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LEARNING, EXPECTATIONS AND MONETARY POLICY

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ABSTRACT. I present a New Keynesian model in which the central bank's anti-inflationary preferences change over time. Agents do not observe the current monetary regime, but rationally learn about it using Bayes theorem. The model provides a structural interpretation for the contractionary effects of monetary policy uncertainty shocks as recently documented in the empirical literature. In addition, the model shows that learning reduces the effects of monetary policy on the economy by softening the link between fundamentals and equilibrium prices and allocations.

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NON-TECHNICAL SUMMARY

The uncertainty that surrounds public perceptions of central bank policy matters for business cycle dynamics. An increasing number of empirical studies document the contractionary effects of increases in monetary policy uncertainty in developed economies.

I offer a theoretical explanation for these contractionary effects. To this end, I present a New Keynesian model in which private agents have limited information about the central bank's reaction function. The latter alternates between periods of active inflation stabilization and periods of passive inflation stabilization. Each quarter, private agents estimate the parameters of the current monetary regime by combining what they see happening in the economy with their own past beliefs.

This belief structure accounts for the contractionary effects of increases in monetary policy uncertainty. Spikes in uncertainty about the current monetary regime obscure the likely path of future nominal interest rates, and hence, of the marginal product of capital. This unpredictability encourages private agents to defer investment decisions until additional information becomes available. The fall in investment then hinders economic activity through the standard channels: a decline in aggregate demand, and the prospect of a protracted period of unusually low capital stock.

In addition, I explore how uncertainty about the current monetary policy regime affects the transmission of monetary policy shocks. By softening the link between fundamentals and equilibrium prices and allocations, learning renders the economy less responsive to monetary policy.

Changes in the monetary policy regime in the model capture a *constrained discretion* strategy [Bernanke and Mishkin (1997)], followed by the Federal Reserve and other central banks. This allows the central bank some flexibility to de-emphasize inflation stabilization to pursue alternative short-term goals, limiting cyclical swings in resource utilization, while retaining a strong commitment to keeping inflation low and stable.

Résumé non technique

Les décisions de politique monétaire sont perçues par le public avec un niveau d'incertitude qui varie à travers le temps, ce qui a des conséquences pour les cycles économiques. Au cours des dernières années, de nombreuses études ont démontré que le niveau d'activité économique tend à diminuer quand cette incertitude est plus grande.

Ce papier propose une explication théorique pour ces contractions économiques à travers une version du modèle keynésien dans laquelle les ménages et les entreprises ont une connaissance imparfaite de la fonction de réaction de la banque centrale. Cette dernière alterne entre des périodes quand la stabilisation de l'inflation est active et des périodes quand elle est passive. Chaque trimestre, les agents privés estiment les paramètres du régime monétaire courant en combinant leurs observations de l'état actuel de l'économie avec ce qu'ils ont appris du passé.

Cette information imparfaite concernant le régime monétaire courant explique les effets contractionnaires d'une hausse de l'incertitude entourant la politique monétaire. Des pics d'incertitude cachent la probable trajectoire des taux d'intérêt nominaux dans le futur et, par conséquent, l'évolution de l'efficacité marginale du capital. Une telle imprévisibilité incite les investisseurs à reporter leurs décisions afin d'attendre des nouvelles informations. La baisse des investissements freine l'activité économique en diminuant la demande globale et en soulevant la possibilité que le stock de capital restera exceptionnellement bas pendant une période prolongée.

Ce papier montre que la perception incertaine de la fonction de réaction de la banque centrale entrave la transmission de la poliltique monétaire à l'économie réelle. En fragilisant le lien entre les fondamentaux économiques, les prix et les allocations d'équilibre, cette incertitude rend l'économie moins sensible aux décisions de politique monétaire.

Dans ce modèle, les changements de régime monétaire représentent la stratégie de «constrained discretion» [Bernanke and Mishkin (1997)] suivie par la Réserve Fédérale Américaine et par de nombreuses autres autorités monétaires. Cette stratégie accorde à la banque centrale une certaine flexibilité dans la stabilisation de l'inflation à court terme, afin de poursuivre des objectifs alternatifs. Par exemple, certaines banques centrales visent à stabiliser les fluctuations cycliques tout en restant fermement engagées à maintenir l'inflation à un niveau bas et stable.

1. Introduction

The uncertainty that surrounds public perceptions of central bank policy actions matters for business cycles. Using time-series econometrics, Creal and Wu (2017) and Husted et al. (2019) document the contractionary effects of increases in monetary policy uncertainty (MPU) in the United States, while Azqueta-Gavaldon et al. (2020) does so for the euro area. Why does MPU dampen economic activity?

Learning dynamics are key. In the context of an otherwise standard New Keynesian model, I consider an environment in which households and firms have limited information about the central bank's reaction function. As in Bianchi and Melosi (2018), the latter alternates between periods of active inflation stabilization, when the Taylor principle is satisfied, and periods of passive inflation stabilization, when the Taylor principle is violated.[1] Each quarter, the central bank sets the nominal interest rate consistent with past inflation, its desired anti-inflationary attitude, and discretionary monetary policy. While private agents know the structure of the economy and observe all endogenous variables, they do not know the current desired anti-inflationary attitude, for it is obscured by discretionary monetary policy. Instead, agents make an inference about the current monetary regime by combining what they see happening in the economy with their own past beliefs using the Hamilton filter [Hamilton (1989)]. Lastly, they include their state likelihood estimates into their expectations, and hence into their decision-making process.

This belief structure explains the contractionary effects of increases in MPU. Spikes in uncertainty about the current monetary regime obscure the likely path of future nominal interest rates, and hence, of the marginal product of capital. This unpredictability of capital returns makes it worth deferring investment decisions to receive additional information. The fall in investment then hinders economic activity through the standard channels: a fall in aggregate demand, and the prospect of a protracted period of unusually low capital stock. This rationale echoes the real option approach that builds on irreversible investment to highlight the importance of delaying investment until uncertainty is resolved [e.g., Bernanke (1983), Abel and Eberly (1994), Bertola and Caballero (1994)]. In addition, it matches the negative link between MPU and firm investment found by Husted et al. (2019).

I also explore how the unknown monetary policy regime affects the transmission of monetary policy shocks. As in Eusepi and Preston (2011), beliefs are a function of historical data, introducing an additional state variable. The model dynamics therefore evolve over time in response to central bank actions and private sector inferences. Relative to a Rational Expectations analysis of the model, learning makes the economy less sensitive to changes in the central bank's inflation

[1] According to Bernanke and Mishkin (1997), many central banks follow a constrained discretion strategy that provides them with some flexibility to de-emphasize inflation stabilization to pursue secondary objectives.

response. Households are aware that they do not know the current monetary regime. As a result, they base their actions on the choices they would make under either monetary regime. By softening the link between fundamentals and equilibrium prices and allocations, learning reduces the effects of monetary policy on the economy.

As mentioned earlier, this paper relates to the growing empirical literature documenting the contractionary effects of MPU shocks; see e.g. Creal and Wu (2017), Husted et al. (2019), and Azqueta-Gavaldon et al. (2020). Specifically, I offer a structural interpretation for these effects.

In addition, my work contributes to the literature on policy uncertainty in general equilibrium; see see e.g. Richter and Throckmorton (2015) and Davig and Foerster (2019) for fiscal policy uncertainty; and Schorfheide (2005), Eusepi and Preston (2010) and Cogley et al. (2015) for monetary policy uncertainty. The closest paper is Bianchi and Melosi (2018), which also develops a general equilibrium model where the central bank deviates from active inflation stabilization, and households face uncertainty about the nature of these deviations. I differ from Bianchi and Melosi (2018) in at least three ways: (i) households in my model are uncertain about the current monetary policy regime, while agents in their model do not know whether the central bank is engaging in a short or a long lasting deviation from active inflation stabilization. (ii) I focus on the effect of MPU shocks, while Bianchi and Melosi (2018) study the welfare consequences of limited information. (iii) I use a global solution method, capturing the endogenous non-linearities linked to the learning process.

Lastly, my paper relates to the stochastic volatility literature [e.g., Bloom (2009), Bloom et al. (2018) and Bachmann and Bayer (2013)]. This body of work mostly focuses on how uncertainty about productivity influences the business cycle, and has one main disadvantage: it is silent on the exact source of uncertainty. In my model, however, the origin of uncertainty is explicit: the monetary policy regime is unobserved. This clarity is essential to obtain structural interpretations.

The remainder of the paper is organized as follows. Section 2 introduces the model, and the different information sets. Section 3 connects the model to the data. Section 4 presents the results. Section 5 concludes.

2. The model

I adopt a New Keynesian model with Rotemberg price-setting frictions to study the consequences of recurring unobserved changes in the central bank's reaction function. The first subsection describes the standard part of the model. Though this description can be found in many sources [e.g. Christiano et al. (2010) and Gavin et al. (2015)], I include it to make my presentation self-contained. The second subsection explains the use of Bayesian learning to endogenize monetary policy uncertainty.

2.1. **Standard part of the model.**

2.1.1. *Households.* A representative household chooses $\{c_t, h_t, b_t, i_t, k_t\}_{t=1}^{\infty}$ to maximize expected lifetime utility given by:

$$\mathrm{E}_t \sum_{t=1}^{\infty} \beta^{t-1} \left[\log c_t - \chi \frac{h_t^{1+\eta}}{1+\eta} \right],$$

where $\beta \in [0,1[$ is the discount factor, $\chi > 0$ is a scale parameter, $1/\eta$ is the Frisch elasticity of labor supply, c_t is real consumption, h_t is labor hours, b_t is the quantity of nominal bonds purchased by the household, i_t is real investment, k_t is the capital stock and E_t is the expectation operator conditional on information available at time t. These choices are constrained by:

$$c_t + i_t + \Gamma(i_t/k_{t-1})k_{t-1} + b_{t+1} = w_t h_t + r_t^k k_{t-1} + \Pi_t + \frac{r_{t-1} b_t}{1+\pi_t},$$

$$k_t = (1-\delta)k_{t-1} + i_t. \tag{1}$$

Here π_t is the rate of inflation from $t-1$ to t, w_t is the real wage, r_t^k is the capital rental rate, Π_t denotes dividends received from ownership of retail firms, r_t denotes the one-period gross nominal rate of interest on a bond purchased in period t, and δ is the depreciation rate of capital. The model features adjustment costs: $\Gamma(i_t/k_{t-1})$ is a positive, increasing and convex function measuring the cost of adjusting the capital stock. Specifically, I assume that $\Gamma(i_t/k_{t-1}) = \gamma(i_t/k_{t-1} - \delta)^2/2$, where γ governs how sensitive investment is to the price of capital.

Solving the household's problem yields standard optimality conditions:

$$w_t = \chi c_t h_t^{\eta}, \tag{2}$$

$$1 = \beta \mathrm{E}_t \left[\frac{c_t}{c_{t+1}} \frac{r_t}{1+\pi_{t+1}} \right], \tag{3}$$

$$q_t = 1 + \gamma \left(\frac{i_t}{k_{t-1}} - \delta \right), \tag{4}$$

$$q_t = \beta \mathrm{E}_t \left\{ \frac{c_t}{c_{t+1}} \left[r_{t+1}^k - \frac{\gamma}{2} \left(\frac{i_{t+1}}{k_t} - \delta \right)^2 + \gamma \left(\frac{i_{t+1}}{k_t} - \delta \right) \frac{i_{t+1}}{k_t} + q_{t+1}(1-\delta) \right] \right\}, \tag{5}$$

Eq. (2) pins down the supply of labor by equating the marginal cost of work, in consumption units, with the marginal benefit, the real wage. Eq. (3), the standard Euler equation for bonds, balances the marginal cost of purchasing a bond with the corresponding expected benefit. Eq. (4) defines an investment demand function relating net investment (i.e. $i_t - \delta k_{t-1}$) to Tobin's q. More precisely, net investment will be positive if and only if Tobin's q exceeds unity. Lastly, eq. (5) defines Tobin's q as the present discounted value of the marginal profits from an extra unit of capital, measured in terms of future marginal products and reductions in installation costs.

2.1.2. *Firms.* The production sector consists of intermediate goods firms producing a continuum of differentiated inputs under monopolistic competition, and a representative final goods firm.

The final goods firm produces gross output, Y_t, using a Dixit-Stiglitz aggregator:

$$Y_t = \left[\int_0^1 y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $y_{i,t}$ is the quantity of differentiated good sold by intermediate firm $i \in [0,1]$, and $\epsilon > 1$ is the elasticity of substitution between the different inputs. Since the final goods producer maximizes profits in a perfectly competitive environment, the demand function facing each intermediate firm i is:

$$y_{i,t} = \left[\frac{p_{i,t}}{P_t}\right]^{-\epsilon} Y_t,$$

where $p_{i,t}$ is firm i's sale price and P_t is the price of the composite good, which is defined by:

$$P_t = \left[\int_0^1 p_{i,t}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

Each intermediate firm i operates an identical technology function given by $y_t^i = h_{i,t}^{1-\alpha} k_{i,t-1}^{\alpha}$, where $\alpha \in [0,1]$. Variables $h_{i,t}$ and $k_{i,t-1}$ are the levels of employment and capital used by firm i. In this context, firm i's real marginal cost, $mc_{i,t}$, evolves according to:

$$mc_{i,t} = \left[\frac{w_t}{1-\alpha}\right]^{1-\alpha} \left[\frac{r_t^k}{\alpha}\right]^{\alpha}. \tag{6}$$

In equilibrium, firm i balances the relative price of its inputs with the corresponding ratio of marginal productivities:

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{k_{i,t-1}}{h_{i,t}}. \tag{7}$$

Every firm relies on the same combination of inputs.

The model features price-setting frictions along the lines proposed by Rotemberg (1982). Each firm faces a real cost to adjust its price, which captures the negative effects that price changes have on customer-firm relationships. This menu cost is given by:

$$\frac{\psi}{2}\left[\frac{p_{i,t}}{p_{i,t-1}} - 1\right]^2 y_t,$$

where $\psi > 0$ determines the size of the adjustment cost. Each firm i chooses its price to maximize expected real profits, given by:

$$\left[\frac{p_{i,t}}{P_t}\right] y_{i,t} - \left[\frac{mc_{i,t}}{P_t}\right] y_{i,t} - \frac{\psi}{2}\left[\frac{p_{i,t}}{p_{i,t-1}} - 1\right]^2 y_t$$
$$+\beta\Lambda_{t,t+1}\left\{\left[\frac{p_{i,t+1}}{P_{t+1}}\right] y_{i,t+1} - \left[\frac{mc_{i,t+1}}{P_{t+1}}\right] y_{i,t+1} - \frac{\psi}{2}\left[\frac{p_{i,t+1}}{p_{i,t}} - 1\right]^2 y_{t+1}\right\},$$

where $\Lambda_{t,t+1}$ is the household intertemporal marginal rate of substitution. Differentiating the objective function with respect to $p_{i,t}$ and imposing symmetry among all retail goods yields a standard New Keynesian Phillips Curve:

$$\psi\pi_t(1+\pi_t) = 1 + \epsilon\left[mc_t - 1\right] + \beta \mathrm{E}_t\left[\frac{c_t}{c_{t+1}}\psi\pi_{t+1}(1+\pi_{t+1})\frac{y_{t+1}}{y_t}\right]. \tag{8}$$

As usual, eq. (8) links current inflation to next period's expected inflation and output gap. If there were no price-setting frictions, the real marginal cost of producing a unit of output would be constant and equal to $\frac{\epsilon-1}{\epsilon}$.

2.1.3. *Aggregate resources and private sector equilibrium conditions.* Rotemberg pricing ensures that labor and capital inputs are equally distributed among the various intermediate goods firms. Aggregate output is thus given by:

$$y_t = h_t^{1-\alpha}k_{t-1}^{\alpha}, \tag{9}$$

where $h_t = \sum_i h_{i,t}$ and $k_{t-1} = \sum_i k_{i,t-1}$. The economy-wide resource constraint is:

$$\left[1 - \frac{\psi}{2}\pi_t^2\right] y_t = c_t + i_t + \Gamma(i_t/k_{t-1})k_{t-1}, \tag{10}$$

where the left hand side represents aggregate production net of price adjustment costs, and the right hand side corresponds to aggregate use of resources.

The private sector equilibrium conditions are equations (1) through (10). These are 10 equations in the following 11 endogenous unknowns:

$$c_t, h_t, i_t, k_t, w_t, r_t^k, q_t, mc_t, y_t, r_t, \pi_t.$$

As it stands, the system is undetermined, because monetary policy is not yet defined. I turn to this in the following subsection.

2.2. **Monetary policy and expectations formation.**

2.2.1. *The Taylor rule.* The monetary authority sets the gross nominal interest rate, r_t, according to the truncated Taylor rule:

$$r_t = \max\left[1, \rho r_{t-1} + (1-\rho)(\bar{r} + \phi_t\pi_{t-1}) + u_t\right], \tag{11}$$

where $\bar{r}$ is the steady-state gross nominal rate, $\rho \in [0,1[$ is a smoothing parameter, the shock, $u \sim \mathcal{N}(0, \sigma_u^2)$, is a proxy for discretionary monetary policy, and ϕ_t is the central bank's response to inflation. The latter evolves according to a 2-state Markov chain with transition matrix, P. For row i and column j of P, element $p_{i,j} = P(\phi_t = \phi_j | \phi_{t-1} = \phi_i)$ for $i, j \in \{L, H\}$, where $0 \leq p_{i,j} < 1$ and $\sum_{j=1}^{2} p_{i,j} = 1$ for all i.

The Markov process models changes in the central bank's reaction function. Under Regime 1, henceforth the *passive regime*, the central bank de-emphasizes inflation stabilization by violating the Taylor principle, i.e., $\phi_L \leq 1$. In contrast, under Regime 2, henceforth the *active regime*, the central bank emphasizes inflation stabilization by respecting the Taylor principle, i.e., $\phi_H > 1$.

This regime changing device captures the *constrained discretion* strategy [Bernanke and Mishkin (1997)], which allows the central bank some flexibility to de-emphasize inflation stabilization to pursue alternative short-term goals. This was particularly true in the 1960s and 1970s, when according to Taylor (2012): "*the Federal Reserve exercised little long-term thinking and a great deal of short-term fine-tunning*".

Starting in the 1980s, the Federal Reserve became more rules-based, but discretion remains an essential element of the Federal Reserve's strategy. According to former Governor Mishkin [Mishkin (2018)]: "*Monetary policy is as much an art as a science [...] the rapid reactions of the Federal Reserve to the [2008] financial crisis and the departure from the Taylor rule were not based on hard data. Instead it was judgment, very often based on anecdotal evidence and conversations with financial market participants, that led the Fed to depart from the Taylor rule*".

As in models considered by Schorfheide (2005) and Bianchi and Melosi (2018), my regime switching framework does not explain why monetary policy shifts over time. I simply assume that there are two regimes subject to constant transition probabilities.

2.2.2. *Expectations formation.* Bayesian learning is introduced as in Richter and Throckmorton (2015), who builds on Hamilton (1989). As mentioned previously, households and firms do not observe the central bank's inflation parameter, ϕ_t, for it is obscured by the discretionary shock, u_t. Instead, they make an inference about its value based on observed realizations of $\{r_t, \pi_{t-1}\}$.

The expectations formation process summarizes the model as:

$$\mathrm{E}_t\left[\mathrm{F}(\mathbf{x}_{t+1}, \mathbf{x}_t)\right] = 0,$$

where F is a vector value function representing the economic environment and the first order conditions of the model, and $\mathbf{x}$ is a vector containing the observed variables in the model:

$$\mathbf{x} = \begin{cases} c_t, h_t, i_t, k_t, w_t, r_t^k, q_t, mc_t, y_t, r_t, \pi_t, \phi_t, & \text{under Rational Expectations,} \\ c_t, h_t, i_t, k_t, w_t, r_t^k, q_t, mc_t, y_t, r_t, \pi_t, \lambda_t, & \text{under Learning.} \end{cases}$$

I introduce the Rational Expectations equilibrium of the model, in which ϕ_t is perfectly observed, to have a clear benchmark. In the Learning model, λ_t is a vector of conditional probabilities that $\phi_t = \phi_i$ for $i \in \{L, H\}$, which agents update to form expectations. Specifically, agents use the nonlinear filter in Hamilton (1989) to compute the conditional probabilities:

$$\lambda_t^i = \mathrm{P}(\phi_t = \phi_i | \Omega_t, \Theta),$$

where $\Omega_t = \{r_t, \cdots, r_0, \pi_{t-1}, \cdots, \pi_0\}$ is the history of the nominal interest rate and inflation, and $\Theta = \left\{P, \phi_1, \phi_2, \sigma^2\right\}$ is the vector of relevant parameters. With each additional observation of $\{r_t, \pi_{t-1}\}$, agents update their inference about ϕ_t.

Under Rational Expectations, ϕ_t is known. Given $\phi_t = \phi_i$, forming expectations is straightforward:

$$\mathrm{E}_t\left[\mathrm{F}(\mathbf{x}_{t+1}, \mathbf{x}_t)\right] = \sum_{j=L}^{H} p_{i,j} \int_{-\infty}^{\infty} \mathrm{F}(\mathbf{x}_{t+1}, \mathbf{x}_t) \vartheta(u_{t+1}) du_{t+1} = 0, \tag{12}$$

where $\vartheta(.)$ is the normal probability density function. In words, the sum operates across the two monetary policy regimes, while the integral operates across realizations of the discretionary shock.

Under learning ϕ_t is unknown, creating a signal extraction problem. Given $\mathbf{q}_t$, the expectation formation process is:

$$\mathrm{E}_t\left[\mathrm{F}(\mathbf{x}_{t+1}, \mathbf{x}_t)\right] = \sum_{i=L}^{H} \lambda_t^i \sum_{j=L}^{H} p_{i,j} \int_{-\infty}^{\infty} \mathrm{F}(\mathbf{x}_{t+1}, \mathbf{x}_t) \vartheta(u_{t+1}) du_{t+1} = 0. \tag{13}$$

According to the first sum operator, agents account for monetary policy uncertainty by weighting each realization of ϕ by its conditional likelihood, λ_t^i.

Comparing eq. (12) and eq. (13) provides three useful insights:

(i) The unknown policy regime generates forecast errors with respect to the Rational Expectations benchmark. These errors distort current decision-making via the three intertemporal optimality conditions.

(ii) As agents' beliefs become increasingly accurate, the learning economy converges to the Rational Expectations benchmark. In other words, deviations from Rational Expectations are a function of how far agents' beliefs are from the truth.

(iii) Learning smooths expectations, because it is unlikely that $\lambda_t^i = 1$ for any i. That is, agents are aware that they do not known ϕ_t, so they put weight on choices they would make in either monetary regime. As will become clear, this smoothing of expectations makes the learning economy less sensitive to monetary policy.

2.2.3. *Measuring MPU.* This section concludes by presenting the model-based MPU index I use throughout the paper. Each period, the Hamilton filter provides two conditional probabilities: $\lambda_t^i = \mathrm{P}(\phi_t = \phi_i | \Omega_t, \Theta)$ for $i \in \{L, H\}$. Periods with $\lambda_t^i = 0.5$ feature absolute uncertainty: agents consider the two possible policy regimes equally likely. In contrast, periods with $\lambda_t^i = 1$ for either i

feature absolute certainty: agents are convinced they know ϕ_t. Following Richter and Throckmorton (2015), I measure uncertainty as:

$$\zeta_t = \frac{\sqrt{0.5} - \sqrt{\sum_{i=L}^{H} (\lambda_t^i - 0.5)^2}}{\sqrt{0.5}}, \tag{14}$$

which ranges from 0 (absolute certainty) to 1 (absolute uncertainty). Importantly: ζ_t reflects how confident agents are about their inferences, not the accuracy of these inferences. .

One final remark. Eq. (11) mean that both the nominal interest rate, r_t, and the vector of conditional probabilities, λ_t, are part of the state space. Consequently, the index ζ_t does not respond to current realizations of control variables such as output and inflation, because it is predetermined.

3. Calibration and Solution Technique

3.1. **Calibration.** Table 1 lists the values assigned to the model parameters. I choose conventional values for the taste and technology parameters. The quarterly discount factor, β, is set to 0.99, corresponding to a 4% annual real interest rate. The Frisch elasticity of labor supply, $1/\eta$, is set to 1, which is in line with Christiano et al. (2010). The leisure-preference parameter, χ, is set to 6.88, implying one third of time is spent working in the deterministic steady state.

Also following convention, the elasticity of substitution between intermediate goods, ϵ, is set to 10, while the price adjustment cost parameter, ψ, is set to 105. This value mimics a Calvo price setting specification where the average duration between price changes is four quarters. Capital's share of output, α, is set to 0.33, and the depreciation rate, δ, is set to 0.025. Finally, the capital adjustment cost, γ, is set to 6, which follows Erceg and Levin (2003) and Gavin et al. (2015).

Concerning monetary policy, the smoothing parameter, ρ, is set to 0.6, which is consistent with Clarida et al. (2000). The four parameters governing the 2-state Markov chain for ϕ_t, $\{\phi_L, \phi_H, p_{L,L}, p_{H,H}\}$, are chosen to be consistent with the following three outcomes. First, deviations from the Taylor principle are short, yet pronounced.[2] Second, the ergodic mean of ϕ is in the neighborhood of 1.5, which is the standard value found in the literature. Third, the calibration is in line with Bianchi and Melosi (2018). The resulting values are: $\phi_L = 0.2$, $\phi_H = 2$, $p_{L,L} = 0.7$, and $p_{H,H} = 0.85$.[3]

Lastly, I set σ_u, the standard deviation of the transitory shock, to 0.001. This parameter is essential. If σ_u approaches 0, the discretionary shock no longer obscures ϕ: each additional observation of $\{r_t, \pi_{t-1}\}$ perfectly reveals it. At the other extreme, if σ_u goes to ∞, the policy rate becomes a

[2] This is in line with Bernanke and Mishkin (1997).

[3] As long as $\phi_L < 1$, $\phi_H > 1$, and $p_{L,L} < p_{H,H}$ (all of them reasonable assumptions), the main qualitative conclusions are unaffected.

TABLE 1. Parameter values.

	Symbol	Value		Symbol	Value
Taste and technology			**Monetary policy**		
Discount factor	β	0.99	Smoothness parameter	ρ	0.60
Inverse Frisch elasticity	η	1.00	Passive regime	ϕ_L	0.20
Leisure parameter	χ	6.88	Active regime	ϕ_H	2.00
Elasticity of substitution	ϵ	10.0	Trans. probability matrix	$p_{L,L}$	0.70
Price adjustment	ψ	105	Trans. probability matrix	$p_{H,H}$	0.80
Capital's share of output	α	0.33	STD discretionary shock (%)	σ_u	0.10
Capital depreciation (%)	δ	2.50			
Capital Adjustment cost	γ	6.00			

random variable, ϕ_t loses its importance, and the signal extraction problem becomes both impossible and pointless. The chosen value for σ_u ensures a role for MPU, while guaranteeing that the learning model is close to the Rational Expectations benchmark. I explore the importance of σ_u further in 4.1.3.

3.2. **Model solution.** I solve the model using a projection method with Chebyshev polynomials and orthogonal collocation.[4] As will become clear, this global solution method captures well the endogenous non-linearities linked to the learning process.

4. Central Results

4.1. **The real effects of monetary policy uncertainty.** This section offers a structural interpretation for the contractionary effects of increases in MPU as documented by Creal and Wu (2017) and Husted et al. (2019).

4.1.1. *Policy functions.* Figure 1 shows the decision rules for output and Tobin's q as a function of the inferred probability of being in the passive monetary policy regime. Capital and the nominal interest rate are held constant at their steady state values.

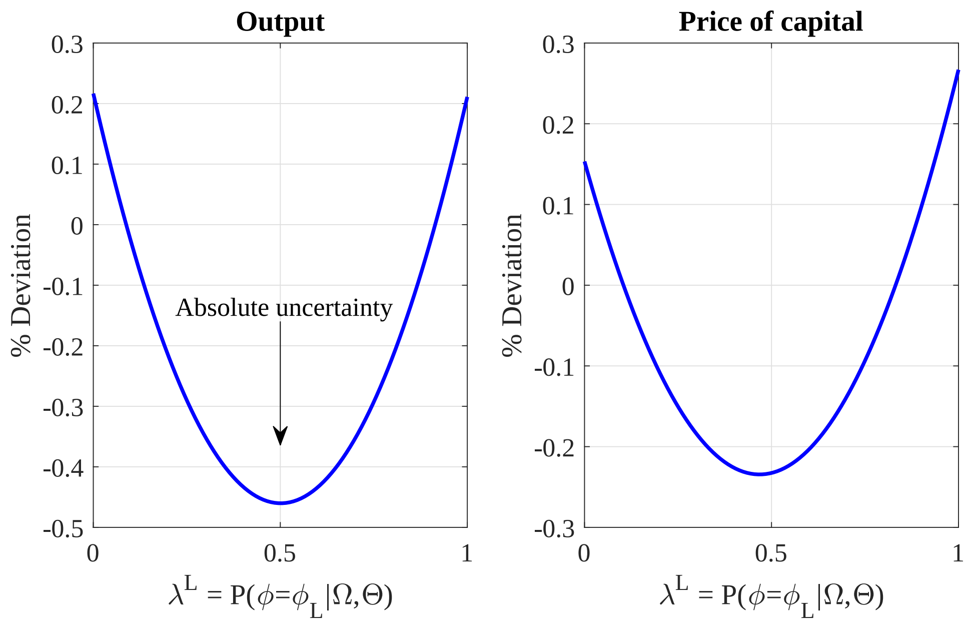
The message is clear: uncertainty about the current monetary policy regime lowers the price of capital; thereby dampening inflation and hindering output. Indeed, periods of high uncertainty, when agents assign each regime equal probability, feature a Tobin's q below one. This makes net investment negative: current investment is below what is necessary to replace depreciated capital. The decline in aggregate demand, together with the prospect of a protracted period of low capital, explain the ensuing fall in production.

The rational behind this result is as follows. Uncertainty about ϕ_t obscures the likely future path of interest rates, and of the marginal product of capital.[5] By blurring the expected return

[4]Please refer to Appendix A for more details.

[5]The nominal interest rate and the rental rate of capital are tightly linked: their correlation is roughly -0.95 in a long run simulation. By construction, the partial derivative of the marginal product of capital with respect to hours worked

FIGURE 1. Policy functions.



Notes. Vertical axes are in percentage deviations from the stochastic steady state. Horizontal axes are in levels.

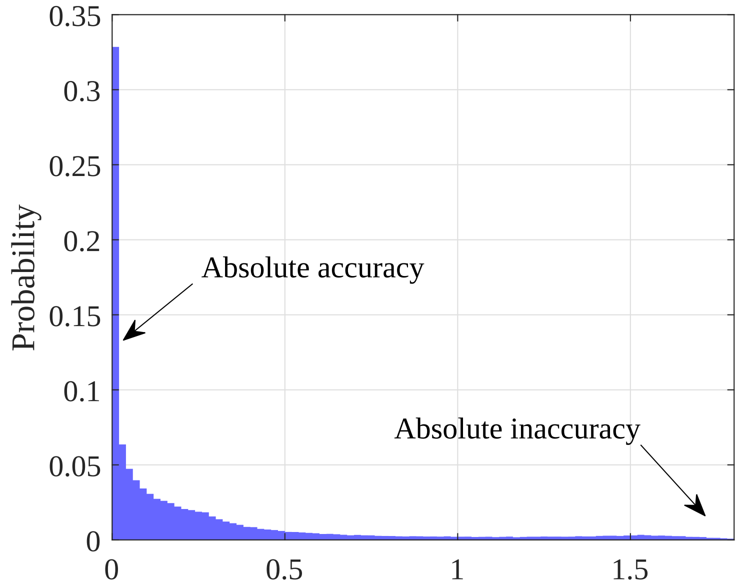
of capital, uncertainty makes it rational to defer investment. This mechanism recalls the seminal work by Bernanke (1983) and Abel and Eberly (1994), emphasizing the importance of waiting until economic uncertainty is resolved.

4.1.2. *Long run properties.* Having explained the main intuition for the effects of MPU, I now explore the dynamic dimension. The results are based on a 100,000-period simulation of the model.

Before tackling uncertainty, I emphasize the accuracy of agents' beliefs. To this end, I analyze the density function of the absolute-value norm between the true policy regime and private sector beliefs: $\theta_t = |\phi_t - (q_t\phi_L + (1-q_t)\phi_H)|$, with $\theta_t \in [0, |\phi_H - \phi_L|]$. As seen in Figure 2, the signal extraction problem does not usually imply a substantial deviation from the Rational Expectation equilibrium of the model. Indeed, most of the time, private sector beliefs are very close to the truth, lowering distortions due to learning. That households usually have a fairly good idea of the central bank's reaction function seems plausible, and to the best of my knowledge, is not rejected by the data.

is positive. Therefore, a decline in the nominal interest rate raises the rental rate of capital by boosting aggregate demand and labor input.

FIGURE 2. Accuracy of agents' beliefs: density function.



Notes. Density function of the absolute-value norm between the true policy regime and agents beliefs: $\theta_t = |\phi_t - (q_t\phi_L + (1-q_t)\phi_H)|$, with $\theta_t \in [0, |\phi_H - \phi_L|]$. Based on a 100,000-period stochastic simulation of the model.

TABLE 2. Correlation coefficients.

	ζ_t	Y_t	q_t	i_t	h_t	c_t	π_t
ζ_t	1.00	-0.54	-0.77	-0.77	-0.54	-0.12	-0.20
ζ_{t-1}	-0.09	0.13	0.17	0.17	0.14	0.06	0.09

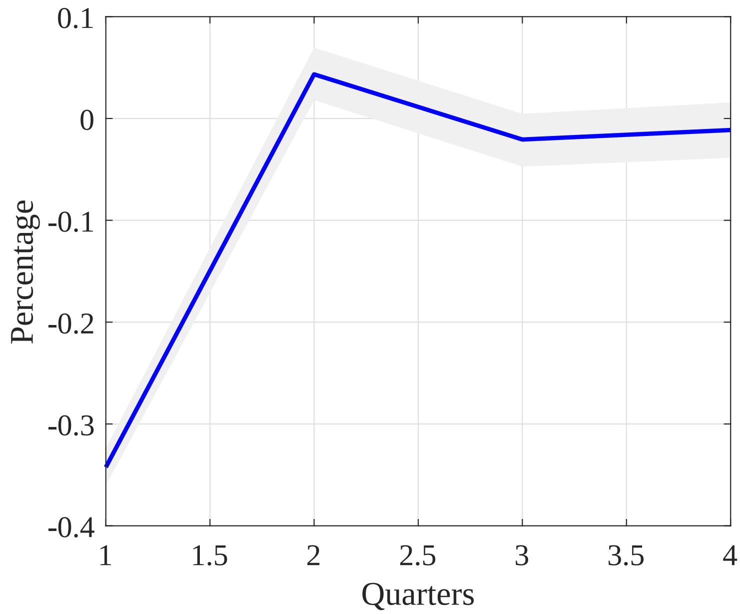
Notes. Based on a 100,000-period stochastic simulation of the model.

I now turn to the link between my MPU index, ζ, and some key aggregates. The first row of Table 2 reports correlations with ζ_t. Understanding these numbers in light of Figure 1 is straightforward. Increases in MPU trigger, first and foremost, a strong decline in investment. To accommodate the ensuing fall in aggregate demand, firms cut labor inputs, lowering output. The reduction in labor income forces households to lower their consumption. But the latter decline is mild, as household attempt to smooth consumption.

The second row reports the correlations with last period's MPU. MPU features a negative autocorrelation: spikes in uncertainty are short-lived, for agents quickly learn about the monetary policy reaction function.[6] The set of lag correlations then conveys a simple message: the resolution of uncertainty leads to a partial investment recovery. New information about the central bank's inflation response improves household forecasts of the future marginal product of capital. This, on average, boosts investment, as the private sector aims to bring the capital stock back to its

[6]This logic is consistent with the small deviations from the Rational Expectations equilibrium mentioned earlier.

FIGURE 3. Output response to a one-standard deviation surprise increase in MPU.



Notes. Impulse response computed via Jorda (2005) Local Projection methods. Shaded areas are the 90% confidence intervals.

steady state level. Once again, this rationale echoes Bernanke: *"Introduction of uncertainty can be associated with slack investment; resolution of uncertainty with an investment boom"* [Bernanke (1983)].

To provide additional evidence of the contractionary effects of MPU, I estimate the predictive relation between my MPU index and output using Local Projection methods [Jorda (2005)]. The empirical specification is:

$$\hat{Y}_{t+h} = \mu_h + \beta_h \zeta_t + \gamma_h X_t + \epsilon_{t+h}, \tag{15}$$

where $\hat{Y}$ is output in percent deviations from the steady state, ζ_t is the MPU index, X_t is a set of control variables[7], ϵ_{t+h} is the projection residual, and μ_h, β_h, γ_h are the projection coefficients. The Local Projection impulse response function of $\hat{Y}$ with respect to ζ is given by $\{\beta_h\}_{h\geq 0}$.[8]

Figure 3 reports the Local Projection impulse response function to a one-standard-deviation increase in ζ_t. On impact, output falls by 0.3%, as the increase in uncertainty reduces aggregate demand by deferring investment. One quarter after the shock, as uncertainty dissipates, output jumps slightly above its steady state value, because, as already mentioned, investment increases to bring the capital stock back to its baseline level.

[7]I include one lag of the dependent variable, the current stock of capital, the current nominal interest rate, and the current Tobin q.

[8]Since output is % deviations from steady state, the coefficient of MPU implies that output deviates form its long run equilibrium by β_h% for every extra unit of MPU.

TABLE 3. Dynamic implications of σ_u.

	Mean	Correlation with ζ_t			
σ_u (%)	θ_t	Y_t	i_t	c_t	π_t
0.07	0.19	-0.07	-0.12	0.05	0.00
0.08	0.21	-0.45	-0.69	-0.07	-0.15
0.09	0.24	-0.46	-0.69	-0.08	-0.16
0.10	0.27	-0.54	-0.77	-0.12	-0.20
0.11	0.30	-0.46	-0.66	-0.09	-0.17
0.12	0.33	-0.17	-0.24	0.02	-0.05
0.13	0.36	-0.17	-0.24	0.01	-0.05

Notes. Based on a 100,000-period stochastic simulation of the model. The second column reports the mean of the inference error, θ. Columns 3 to 6 report the correlation between the MPU index, ζ_t, and output, investment, consumption, and inflation.

4.1.3. *Remarks.* The previous subsections implicitly suggest that the crucial parameter for MPU to have contractionary effects is the standard deviation σ_u. When σ_u approaches 0, the evolution of $\{r_t, \pi_{t-1}\}$ conveys perfect information about ϕ_t. In this instance, agents' inferences are totally precise, as the signal extraction problem becomes trivial. As a result, the link between MPU and economic activity disappears.

In contrast, when σ_u goes to ∞, ϕ_t loses its importance, for r_t is only driven by the discretionary shock, u_t. Furthermore, $\{r_t, \pi_{t-1}\}$ no longer conveys useful information about ϕ_t; agents' beliefs thus converge to their ergodic distribution: $q_t^i \to \mathrm{P}(\phi_t = \phi_i)$ for $i \in \{L, H\}$. In this case, the relevance of MPU also vanishes.

In sum, the negative link between MPU and economic activity only occurs for an intermediate set of values for which the signal extraction problem is neither trivial nor impossible. Table 3 provides a numerical illustration of this argument by reporting contemporaneous correlations for various values of σ_u. As expected, the mean of the inference error, $\bar{\theta}$, increases monotonically with σ_u, as the signal extraction problem becomes increasingly challenging. Likewise, low values of σ_u make learning too easy, also reducing the importance of MPU.

4.2. **Learning and the transmission mechanism.** This second section evaluates the impact of learning on the monetary policy transmission mechanism. The key insight is clear: learning makes the economy less sensitive to the central bank's reaction function. Spikes in MPU intensify this phenomenon, for agents are forced to base their actions on the choices they would make conditional on *both* monetary regimes.

4.2.1. *Correlations.* I first compare the links between the exogenous policy processes (ϕ and u) and output and inflation. Table 4 reports the contemporaneous correlations in a 100,000-period simulation of the model under learning and under Rational Expectations.

TABLE 4. Correlation coefficients.

	Correlation with ϕ_t		Correlation with u_t	
	Learning	Rational Expectations	Learning	Rational Expectations
Y_t	0.59	0.81	-0.68	-0.45
π_t	0.77	0.87	-0.54	-0.38

Notes. Based on a 100,000-period stochastic simulation of the model. Learning stands for the model with limited information. Rational Expectations stands for the model with perfect information.

The table conveys two messages. First, learning renders prices and quantities *less* sensitive to the central bank's inflation response. The correlations with ϕ_t are significantly closer to 0 in the learning economy. The intuition is straightforward: since agents know that their beliefs can be inaccurate, they put equal weight in expectation on choices they would make in either regime (see subsection 2.2.2). Their behavior is thus tempered by uncertainty, making them less responsive to the monetary regime.

Second, learning makes the economy *more* sensitive to discretionary shocks. In absolute value, correlations with u_t are, in absolute value, larger in the learning economy. The logic is as follows. Under learning, transitory shocks affect agents' beliefs, λ. As a result, they have stronger effects on the expected path of future interest rates than in the Rational Expectations model (where agents perfectly observe the transitory disturbances). These stronger effects, combined with the three optimality inter-temporal conditions, account for the amplified responses to transitory shocks.

4.2.2. *Growth rates and regime changes.* This final subsection digs deeper into the effects of uncertainty on the responses to regime changes. I first identify all quarters featuring a regime change in the simulation. Then I compute the absolute value of the growth rate of output and inflation in those quarters, and compare them with their unconditional means. I take the absolute value of the growth rates, for I am interested in the magnitudes of the changes, not in their signs. Formally, I first identify all periods t^* in which $\phi_{t^*} = \phi_i$ and $\phi_{t^*-1} = \phi_j$ for $i \neq j$, where $i, j \in \{L, H\}$. Next, I compute:

$$\eta_x = \frac{\text{mean}\left[\left|\log(x_{t^*}) - \log(x_{t^*-1})\right|\right]}{\text{mean}\left[\left|\log(x_t) - \log(x_{t-1})\right|\right]},$$

where $x = \{Y, \pi\}$. In words, the numerator is the mean of the absolute value of the growth rate of x conditional on a regime change. The denominator is the mean of absolute value of the growth rate of x in the entire 100,000-period simulation.

Columns 2 and 3 in Table 5 support previous findings. The Rational Expectations economy reacts much more strongly to changes in ϕ. For example, conditional on a regime change, the absolute growth rate of output is 2.45 times larger than its mean. In the learning model, that number is 1.49. Inflation features similar dynamics.

TABLE 5. Correlation coefficients.

	Rational Expectations	Learning		
		Unconditional	Conditional on	
			High ζ_t	Low ζ_t
Output, η_y	2.45	1.49	1.31	1.69
Inflation, η_π	2.74	1.84	1.35	2.40

Notes. Based on a 100,000-period stochastic simulation of the model. Learning stands for the model with limited information. Rational Expectations stands for the model with perfect information. In column 4 (5), the numerator of η_x is conditioned on ζ_t being higher (lower) than its 0.8 (0.2) percentile.

Because uncertainty tempers agents' behavior, regime changes conditional on high MPU must feature lower η_x statistics. Column 4 confirms this intuition. When the numerator of η_x is conditioned on ζ_t being higher than its 0.8 percentile, the resulting statistics are way below those reported in column 3. In contrast, when MPU is low (ζ_t lower than its 0.2 percentile), responses in the learning economy are closer to those in the Rational Expectations economy (see columns 2 and 5).

5. CONCLUSION

I present a New Keynesian model in which the central bank's response to inflation varies over time. Agents do not observe the current monetary policy regime, but learn about it using Bayes theorem. I solve the model using a textbook projection method to account for the endogenous non-linearity introduced by imperfect information.

The model provides a structural interpretation of the contractionary effects of monetary policy uncertainty (MPU) shocks documented by Husted et al. (2019). The mechanism is straightforward: spikes in MPU obscure the likely future path of capital returns. This unpredictability makes it rational to defer investment (to receive additional information). The ensuing fall in aggregate demand reduces output through standard channels.

I also explore how MPU affects the monetary policy transmission mechanism. The key insight is intuitive: by softening the link between fundamentals and equilibrium prices and allocations, learning renders the economy less responsive to the monetary policy inflation response.

Introducing both a financial system and uncertainty about unconventional monetary measures are important topics for future research. Likewise, considering a fully-fledged dynamic stochastic general equilibrium model would sharpen the model's quantitative predictions.

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Appendix A. Model solution

I solve the model using a textbook projection method with Chebyshev polynomials and orthogonal collocation. For a detailed review on this technique, see Fernandez-Villaverde et al. (2016). I first describe the solution under Rational Expectations, then move to the introduction of limited information, and lastly, discuss accuracy.

A.1. The Full Information economy.

The state space of the model consists of two continuous variables, $\{k_{t-1}, r_t\}$, and one discrete variable, ϕ_t. I approximate the decision rules for labor, inflation, and the price of capital for all possible values of ϕ_t using 3 Chebyshev polynomials for both k_{t-1} and r_t. I need to estimate 54 parameters ($3x3x3x2$).

Evaluating the residuals of eq. (3), eq. (5) and eq. (8) at each of the 3 zeros of the Chebyshev of order 3 for k and r (i.e., collocation points), and the 2 levels of ϕ gives 54 equations to solve for those 54 coefficients. Given a good initial guess (for which I use a linear solution of a simpler model) a Newton solver easily deals with this system.

A.2. The Limited Information economy.

In Case 2, variable $\lambda_t^L = \text{P}(\phi_t = \phi_L | \Omega_t, \Theta)$ substitutes ϕ_t in the state space. I still approximate the decision rules for labor, inflation, and the price of capital, but I now do it using 3 Chebyshev polynomials for $\{k_{t-1}, r_t, \lambda_t^L\}$. As before, I choose the resulting 81 parameters to minimize both residual functions at the 81 collocation points. To update λ_t^L, agents use the filter in Hamilton (1989) as described in Appendix B.

A.3. Accuracy.

Following Fernandez-Villaverde et al. (2016) and Fernandez-Villaverde and Levintal (2018), I assess accuracy by computing the mean and maximum unit-free Euler error across the ergodic set of the model. Overall, the solutions for both models are accurate. The mean Euler error in log10 units is -3.2 under Rational Expectations, and -2.8 under limited information, while the maximum Euler error are -2.9 and -2.1, respectively. To put these numbers into perspective, a value of -2 means $1 mistake for each $100, a value of -3 means $1 mistake for each $1000, and so on.

Appendix B. Beliefs and the Zero Lower Bound

Subsection 2.2.2 showed that under limited information agents observe r_t directly but con only make an inference about the value of ϕ_t based on what they see happening with $\{r_t, \pi_{t-1}\}$. This inference takes the form of the vector of conditional probabilities λ_t.

The Hamilton filter -an application of Bayes theorem- performs such an inference iteratively for $t = 1, 2, 3, ...$ with step t taking as input λ_{t-1}. The crucial magnitudes agents use to perform iteration t are the probability densities under the two regimes

$$\eta_{j,t} = f(r_t|\phi_t = \phi_j, \Omega_t, \Theta)$$

for $j = H, L$, and where Ω_t is the history of the nominal interest rate and inflation, and Θ is the vector of relevant parameters (see Subsection 2.2.2). The density functions $f(.)$ are specified below.

Given $\{\lambda_{t-1}, \eta_{j,t}\}$, the conditional density of the tth observation is given by

$$g(r_t|\Omega_t, \Theta) = \sum_{i=1}^{2}\sum_{j=1}^{2} p_{i,j}\lambda_{t-1}^{i}\eta_{j,t},$$

and the desired output is

$$\lambda_t^j = \frac{\sum_{i=1}^{2} p_{i,j}\lambda_{t-1}^{i}\eta_{j,t}}{g(r_t|\Omega_t, \Theta)}.$$

The elements of η_t need not be Gaussian densities or belong to the same family of distributions.This property is essential, because the max operator in (11) defines two completely different scenarios.

When $r_t > 0$ the elements of η_t are normal densities

$$\eta_{j,t} = f(r_t|\phi_t = \phi_j, \Omega_t, \Theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(r_t - \rho r_{t-1} - (1-\rho)(\bar{r} + \phi_j \pi_{t-1}))^2}{2\sigma^2}},$$

for $j = H, L$. In these instances, the latest observation of the nominal interest rate and inflation contain relevant information about ϕ_t, which agents insert into the Hamilton filter.

When $r_t = 0$, agents face a degenerate distribution, which entails a probability mass function $\eta_{j,t} = 1$ for $j = 1, 2$. It is trivial to verify that under $r_t = 0$ agents update their beliefs according to

$$\lambda_t^i = p_{i,i}\lambda_{t-1}^{i} + p_{i,j}\lambda_{t-1}^{j}.$$

Iterating the above expression forward shows that λ_t^i converges to the unconditional probability $P(\phi = \phi_i)$. This result is fairly intuitive. At the Zero Lower Bound, neither the nominal interest rate nor the inflation rate have useful information about the value of ϕ_t. Agents' best inference thus dictates that ϕ_t converges to its ergodic distribution.

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