by:

$$\pi_B = N(-\frac{\ln(V^B / X) + (r - \frac{1}{2}\sigma_B^2)(T-t)}{\sigma_B\sqrt{T-t}}).$$

Similarly the book-value risk neutral PDs of the Delianedis and Geske model can be estimated by substituting V_B and σ_B into k_1 and k_2 in the Delianedis and Geske model. Given σ_B, the critical book value of total assets $\overline{V}^B$ at T_1 is calculated first. Similarly, this study takes all short term obligations due in one year as a one-year maturity debt, and all long-term debt as a ten-year maturity debt.

III. Data

This study is applied to 20 major European banking groups which have 33 subsidiaries and branches active in Luxembourg, to those 33 Luxembourg-registered subsidiaries and branches, and to two 100% Luxembourg banks. Market data used for the major European banking groups include government bond yields, the S&P 500 index, equity prices, the number of outstanding shares, and book value data from Bloomberg. However, short-term borrowings (BS047) and long-term debt (BS051) from Bloomberg have annual, semi-annual, and quarterly frequencies. To make them consistent, four filtering rules as described in Appendix A are used. To get the “actual” PDs from neutral PDs, the expected returns are estimated using a capital-asset pricing model. The implied equity risk premiums data (Damodaran 2011) are downloaded from Damodaran Online at http://pages.stern.nyu.edu/~adamodar/. Given the relatively deeper nature of the US market and the global business of the banking groups used in this study, stock market returns are represented by the returns on the S&P 500 index. The sample period is from May 30, 2000 to December 31, 2010.

All the Luxembourg banks are unlisted, so quarterly book value data from the BCL database going back to 2003Q1 are used. The 33 subsidiaries and branches registered in Luxembourg represent about two thirds of the total assets of the Luxembourg banking industry. When the two 100% Luxembourg banks are added to the list, the database represents over 70 percent of the total assets of the industry. For all the selected Luxembourg banks, the short term debt includes demand and time deposits of up to one-year maturity, short term funding, and repos, while the long term debt includes time deposits of over one-year maturity and long term funding.

IV. Empirical Results

The results of the estimation of the five models described in the previous section are presented in several figures and tables. The purpose of this section is twofold: first, to show how the estimated models can be used to build three proxies of banking "systemic risk", even before accounting for default dependency across financial institutions; second, to illustrate possible uses and differences across models of default and their assumptions that are important for assessing and tracking default risk, while putting a premium on indicators that lead market developments.

1. An asset-weighted index of systemic banking risk

a. The Asset-weighted PD index for Banking Groups

PDs estimated using the different models described in Section II can be put together in an index of systemic risk by aggregating the individual PD estimates weighted by their respective estimated implied asset values.[15] Figure 1a illustrates variants of such indicator of systemic risk for banks. In general, variants of the index track surprisingly well the major events of Lehman Brothers' default and the sovereign crisis. In addition, consistent with all models' results, it seems that the US Treasury Financial Stabilization Fund had a major positive impact on markets as all PDs started falling after it was announced.[16] Table 1 provides descriptive statistics of the PDs estimated with the different models used in building the index both for banking groups and Luxembourg banks. Given that there are differences across models in terms of the timeliness with which they reflect market events, their computation costs, and the richness of the information they provide, an analysis of them according to those criteria is opportune.

Given that the timeliness of an indicator for monitoring risk is a desirable feature, a first important point to bring out from Figure 1a is the capacity of the Merton GARCH-MIDAS model and the Heston & Nandi model to reflect significant market events relatively

[15] Only neutral probabilities of default are used for the comparisons across models.

[16] The Plan involved Treasury purchases of convertible preferred stock in eligible banks, the creation of a Public-Private Investment Fund to acquire troubled loans and other assets, expansion of the Federal Reserve's Term Asset-Back Securities Loan Facility (TALF), and other initiatives. The Fed also announced it would expand the TALF to as much to one trillion US dollars.

earlier than the other models. This is the case, for instance, at the time of Lehman Brothers' default, and also later at the time of the sovereign distress started toward the end of 2009. As discussed in Section II, for the GARCH-MIDAS plus Merton model, this is the outcome of nesting GARCH-like time-varying volatility indirectly into the model structure (volatility is constant in the Merton model). For the Heston & Nandi model, timeliness is due to its ability to simultaneously capture the volatility correlation with spot returns and volatility path dependence.[17] Also, as Heston & Nandi model does not assume a normal distribution (shown to be incompatible with observed volatility regularities), it is theoretically expected to track changes in credit risk more timely. In addition, the combined model, GARCH-MIDAS plus Merton, lends itself to modelling the long-run component of PDs not only by showing the steadily accumulated credit risk, but also by improving its forecasting ability as shown by Engle et al. (2008).[18] This feature goes beyond the purpose of this study, however, and it is not further addressed here.

A second and very much related observation is that the asset-weighted estimated PD profiles from the Merton model seem similar to those estimated from the Heston & Nandi model. However, the levels are different, especially during the periods of relatively higher volatility such as the recent financial crisis. Given that the aggregate level might hide some important properties of the models, looking at the estimates for individual banks may help improving the understanding of the different PDs profiles generated. Therefore, Figure 1b compares the results of the standard Merton model with those of the Heston & Nandi model for the European banking group A. As with other banking groups (not shown on the graph), an increase in banking group A's PDs accompanied the rise of tensions following the Lehman Brothers' event. As indicated earlier, the way in which volatility is traditionally estimated (e.g. using a 1-year rolling volatility) tends to make those estimates backward looking. Figure 1b also illustrates how more timely are Heston & Nandi model PD estimates. However, the computing time for the Heston & Nandi model is significantly longer than for the Merton model. As a result, a policymaker may have to trade off the timeliness of the Heston & Nandi model with its much higher estimation cost, given that PD profiles are similar. So, a systemic risk index based on the Heston & Nandi model may be useful for a macroprudential supervisor with a clear preference for measures that reflect credit risk events relatively earlier.

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[17] Jin *et al* (2011) evaluate econometrically the ability of the models studied in this paper to correctly and timely identify changes in credit risk.

[18] Engle et al. (2008) show that when the long-run volatility component is driven by output growth and inflation, their model out-performs traditional time series volatility models in out-of-sample forecasting. Even at short horizons, those macroeconomic variables play a significant role in explaining one-day-ahead volatility prediction.

Finally, the short-term PD of Delianedis and Geske model has a similar profile to Merton's and to Heston & Nandi's. However, Delianedis and Geske model is much richer as it also produces the probability of default after one year conditional on not having defaulted the first year. This can provide valuable information for macroprudential policy as illustrated in the next section. Suffice it to note here the evolution of the conditional probability of default on long-term debt that accompanied banks' efforts to increase the maturity of their debt profile and policy measures that increased capital (Figure 1a).

b. The Asset-weighted PD index for Luxembourg banks

Figure 1c illustrates the usefulness of the asset-weighted PD index to track Luxembourg banks' time structure of PDs applying the Merton and Delianedis and Geske models to book-value information. A first observation is that the increase in the PD index is more modest for Luxembourg banks than for the European banking groups, but the pattern is still consistent with the timeline of major events leading to the crisis, such as the Lehman Brothers' collapse.

A second observation refers to the PD results using alternative methods to estimate asset volatility with book-value information. As expected, the PD index estimated by NRW is much lower than that estimated by RW or RM because only negative shocks are taken into account in estimating volatility. In addition, the PD estimated by NRW responds relatively slowly, with a quarter lag, to the financial crisis, but also drops quickly afterward. The PD index estimated with RM looks similar to that estimated with RW, but it is more persistent and somewhat higher on average; it also displays a clear upward trend at the end of 2010.

Third, in order to show the difference brought by the RiskMetrics filter, Figure 1d plots the PD index for Luxembourg banks using the Merton model with different volatility estimates. Clearly, the PD index estimated using RW is relatively more volatile, and it can be closely approximated by using RM with a smoothing parameter of 0.5. As indicated in Section II.5, however, the smoothing parameter estimated cross-sectionally is 0.82, and it is thus to be preferred.[19] This is the estimate showed in what follows under the acronym of RM. It provides a PD index which is obviously quite close to the one resulting from the use of RiskMetrics with the fixed parameter of 0.94.[20]

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[19] The distribution is not theoretically known. However, an estimate of the standard error of the estimate can be obtained using the information contained in the Hessian matrix. This equals 0.0089, a very low number indeed.

[20] RiskMetrics' 0.94 fixed parameter is for daily returns. The estimated parameter in this study, 0.82, results from using a sample of quarterly frequency.

To conclude, given the lagged behaviour of estimates of the PD index for Luxembourg banks using NRW in a context in which early detection of stress carries a premium, and to conserve space, only the results obtained estimating volatility using the RM filter with the estimated coefficient will be reported in what follows. Note, however, that the PDs estimated using the NRW measure of volatility can, by construction, signal earlier than other measures the end of a period of distress. From a policymaking viewpoint, using both NRW and RM can be a good compromise between, other things equal, missing the start of distress and having too tight a policy stance due to the sole use of NRW estimates, and missing the end of a period of distress and having too accommodative a policy stance due to the sole use of RM estimates.

c. Comovement between Banking Groups and Luxembourg Banks' PD Indexes

While the PD index levels cannot be compared directly, it is fruitful to have some measure of the order of magnitude of changes in PDs for banking groups compared to changes in PDs for Luxembourg banks; in particular, it is useful to analyze how the PD indexes of Luxembourg banks comove over time with the PD indexes of the European banking groups they belong to. For this purpose, a default index can be constructed normalizing, say, by the PD of 2008Q1. Table 2 reports the first four moments, the first order autocorrelation coefficient, the minimum and the maximum of such default index. Clearly, the range of variation of the index for Luxembourg banks is smaller than the range of variation of the index for banking groups, for both short-term and long-term PD estimates. This may suggest a relatively more stable banking industry at home. In addition, while the persistence of PD measures of banking groups is similar for the short- and the long-run PDs estimates, the persistence of short-term PD estimates is much higher than the persistence of long-term PD estimates for Luxembourg banks (see first order autocorrelation coefficients).[21]

To obtain some insights on the comovement of PDs between European banking groups and Luxembourg banks, Table 3 displays PDs and changes in PDs' Kendall rank correlations from 2004 to 2010. All correlations are strongly significantly positive in the range of 0.3 - 0.6. A few observations are noteworthy, especially when Figures 1a and 1c are also taken into account. First, the PD indexes of Luxembourg banks seem to commove more closely with those of the banking group's since 2008, consistent with the timeline of major events as the Lehman Brothers' collapse, the US Treasury Financial Stabilization Fund, and the sovereign crisis. Second, concentrating on PDs estimated using RM measures of volatility (shaded area of the table), the Luxembourg short- and

[21] Without further analysis of the determinants of PDs, nothing else can be said at this stage. Ongoing research modeling volatility using macroeconomic variables may shed some light on this matter.

long-run PDs are more correlated with banking groups' short-run PDs than with banking groups' long-run PDs for the whole period. This is consistent with the different business lines that Luxembourg banks follow compared to banking groups, specially the importance of Luxembourg banks in the short-term funding of their parents.[22] Finally, toward the second half of the sample period, the PD index of Luxembourg banks dies off slower than the PD index of European banking groups, partly because the sovereign crisis impacts not only the short-term PDs, but also relatively more the long-term PDs of Luxembourg banks.

2. A cumulative asset share indicator of systemic banking risk

Figure 2a displays an indicator of banking risk for the 20 banking groups using the Delianedis and Geske model and market information. It displays cumulative banks' estimated implied asset values versus banks' PDs--sorted from lowest to highest. For example, at the end of 2008, 90 percent of the assets of banking system had a short-term PD larger than 10 percent (see vertical line on the graph). The situation improved at end-2009 as only 19 percent of the banking system had a short-term PD larger than 10 percent. At the end of 2010, it had further improved as just 1 percent of the banking system had a short-term PD larger than 10 percent. The long-term PDs at a level larger than 10 percent behave similarly. The share of assets with a PD larger than 10 percent dropped to 3 percent at end-2009 from 5.5 percent at end-2008, and further to nil at end-2010. This is consistent with results presented on Figure 1a.

Figure 2b shows the same measure for Luxembourg banks using the book-value version of the Delianedis and Geske model together with the RM measure of volatility. Overall, the reduction of systemic risk was slower in Luxembourg banks than in their parent companies, albeit the latter suffered more from the crisis than the former. For instance, at end-2008 and end-2009, the share of assets with a PD larger than 10 percent remained constant at the level of about 69 percent (see vertical line on the graph). It improved by decreasing to about 62 percent at end-2010. Regarding long-term PDs, there was a mild deterioration from end-2008 to end-2009 as the share of assets with a PD larger than 10 percent increased from about 6 percent to about 7.5 percent, most likely consistent with the negative impact of the sovereign crisis already mentioned. However, at end-2010 it was again about 6 percent.

Overall, systemic risk for the banking groups dramatically declined from 2008 to 2010. It did not change too much over this period for Luxembourg banks. However, there was a

[22] The detailed relationship between Luxembourg systemic risk and European systemic risk in both tranquil and distressed periods is, however, explored in another study and is not further addressed here.

marginal decrease during the second half of 2010 for both short-run PDs and conditional forward PDs.

3. Distance to distress as an indicator of systemic banking risk

Another indicator of systemic risk is based on the distance to distress (DD). Figure 3a shows the estimated time line of the DD for the 20 banking groups in the sample. This is done for individual banking groups weighting the estimated DD by the respective estimated implied asset values. In addition, the portfolios of all banks are treated as "one bank", the system, and its DD is estimated. The DD difference between the system and the average banking group falls during periods of market stress, for example since the middle of 2007. This is an indication that the correlation among banks is increasing as it is normally the case in these circumstances.[23] Figure 3b shows the same estimates (the expected rate of return μ is substituted by the risk-free rate r) for the 33 Luxembourg-registered subsidiaries and branches of the European banking groups. The results are similar to those of the European banking groups. Finally, it is noteworthy that DD differences in Figures 3a and 3b suggest a minimum point around March 2009, which is also the peak of the PD index displayed in Figures 1a – 1c.

4. Other applications of models of PDs for macroprudential objectives

This section illustrates three possible applications of the estimated structural models that highlight their richness. They include an event study, a static simulation of changes in the maturity structure, and an analysis of the importance of banks' business lines.

a. Event study

Figure 4a plots the PDs using the Merton and the Delianedis and Geske models applied to bank group B. As indicated before, the Delianedis and Geske's short-term PD tends to coincide with Merton's. They both track the crisis main events closely, including the additional market distress started toward the end of 2009 with the sovereign crisis. The value added of the Delianedis and Geske model, however, is clear: when banking group B launched a take-over in 2007, the conditional probability of defaulting in its long-term debt increased. It seemed to have raised further with the important expansion of bank group B in Europe in 2008 and in the US in 2009. It is noteworthy that the conditional probability increased at the same time the one-year probability of default was falling. This cannot be captured by the Merton model.

[23] Note that this is an upper-bound proxy for the difference DD as it is calculated without taking into account banks' default dependency.

b. Simulation of maturity structure changes

Another use of the Delianedis and Geske's framework is to perform an (admittedly partial equilibrium) analysis by a macroprudential supervisor interested in recommending structural changes to the debt maturity of a systemic bank in distress. Suppose, for example, that during mid-2009, systemic bank A in Luxembourg requires restructuring its debt maturity to comply with the conditions set for state aid or to accompany the reception of liquidity support. In addition, suppose that the historical risk tolerance of the supervisor for this bank may have been set at a PD of 10 percent. However, as shown in Figure 4b, the current level of bank A's PD is 40 percent. This is the outcome of its short-term debt share in total debt of almost 90 percent at the time. As a result, the supervisor may request a reduction of the share of total debt, other things equal, consistent with a book-value short-term PD of 10 percent. The simulation suggests a refinancing-need time structure consistent with a reduction of short-term debt to 30 percent. Note how this is accompanied by a simultaneous increase in the conditional probability of default on long-term debt, given that the total of the balance sheet has not changed.

c. The importance of the business line

Bank B in Luxembourg is active in the mortgage bond market with a correspondingly dominating share of long-term debt in its balance sheet. If the Merton model for estimating PDs is used to assess the solvency of this bank, PDs will be most of the time close to zero (Figure 4c). The use of the Delianedis and Geske model instead evinces the importance of changes in the probability that bank B will default on its long-term debt conditional on not defaulting on its short-term debt. In fact, starting in 2004, bank B business grew significantly. Further, in early 2006, the first covered bond was issued in Luxembourg (Luxembourg mortgage bond). During this period, the conditional probability of default increased as markets may have considered bank B's expansion as posing an increased long-term credit risk. The conditional probability of default declined afterward, but rose again during the crisis, a feature that the Merton model does not capture. The acquisition of bank B by a large banking group may have also been perceived as increasing its conditional long-term default probability. Finally, the sovereign crisis that started at the end of 2009 pushed up bank B's default probability much more than for other Luxembourg banks (not shown here), likely as a result of bank B's heavy involvement in the covered bond market. This illustrates how this model of PD can be used to better assess the impact of distress on banks as it is more flexible regarding the time-structure of debt, which is also a function of banks' business lines. Therefore, a ranking of banks using this model produces richer results quite valuable for macroprudential analysis.

V. Conclusions and policy implications

The importance of relaxing the Merton model's assumptions has long been discussed in the theoretical literature. This study estimates several models of default probabilities including two models that either relax the assumption of constant volatility, i.e., the Heston and Nandi model, or the assumption of no debt maturity structure, i.e.,the Delianedis and Geske model. In addition, this study deals with the constraint imposed by the use of market data, a serious issue in Luxembourg where banks are not quoted. In an application to 20 major European banking groups active in Luxembourg and to 35 Luxembourg banks, the results highlight the importance of relaxing those assumptions for policymakers interested in macroprudential policy. The Merton GARCH-MIDAS model, by modeling equity volatility separately, permits a more timely identification of changes in default risk. The Delianedis and Geske model, by making the default barrier more flexible than the Merton model, is useful, for example, for analyzing banks' restructuring issues and for better assessing banks' solvency risk while been able to reflect banks' business lines' implications on their PDs. In addition, this study proposes a series of simple indicators of systemic default risk which are useful for assessing and tracking over time this key component of financial stability.

From this work, two future avenues of research that are important for policymakers evince clearly. First, volatility can be modeled structurally so as to separate the role of system developments from individual banks' idiosyncratic features. This will be an important step toward building macro-financial models with a realistic characterization of episodes of financial instability. Ideally, these models will contain early-warning features. Second, systemic risk has not properly been accounted for in this study given that default dependency has been ignored. Therefore, future work will incorporate the externalities that financial intermediaries exert on the rest of the financial system and on the economy in general. This includes developing contagion models, introducing aggregate shocks and widespread imbalances in macro-financial models in which PDs can be different from zero.

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Appendix

A. Filtering Rules for BS_ST_BORROW and BS_LT_BORROW:

Short-term borrowing (BS047) and long-term debt (BS051) from Bloomberg have annual, semi-annual, and quarterly data. The four filtering rules applied to make them consistent are the following:

I. Take any zero as missing data.

II. If annual data exist and are not equal to the semi-annual/quarterly data, then let the semi-annual/quarterly data be equal to the annual data—this gives priority to annual data assuming them to be relatively more reliable.

III. If annual data do not exist for the current fiscal year and both the semi-annual/quarterly data and annual data exist for the previous and next fiscal years, but semi-annual/quarterly data are very different to the corresponding annual data at the previous and next fiscal year, then treat the semi-annual/quarterly as missing data—this is done to avoid unreliable semi-annual /quarterly data.

IV. If annual data do not exist for the current fiscal year and only annual data exist at both previous and next fiscal year, but they are very different to the semi-annual /quarterly data, then treat the semi-annual/quarterly as missing data—this should avoid having unreliable and too choppy semi-annual/quarterly data between previous and next fiscal year.

Table 1: Descriptive statistics of PDs for both banking groups and Luxembourg banks [1]

Banking Groups Monthly
5/31/2001 - 12/31/2010

	Merton	Merton Garch-MIDAS ST	Merton Garch-MIDAS LT	Delianedis and Geske Total	Delianedis and Geske ST	Delianedis and Geske LT	Heston & Nandi
Mean	0.051	0.034	0.035	0.063	0.054	0.012	0.044
STD	0.089	0.064	0.052	0.100	0.090	0.015	0.073
Skewness	2.565	2.538	2.239	2.336	2.511	2.227	2.627
kurtosis	9.263	8.964	7.389	7.998	9.012	7.891	9.939
1st Order Auto-Correlatior	0.966	0.783	0.959	0.968	0.966	0.947	0.960
1/30/2004 - 12/31/2010							
Mean	0.053	0.033	0.033	0.064	0.055	0.010	0.046
STD	0.102	0.069	0.059	0.114	0.103	0.017	0.085
Skewness	2.292	2.615	2.225	2.152	2.267	2.398	2.307
kurtosis	7.219	8.840	6.653	6.477	7.120	7.675	7.595
1st Order Auto-Correlatior	0.969	0.841	0.964	0.971	0.969	0.953	0.964

Luxembourg Banks Quartertly
3/31/2004 - 12/31/2010

	Lux Merton NRW	Lux Delianedis and Geske NRW Total	Lux Delianedis and Geske NRW ST	Lux Delianedis and Geske NRW LT	Lux Merton RW	Lux Delianedis and Geske RW Total	Lux Delianedis and Geske RW ST	Lux Delianedis and Geske RW LT	Lux Merton RM	Lux Delianedis and Geske RM Total	Lux Delianedis and Geske RM ST	Lux Delianedis and Geske RM LT
Mean	0.083	0.101	0.097	0.006	0.175	0.216	0.202	0.023	0.187	0.229	0.213	0.023
STD	0.040	0.049	0.045	0.008	0.026	0.036	0.031	0.013	0.029	0.036	0.031	0.012
Skewness	1.187	1.110	1.146	1.309	0.329	0.212	0.170	2.443	0.468	0.257	0.311	2.290
kurtosis	3.668	3.200	3.445	3.390	2.198	2.081	2.009	10.535	1.811	1.972	1.866	9.822
1st Order Auto-Correlatior	0.814	0.850	0.829	0.848	0.648	0.682	0.668	0.335	0.863	0.851	0.842	0.481

[1] The table reports the first four sample moments and the first order autocorrelation of PDs for both banking groups and Luxembourg banks.

Table 2: Descriptive statistics of default index for banking groups and Luxembourg banks

	Mean	STD	Skewness	kurtosis	Min	Max	1st Order Auto-Correlation	Mean	STD	Skewness	kurtosis	Min	Max	1st Order Auto-Correlation
	Default Index 3/31/2004 - 12/31/2010							Default Index Change 6/31/2004 - 12/31/2010						
Group Delianedis and Geske Total	2.26	4.03	2.16	6.44	0.00	15.10	0.84	0.05	2.31	0.95	7.67	-5.19	8.11	0.52
Group Delianedis and Geske ST	2.12	3.98	2.26	6.94	0.00	15.09	0.83	0.05	2.39	0.99	8.34	-5.35	8.56	0.47
Group Delianedis and Geske LT	4.29	7.03	2.32	7.21	0.02	27.81	0.79	0.06	4.65	-0.96	7.90	-15.99	12.02	0.42
Lux Delianedis and Geske RM Total	1.01	0.16	0.26	1.97	0.78	1.31	0.85	0.01	0.09	0.23	4.00	-0.18	0.24	-0.13
Lux Delianedis and Geske RM ST	1.02	0.15	0.31	1.87	0.80	1.26	0.84	0.01	0.08	0.59	3.80	-0.14	0.24	-0.14
Lux Delianedis and Geske RM LT	0.88	0.47	2.29	9.82	0.39	2.73	0.48	0.02	0.48	-1.43	9.41	-1.78	1.22	-0.37

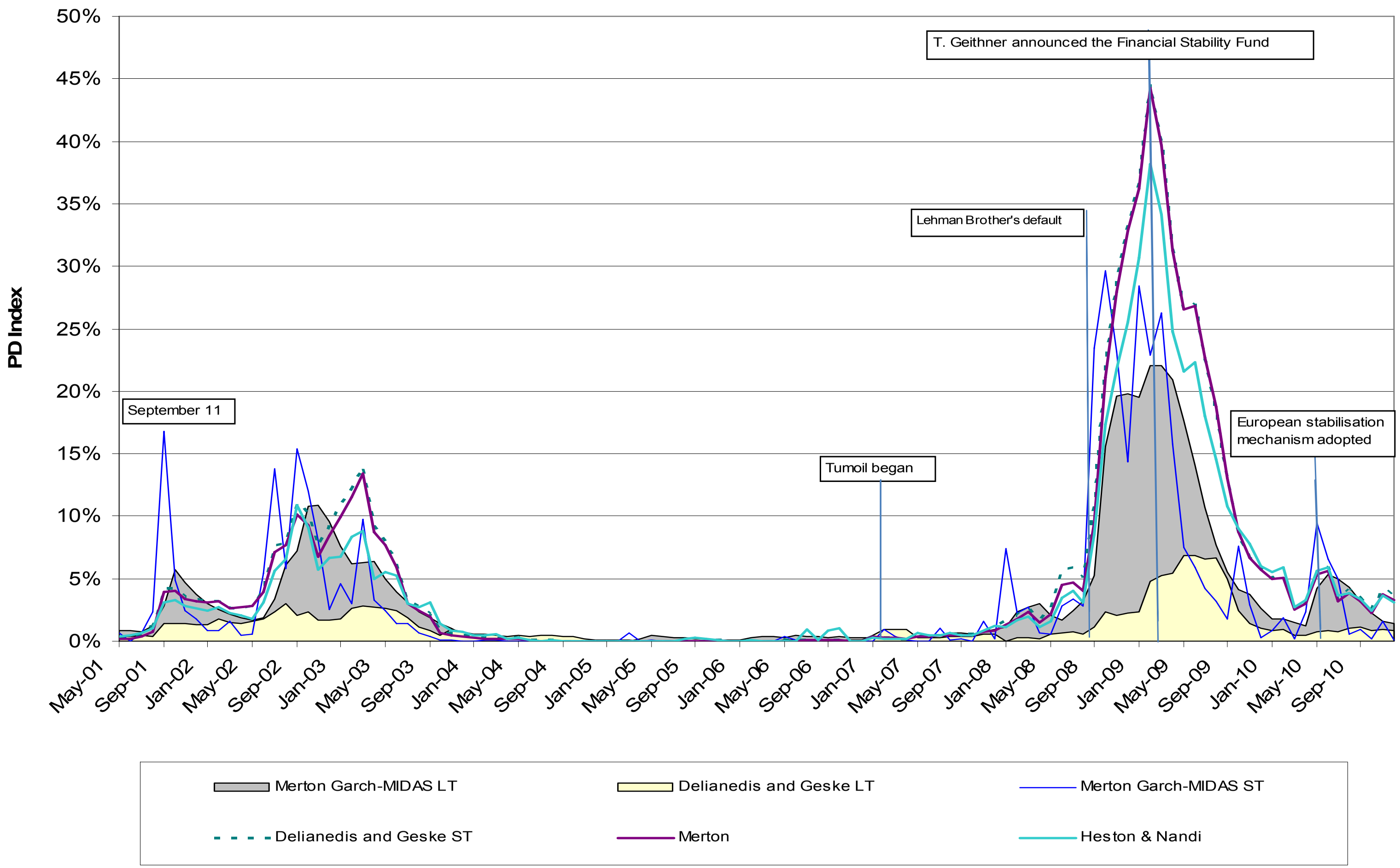
[1] The table reports the first four sample moments, the first order autocorrelation, minimum, and maximum of the default index for both banking groups and Luxembourg banks from 2004 to 2010. The default index is constructed by dividing full PDs by the PD on the first quarter of 2008.

Table 3: PDs and changes in PDs rank correlation between European banking groups and Luxembourg banks [1]
January 30, 2004 to December 31, 2010

	Banking groups						
	PDs						
Luxembourg Banks	Merton	Merton Garch-MIDAS ST	Merton Garch-MIDAS LT	Delianedis and Geske Total	Delianedis and Geske ST	Delianedis and Geske LT	Heston & Nandi
Merton RM	**0.60**	**0.58**	**0.60**	**0.55**	**0.59**	**0.46**	**0.60**
Delianedis and Geske RM Total	**0.51**	**0.50**	**0.53**	**0.45**	**0.49**	**0.36**	**0.51**
Delianedis and Geske RM ST	**0.50**	**0.49**	**0.53**	**0.45**	**0.49**	**0.36**	**0.50**
Delianedis and Geske RM LT	**0.53**	**0.52**	**0.52**	**0.47**	**0.52**	**0.33**	**0.53**
	Changes in PDs						
Merton RM	**0.16**	**0.14**	0.09	0.12	**0.15**	-0.03	**0.14**
Delianedis and Geske RM Total	0.11	0.13	0.05	0.11	0.12	0.00	0.06
Delianedis and Geske RM ST	**0.21**	**0.14**	0.09	**0.20**	**0.21**	0.07	**0.14**
Delianedis and Geske RM LT	0.08	0.07	-0.02	0.06	0.08	0.09	0.11

[1]The table reports the Kendall correlation matrix of the monthly PDs and changes in PDs between European banking groups and Luxembourg banks from 2004 to 2010. For Luxembourg banks, monthly PDs are assumed to be same within each quarter. A bold value with underscore indicates significance at the 95% level, whereas a bold value without underscore indicates significance at the 90% level.

Figure 1a - Asset Weighted PD Index for European Banking Groups (Merton / Merton Garch-MIDAS / Heston & Nandi / Delianedis and Geske Models)



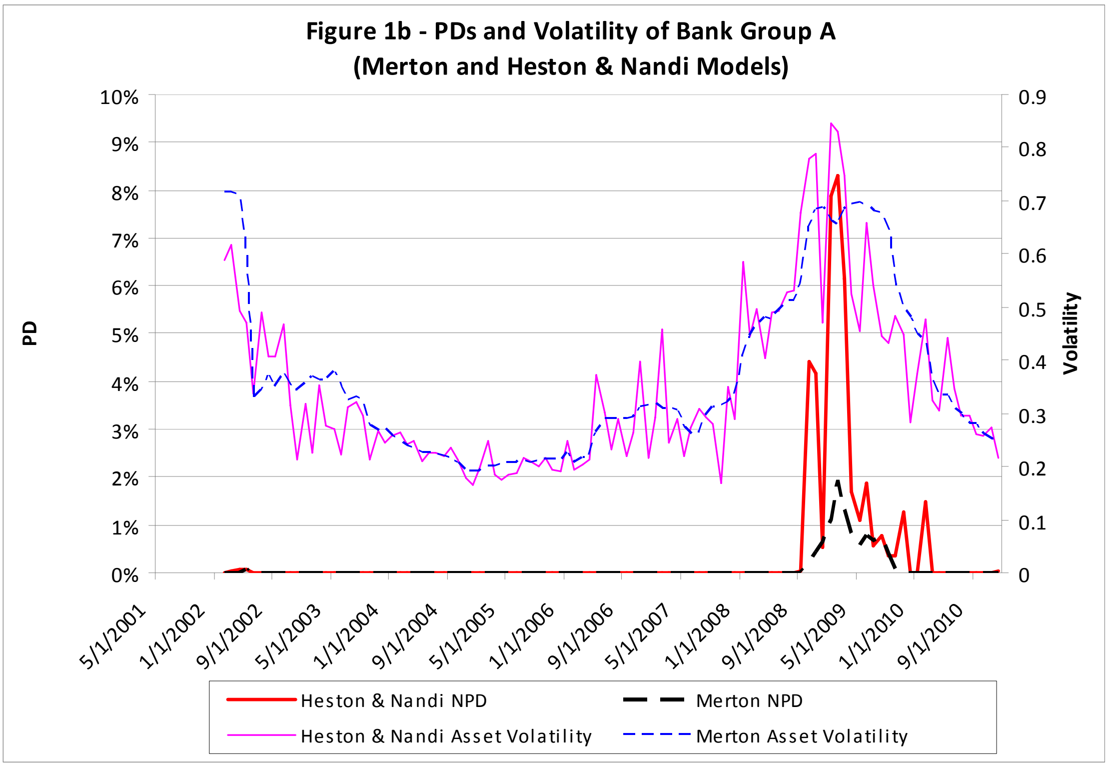


Figure 1b - PDs and Volatility of Bank Group A
(Merton and Heston & Nandi Models)
PD
Volatility
10%
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5/1/2001
1/1/2002
9/1/2002
5/1/2003
1/1/2004
9/1/2004
5/1/2005
1/1/2006
9/1/2006
5/1/2007
1/1/2008
9/1/2008
5/1/2009
1/1/2010
9/1/2010
Heston & Nandi NPD
Merton NPD
Heston & Nandi Asset Volatility
Merton Asset Volatility

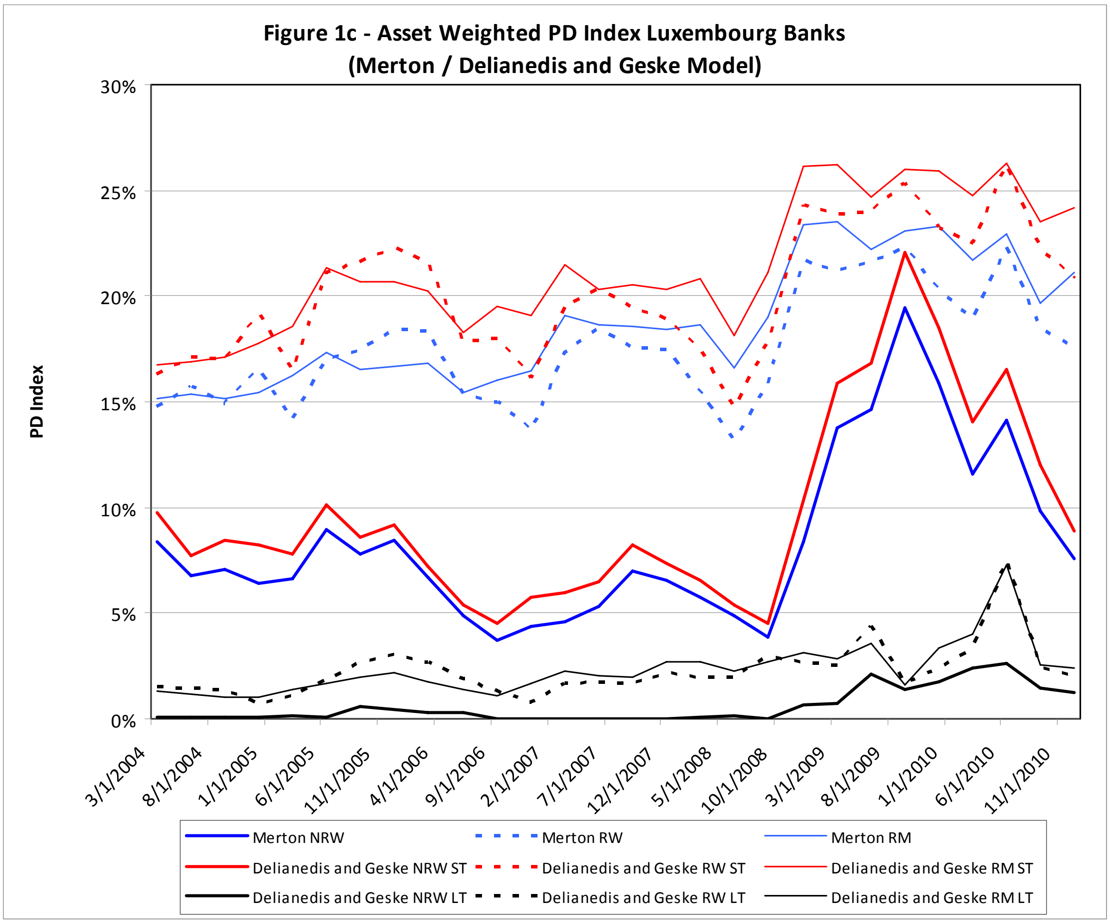


Figure 1c - Asset Weighted PD Index Luxembourg Banks
(Merton / Delianedis and Geske Model)
PD Index
30%
25%
20%
15%
10%
5%
0%
3/1/2004
8/1/2004
1/1/2005
6/1/2005
11/1/2005
4/1/2006
9/1/2006
2/1/2007
7/1/2007
12/1/2007
5/1/2008
10/1/2008
3/1/2009
8/1/2009
1/1/2010
6/1/2010
11/1/2010
Merton NRW
Merton RW
Merton RM
Delianedis and Geske NRW ST
Delianedis and Geske RW ST
Delianedis and Geske RM ST
Delianedis and Geske NRW LT
Delianedis and Geske RW LT
Delianedis and Geske RM LT

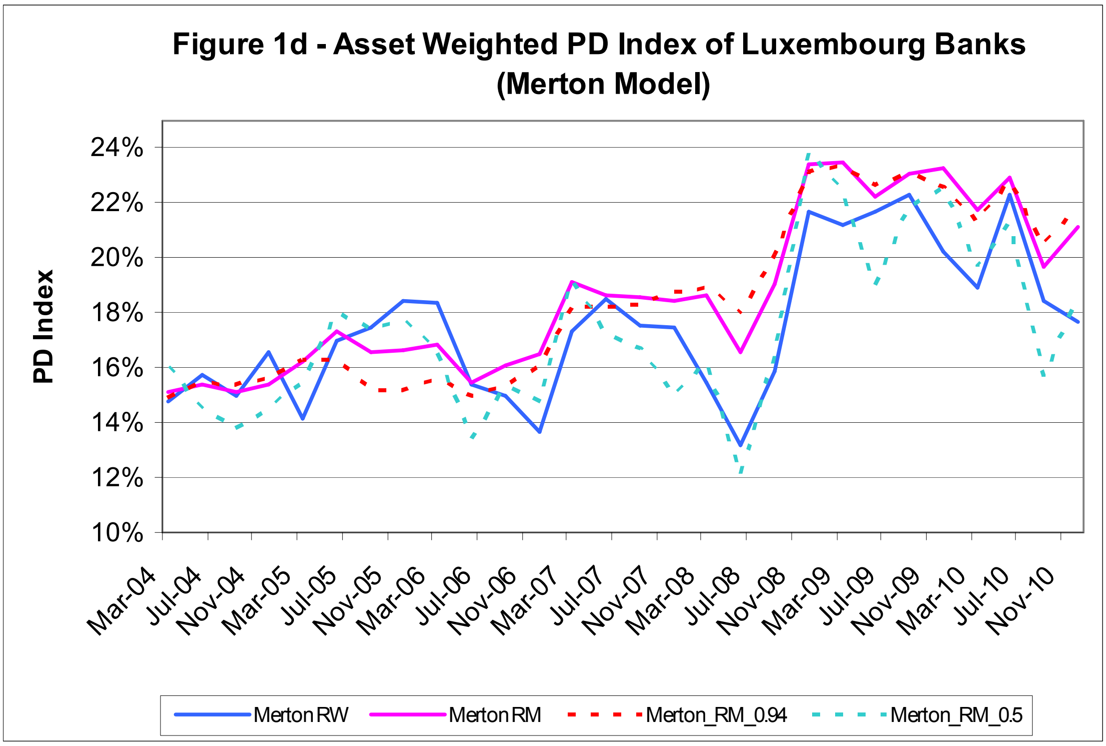


Figure 1d - Asset Weighted PD Index of Luxembourg Banks (Merton Model)
PD Index
24%
22%
20%
18%
16%
14%
12%
10%
Mar-04
Jul-04
Nov-04
Mar-05
Jul-05
Nov-05
Mar-06
Jul-06
Nov-06
Mar-07
Jul-07
Nov-07
Mar-08
Jul-08
Nov-08
Mar-09
Jul-09
Nov-09
Mar-10
Jul-10
Nov-10
Merton RW
Merton RM
Merton_RM_0.94
Merton_RM_0.5

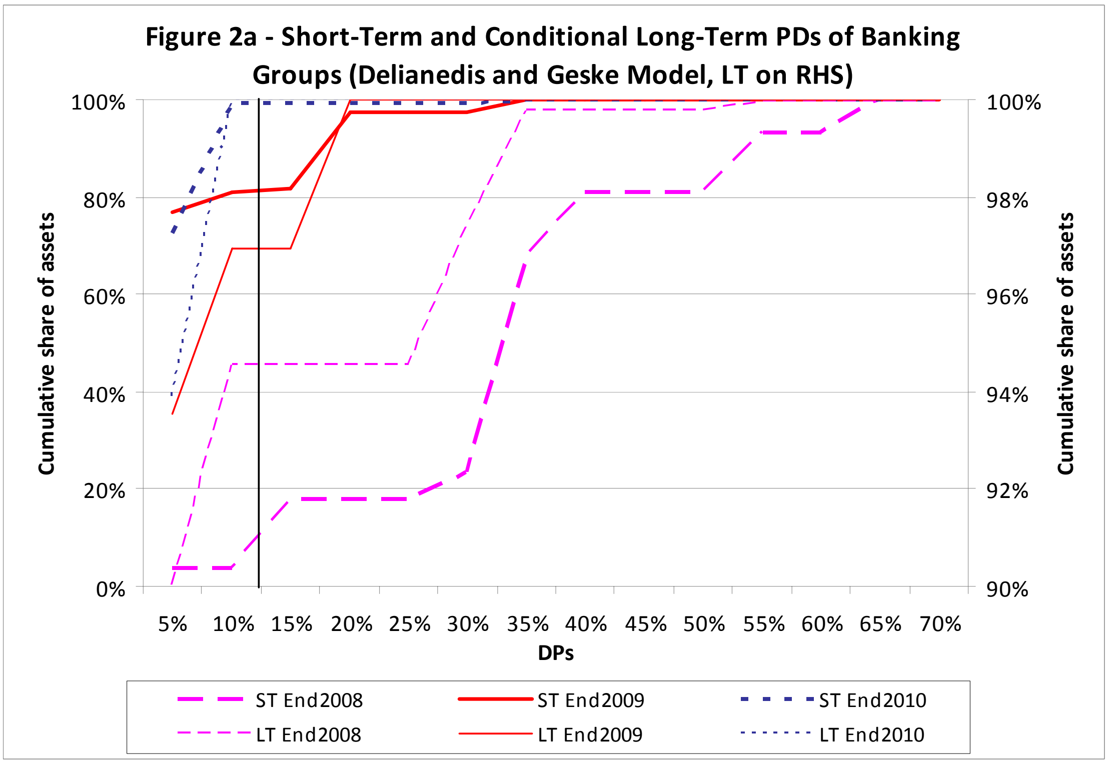


Figure 2a - Short-Term and Conditional Long-Term PDs of Banking Groups (Delianedis and Geske Model, LT on RHS)
Cumulative share of assets
100%
80%
60%
40%
20%
0%
100%
98%
96%
94%
92%
90%
5%
10%
15%
20%
25%
30%
35%
40%
45%
50%
55%
60%
65%
70%
DPs
ST End2008
ST End2009
ST End2010
LT End2008
LT End2009
LT End2010

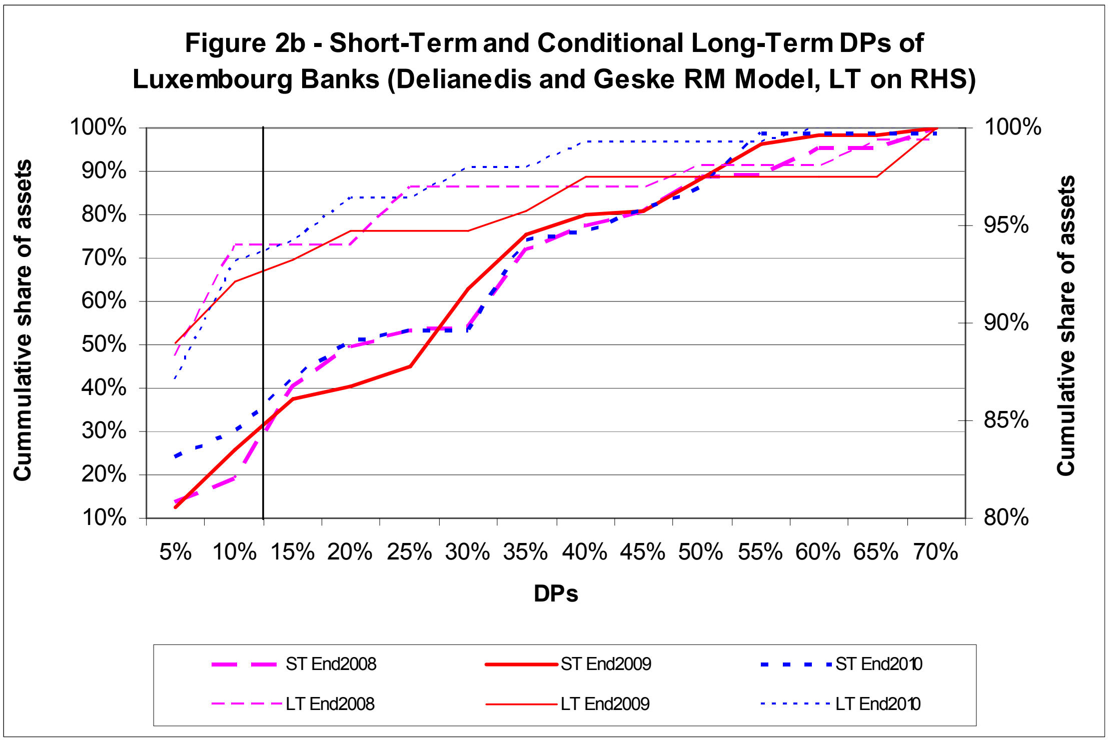


Figure 2b - Short-Term and Conditional Long-Term DPs of Luxembourg Banks (Delianedis and Geske RM Model, LT on RHS)
Cummulative share of assets
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
Cumulative share of assets
100%
95%
90%
85%
80%
5%
10%
15%
20%
25%
30%
35%
40%
45%
50%
55%
60%
65%
70%
DPs
ST End2008
ST End2009
ST End2010
LT End2008
LT End2009
LT End2010

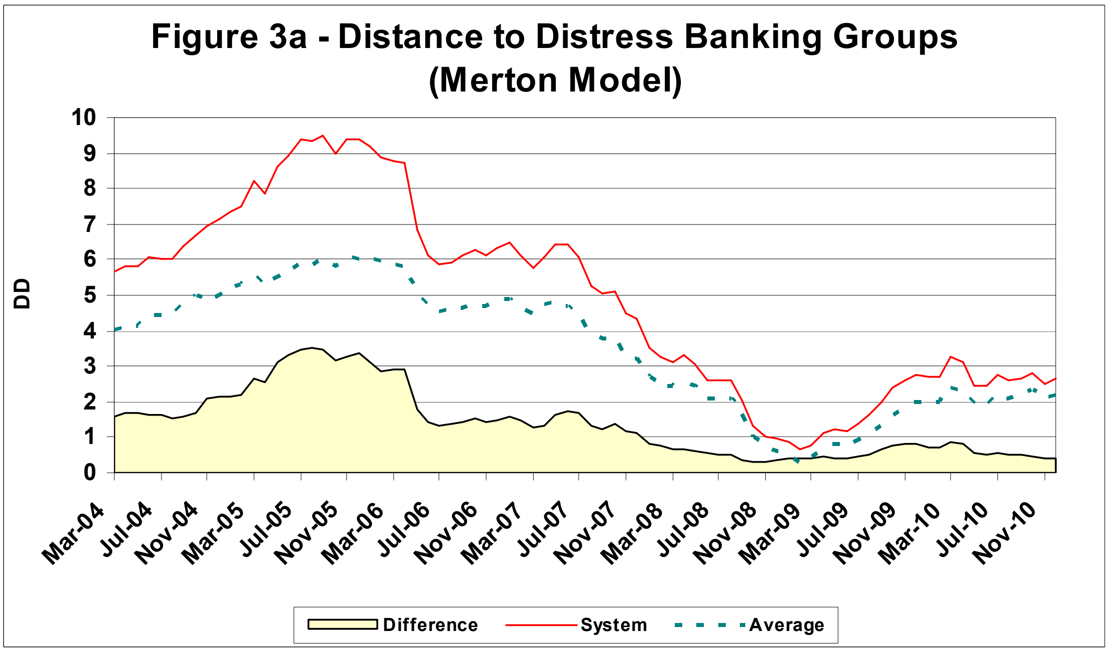


Figure 3a - Distance to Distress Banking Groups (Merton Model)
DD
10
9
8
7
6
5
4
3
2
1
0
Mar-04
Jul-04
Nov-04
Mar-05
Jul-05
Nov-05
Mar-06
Jul-06
Nov-06
Mar-07
Jul-07
Nov-07
Mar-08
Jul-08
Nov-08
Mar-09
Jul-09
Nov-09
Mar-10
Jul-10
Nov-10
Difference
System
Average

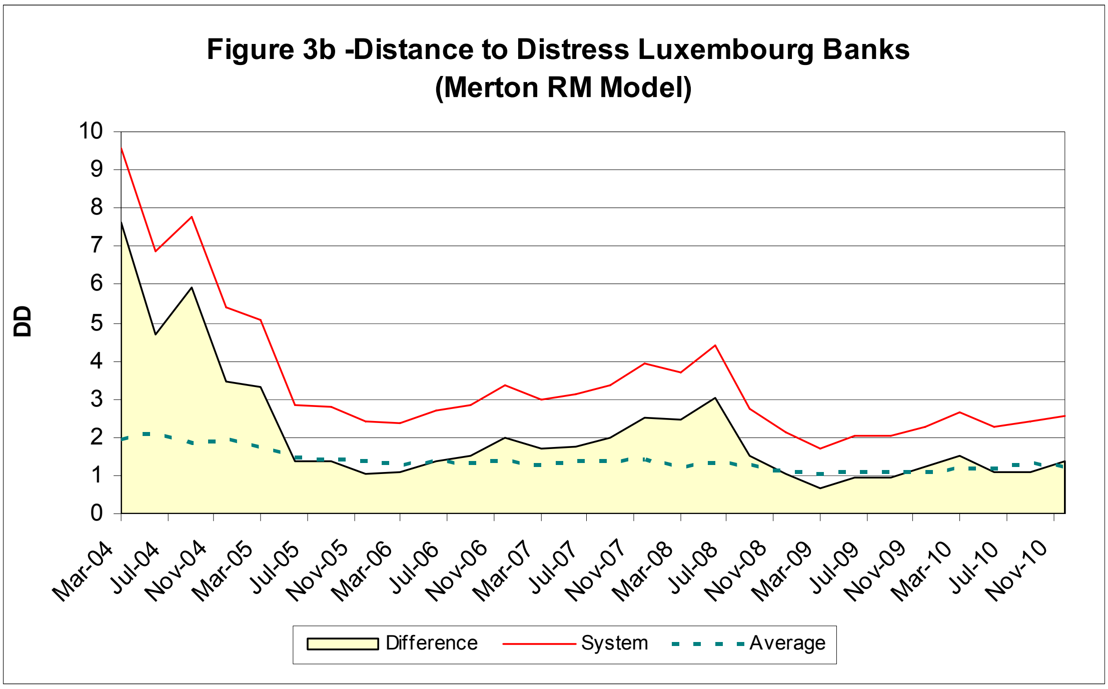


Figure 3b -Distance to Distress Luxembourg Banks
(Merton RM Model)
DD
10
9
8
7
6
5
4
3
2
1
0
Mar-04
Jul-04
Nov-04
Mar-05
Jul-05
Nov-05
Mar-06
Jul-06
Nov-06
Mar-07
Jul-07
Nov-07
Mar-08
Jul-08
Nov-08
Mar-09
Jul-09
Nov-09
Mar-10
Jul-10
Nov-10
Difference
System
Average

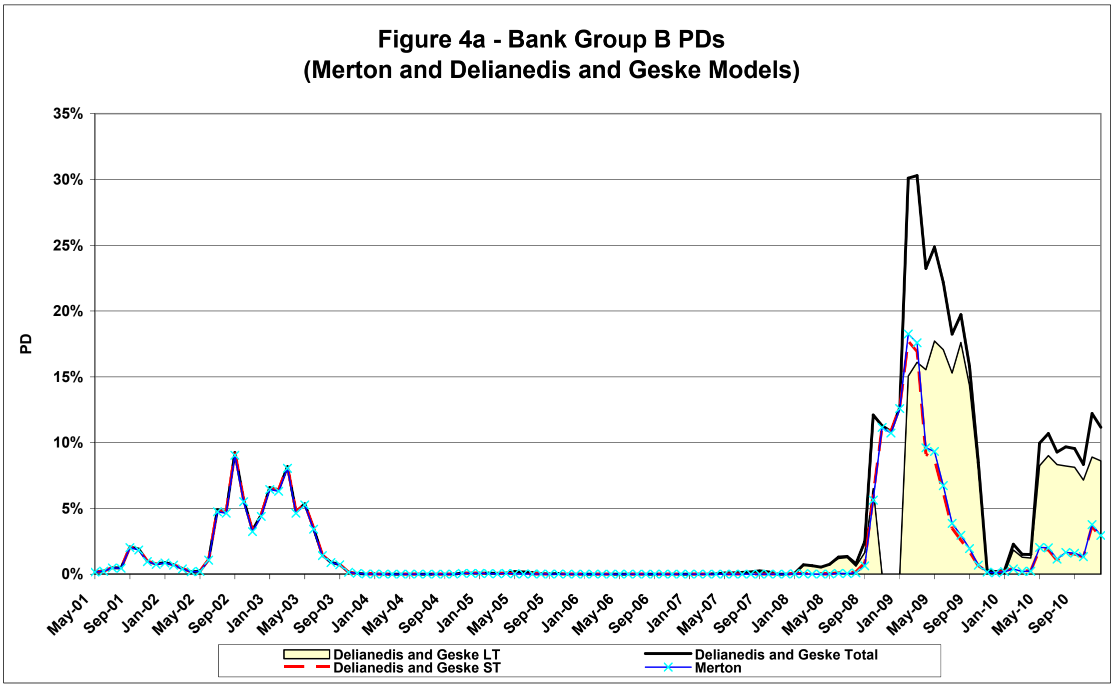


Figure 4a - Bank Group B PDs
(Merton and Delianedis and Geske Models)
PD
35%
30%
25%
20%
15%
10%
5%
0%
May-01
Sep-01
Jan-02
May-02
Sep-02
Jan-03
May-03
Sep-03
Jan-04
May-04
Sep-04
Jan-05
May-05
Sep-05
Jan-06
May-06
Sep-06
Jan-07
May-07
Sep-07
Jan-08
May-08
Sep-08
Jan-09
May-09
Sep-09
Jan-10
May-10
Sep-10
Delianedis and Geske LT
Delianedis and Geske ST
Delianedis and Geske Total
Merton

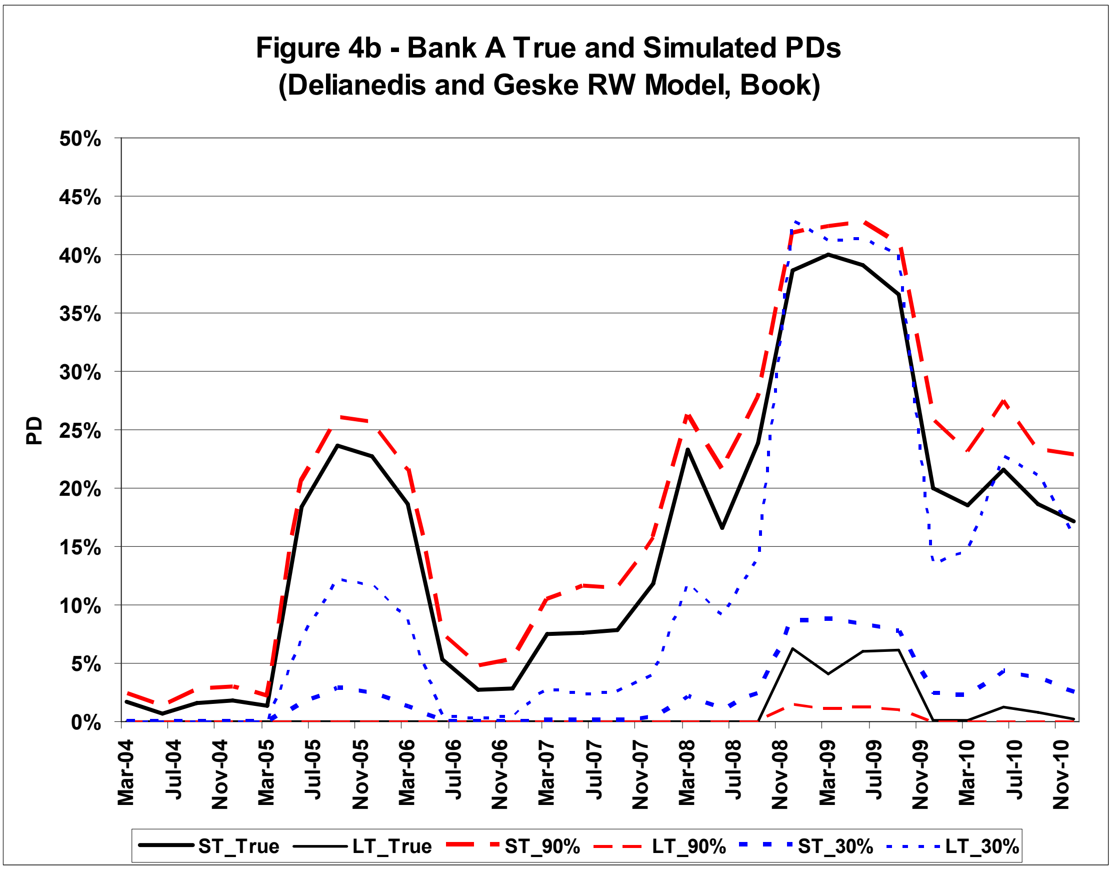


Figure 4b - Bank A True and Simulated PDs
(Delianedis and Geske RW Model, Book)
PD
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
Mar-04
Jul-04
Nov-04
Mar-05
Jul-05
Nov-05
Mar-06
Jul-06
Nov-06
Mar-07
Jul-07
Nov-07
Mar-08
Jul-08
Nov-08
Mar-09
Jul-09
Nov-09
Mar-10
Jul-10
Nov-10
ST_True
LT_True
ST_90%
LT_90%
ST_30%
LT_30%

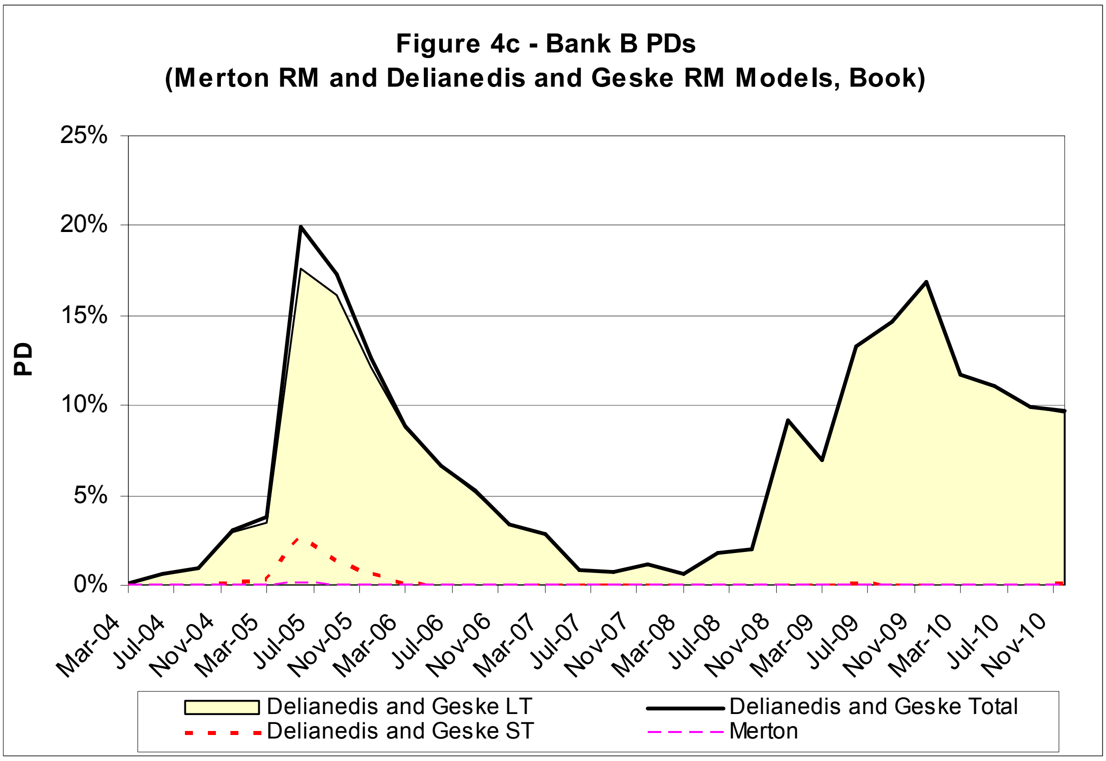


Figure 4c - Bank B PDs
(Merton RM and Delianedis and Geske RM Models, Book)
PD
25%
20%
15%
10%
5%
0%
Mar-04
Jul-04
Nov-04
Mar-05
Jul-05
Nov-05
Mar-06
Jul-06
Nov-06
Mar-07
Jul-07
Nov-07
Mar-08
Jul-08
Nov-08
Mar-09
Jul-09
Nov-09
Mar-10
Jul-10
Nov-10
Delianedis and Geske LT
Delianedis and Geske Total
Delianedis and Geske ST
Merton

BANQUE CENTRALE DU LUXEMBOURG

EUROSYSTÈME

2, boulevard Royal
L-2983 Luxembourg

Tél.: +352 4774-1
Fax: +352 4774 4910

www.bcl.lu • info@bcl.lu