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HOW WELL DO DSGE MODELS WITH REAL ESTATE AND COLLATERAL CONSTRAINTS FIT THE DATA?

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Abstract. Not so well. We reach this conclusion by evaluating the empirical performance of a benchmark DSGE model with real estate and collateral constraints. We estimate the model from U.S. data using Bayesian methods and assess its fit along various dimensions. We find that the model is strongly rejected when tested against unrestricted Bayesian VARs and cannot replicate the persistence of real estate prices and various comovements between aggregate demand, real estate prices, and debt. Performance does not improve with alternative definitions of real estate prices, estimation samples, or detrending approaches. Our results raise doubts about the ability of current DSGE models with real estate and collateral constraints to deliver credible policy insights and identify the dimensions in need of improvement.

JEL Codes: C52, E32, E44.

Keywords: real estate; housing; DSGE models; collateral constraints; model evaluation.

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Résumé Non Technique

Au cours de l'année 2007, une chute du marché immobilier américain s'est muée en crise financière généralisée, provoquant finalement une récession internationale. Cet épisode a ravivé l'intérêt des économistes pour les interactions entre le marché immobilier, la stabilité financière et la macroéconomie. Un grand nombre d'études publiées dans ce contexte développent des modèles DSGE (Dynamic Stochastic General Equilibrium) dans lesquels les actifs immobiliers contribuent au bien-être des ménages (dimension résidentielle) et constituent un facteur de production pour les entreprises (dimension commerciale). Ces modèles décrivent également la capacité des actifs immobiliers à servir de collatéral, c'est-à-dire à permettre aux ménages et aux entreprises d'obtenir plus facilement des prêts. Ces études étudient par exemple comment une chute des prix de l'immobilier affecte la consommation des ménages ou l'investissement des entreprises. Certaines analysent aussi la capacité des politiques macro-prudentielles à limiter les crises immobilières.

Cependant, aucune de ces études n'établit clairement la capacité de ce type de modèles de reproduire les principales caractéristiques des données. Par exemple, ces modèles sont-ils capables de générer une forte persistance des prix de l'immobilier, comme observé dans les données ? Sont-ils capables de reproduire la forte corrélation entre les prix de l'immobilier et l'encours des crédits, toujours comme observé dans les données ? Il s'agit pourtant d'une question importante car un modèle incapable de reproduire les données perd en crédibilité pour évaluer la politique économique.

Ce papier propose une évaluation empirique de ce type de modèles. Nous considérons un modèle DSGE avec immobilier (résidentiel et commercial) et des contraintes de collatéral, dans la lignée des travaux de référence de Iacoviello (American Economic Review, 2005), Iacoviello et Neri (American Economic Journal : Macroeconomics, 2010) et Liu *et al.* (Econometrica, 2013). Nous estimons ce modèle sur données américaines, avant d'évaluer sa concordance avec les observations disponibles. Nos résultats soulignent l'incapacité du modèle à reproduire tant la persistance des prix de l'immobilier que leur forte corrélation avec le niveau de dette dans l'économie. Ce dernier point suggère que la formulation habituelle de la contrainte de collatéral n'est pas satisfaisante empiriquement.

Deux conclusions découlent de nos résultats. D'abord, l'incapacité de ce type de modèles à reproduire correctement les données signifie qu'il est hasardeux de mettre trop de confiance dans leurs implications quantitatives, par exemple quant à l'évaluation de politiques économiques. Ensuite, en identifiant les principales faiblesses de ces modèles, notre étude souligne les dimensions à améliorer dans le futur : l'introduction de source(s) de persistance pour les prix de l'immobiliers et des contraintes de collatéral plus en ligne avec la réalité. A cet égard, certaines tentatives d'amélioration ont récemment été avancées (modèles de recherche, anticipations non rationnelles, collatéral limité et dette de long terme, etc.), mais

dans le cadre de petits modèles en équilibre partiel. Intégrer et tester ces modifications dans des modèles DSGE constitue la prochaine étape dans l'agenda de recherche.

1. Introduction

During the second half of the 2000s, the U.S. economy experienced a boom-bust cycle in the housing market. The collapse of house prices triggered a widespread financial crisis, which turned into a deep recession with worldwide consequences (see, e.g., Mishkin, 2011). This sequence of events spurred research interest about the interactions between real estate (RE) markets and the macroeconomy. (In this paper, we use *housing* and *real estate* equivalently.)

Building upon Iacoviello's (2005) insight that collateral constraints tied to the value of real estate provide a powerful amplification mechanism, a number of studies have since evaluated the spillovers between the real economy and housing markets, focusing on the drivers of RE price movements and their influence on household consumption (Iacoviello and Neri, 2010), the impact of RE price fluctuations on business investment (Liu, Wang, and Zha, 2013), the joint dynamics of household debt and house prices during the late 2000s (Justiniano, Primiceri, and Tambalotti, 2015, 2016, 2019), the effects of fixed-rate and adjustable-rate mortgages on aggregate dynamics (Kydland, Rupert, and Sustek, 2016; Garriga, Kydland, and Sustek, 2017, 2021), the appropriate policy reactions to household indebtedness (Alpanda and Zubairy, 2017), or the welfare losses caused by collateral constraints (Catherine, Chaney, Huang, Sraer, and Thesmar, 2022).[1] From a broad perspective, these studies all rely on dynamic stochastic general equilibrium (DSGE) models in which households and/or entrepreneurs hold real estate as collateral to relax financing constraints, in addition to its productive or utilitarian role.[2]

Surprisingly, empirical evidence about the ability of this type of DSGE models to reproduce the behavior of aggregate data remains scarce. For instance, while Iacoviello and Neri (2010), Liu, Wang, and Zha (2013), and Alpanda and Zubairy (2017) all use estimated models, only Iacoviello and Neri assess the cyclical fit of their artificial economy. Furthermore, their evaluation remains crude: they look only at HP-filtered moments (which prevents them from studying persistence in growth rates) and they do not consider debt (which prevents them evaluating the fit of collateral constraints). This gap in the literature is problematic at two levels. First, in order to trust the quantitative implications of DSGE models with real estate and collateral constraints, it is important to know how close these models come to reality. This is especially true when the focus is on key policy issues, such as evaluating the effects of changes in regulatory loan-to-value ratios (Alpanda and Zubairy, 2017). Second, identifying current shortcomings of the models is important to guide future research.

[1] While we focus on DSGE models, empirical work about housing and real estate is not limited to this framework. For instance, Mian and Sufi (2011, 2014), Mian, Rao, and Sufi (2013), and Chaney, Sraer, and Thesmar (2012) use disaggregated data and regression techniques to identify the relationship between RE prices, business and household debt, and aggregate demand.

[2] DSGE models with housing but without collateral constraints exist, for instance Davis and Heathcote (2005). However, their weak propagation mechanisms limit their empirical use.

In this paper, we consider a standard DSGE model with real estate and collateral constraints, take it to U.S. data using Bayesian methods, and perform a thorough evaluation of its empirical performance. Our main finding is that the model fails to provide a satisfactory representation of the data: it is strongly rejected when tested against less restrictive Bayesian vector autoregressions (BVARs) and it appears unable to reproduce basic properties of aggregate time series, including the variances of investment, RE prices, and debt; the persistence of RE prices; and comovements between aggregate demand, debt, and RE prices. In our view, these results signal serious misspecifications in the current generation of DSGE models with real estate and collateral constraints. Necessarily, they also raise doubts about the ability of these models to deliver credible quantitative insights.

The model we consider features three economic agents: a patient household, an impatient household, and an impatient entrepreneur. Households work, consume, and accumulate real estate for residential purposes, while entrepreneurs produce, consume, and accumulate business capital and real estate for production purposes. Discount-rate differences induce equilibrium saving by patient agents and equilibrium borrowing by impatient agents, and we follow Iacoviello (2005) and others in limiting access to external finance by collateral constraints tied to RE holdings. The model is driven by standard shocks to preferences, technology, and borrowing limits. To keep the setup as simple and transparent as possible, we fix the supply of real estate at an exogenous level as in Iacoviello (2005, 2015) or Liu, Wang, and Zha (2013). This assumption does not drive our results, as we verified in preliminary work that endogenizing the production of real estate (as in Iacoviello and Neri, 2010) actually *worsens* the fit of the model, by imposing additional arbitrage restrictions that seem rejected by the data.[3] We also focus on the real economy, omitting nominal frictions.

We estimate the model from data on standard macro variables (consumption, investment, hours worked, the relative price of investment), financial variables (business and household debt), and RE prices. Then, we evaluate its fit from both a relative perspective, based on comparing the DSGE economy with various BVARs, and an absolute perspective, based on posterior predictive checks (see, e.g., An and Schorfheide, 2007). This empirical evaluation yields strong evidence against the DSGE model and identifies important discrepancies between the model and the data. We also perform a thorough robustness analysis, considering alternative observables for RE prices (our benchmark measure is the price of residential housing; we also consider the prices of commercial real estate and residential land), alternative samples (we exclude the Great Recession and its aftermath to focus on "normal" circumstances), and alternative moments (we look at HP-filtered series, as in Iacoviello and Neri, 2010). These alternatives all confirm our main findings, implying that our analysis identifies a shortcoming of the model, rather than some issue in the way we perform the evaluation.

[3]These results are available upon request.

Overall, our results call for additional research to develop alternative specifications for real estate in DSGE models. Our analysis can guide researchers in the modeling process by identifying the two main points deserving more attention: the design of collateral constraints and the source of persistence in RE prices. Ongoing work tackles both issues. For instance, Kydland, Rupert, and Sustek (2016) and Garriga, Kydland, and Sustek (2017, 2021) propose an alternative representation of borrowing constraints, limiting home equity extraction and featuring long-term debt. Burnside, Eichenbaum, and Rebelo (2016) and Pancrazi and Pietrunti (2019) modify expectation formation, allowing for social dynamics or natural expectations able to generate momentum in RE prices. Guren (2018) shows that introducing price-setting complementarities in a search model also creates price momentum. Our contribution highlights the importance of these new developments and suggests that the empirical evaluation of new models will be important to settle on the best alternative.

We organize the paper as follows. Section 2 describes the DSGE model with real estate and collateral constraints. Section 3 presents the estimation exercise. Section 4 reports our main results by evaluating the empirical performance of the model. It also discusses some economic implications, including impulse responses (IRFs) and shock decompositions. Finally, Section 5 presents our robustness exercises and Section 6 concludes.

2. Model

We consider a discrete-time real economy with three agents: a patient household, an impatient household, and an entrepreneur. Households work, consume, and accumulate real estate for residential purposes, while the entrepreneur accumulates business capital and real estate for production purposes, hires household labor, and produces the final good. Due to differences in discount rates, the patient household saves in equilibrium, while both the impatient household and the entrepreneur borrow. Following the literature, we assume that borrowing is limited by collateral constraints that bind in the neighborhood of the steady state of the economy.[4] To save space, we relegate all optimality conditions to Online Appendix I.

2.1. Households.

[4]Our setup compares to existing papers as follows. We build upon Iacoviello (2005, 2015) in considering both patient and impatient households and impatient entrepreneurs. In contrast, Iacoviello and Neri (2010) focus on household debt and do not allow entrepreneurs to borrow, while Liu, Wang, and Zha (2013) focus on business debt and ignore impatient households. Our choice to keep real estate supply fixed at an exogenous level follows much of the literature, with Davis and Heathcote (2005) and Iacoviello and Neri (2010) being exceptions that endogenize housing production. Finally, we follow Davis and Heathcote (2005), Liu, Wang, and Zha (2013), and Iacoviello (2015) in considering a real model, while Iacoviello (2005) and Iacoviello and Neri (2010) study economies with nominal frictions and endogenous monetary policy.

2.1.1. *Patient household.* The patient household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta_S^t A_t \left[\ln \left(C_{S,t} - \gamma C_{S,t-1} \right) + \theta_{H,t} \ln(H_{S,t}) - \theta_{N,t} N_{S,t} \right],$$

where $C_{S,t}$ is consumption, $H_{S,t}$ is real estate, and $N_{S,t}$ is labor supply. We use the subscript S to indicate that the patient household is net saver in equilibrium. β_S is the discount factor, γ measures consumption habits, A_t is a discount rate shock, $\theta_{H,t}$ is a housing demand shock, and $\theta_{N,t}$ is a labor supply shock. Following Liu, Wang, and Zha (2013), we assume that households experience linear disutility of work.[5]

We assume that $\ln(A_t/A_{t-1})$, $\ln(\theta_{H,t})$, and $\ln(\theta_{N,t})$ follow stationary AR(1) processes with means 0, $\ln(\overline{\theta}_H)$, and $\ln(\overline{\theta}_N)$, persistence parameters ρ_A, ρ_H, and ρ_N, and innovations $\epsilon_{A,t}$, $\epsilon_{H,t}$, and $\epsilon_{N,t}$ with standard deviations σ_A, σ_H, and σ_N.

The patient household faces a budget constraint given by

$$C_{S,t} + P_{H,t}(H_{S,t} - H_{S,t-1}) + \frac{S_t}{R_t} \leq W_t N_{S,t} + S_{t-1},$$

where $P_{H,t}$ is the price of real estate in consumption units, S_t denotes the purchases of period-t bonds paying off one unit of consumption good in all states of nature in period $t+1$, R_t is the gross real interest rate, and W_t is the real wage. As in Iacoviello (2005) and Liu, Wang, and Zha (2013), we assume that real estate does not depreciate and that its aggregate supply is fixed. This assumption can be motivated by interpreting real estate as land, or simply as an approximation to the very low depreciation rates for installed structures.

2.1.2. *Impatient household.* The impatient household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta_B^t A_t \left[\ln \left(C_{B,t} - \gamma C_{B,t-1} \right) + \theta_{H,t} \ln(H_{B,t}) - \theta_{N,t} N_{B,t} \right],$$

where the notation is transparent. We use the subscript B to indicate that the impatient household is net borrower in equilibrium. Higher impatience relative to the patient household requires $\beta_S > \beta_B$, an assumption ensuring that the borrowing constraint described below binds in local equilibria around the steady state. Patient and impatient households experience the same preference shifters and share a common degree of consumption habits.

The impatient household faces a budget constraint given by

$$C_{B,t} + P_{H,t}(H_{B,t} - H_{B,t-1}) + B_{B,t-1} \leq W_t N_{B,t} + \frac{B_{B,t}}{R_t},$$

where $B_{B,t}$ denotes the sales of period-t bonds by the household, i.e. new debt.

[5]This assumption has a long tradition in the business-cycle literature, in which it generates amplification (see Hansen, 1985). In the present context, it also ensures that we can characterize the steady state of our economy in closed form, which is useful for the quantitative analysis. In the Online Appendix, we consider a more general model with power disutility on labor and show how imposing linear disutility helps solve for the steady state.

The household also faces a borrowing constraint given by

$$B_{B,t} \leq \rho_B B_{B,t-1} + (1-\rho_B)\psi_{B,t} E_t P_{H,t+1} H_{B,t}.$$

When $\rho_B = 0$, the total value of household debt cannot be larger than a fraction $\psi_{B,t}$ of the expected value of current RE holdings.[6] As in Iacoviello (2015), allowing for a positive value $\rho_B \in [0,1)$ slows down the adjustment of debt over time, capturing the idea that lenders may not adjust borrowing limits every quarter. $\psi_{B,t}$ is a collateral shock reflecting shifts in credit availability for households. We assume that $\ln(\psi_{B,t})$ follows a stationary AR(1) process with mean $\ln(\overline{\psi}_B)$, persistence parameter $\rho_{\psi B}$, and innovation $\epsilon_{\psi B,t}$ with standard deviation $\sigma_{\psi B}$.

This specification of the borrowing constraint, which follows Iacoviello (2005) and others, imposes strong restrictions on the model: debt matures one period ahead, the full expected value of the RE stock serves as collateral at each period, and the loan can finance consumption in addition to RE purchases. As noted by Kydland, Rupert, and Sustek (2016), these features bring the model loan closer to a home equity withdrawal than to a standard mortgage. Kydland, Rupert, and Sustek propose an alternative specification of the borrowing constraint, with long-term debt and only new RE purchases serving as collateral. In this paper, we only consider the standard Iacoviello specification, leaving the evaluation of the alternative constraint to future work.

2.2. **Entrepreneur.** The entrepreneur maximizes

$$E_0 \sum_{t=0}^{\infty} \beta_E^t \ln\left(C_{E,t} - \gamma C_{E,t-1}\right),$$

where the notation is again transparent. To ensure that the entrepreneur faces a binding borrowing constraint in local equilibria around the steady state, we assume that $\beta_S > \beta_E$.

The entrepreneur produces final goods using business capital, real estate, and labor as inputs. The production function is given by

$$Y_t = Z_t \left(H_{E,t-1}^{\phi} K_{t-1}^{1-\phi}\right)^{\alpha} N_t^{1-\alpha},$$

where Y_t denotes output, $H_{E,t-1}$, K_{t-1}, and N_t denote the real estate, capital, and labor inputs, and the parameters α and ϕ define the production elasticities. Following Liu, Wang, and Zha (2013), we assume that neutral technology features a permanent component $Z_{P,t}$ and a transitory component $\nu_{Z,t}$: $Z_t = Z_{P,t}\nu_{Z,t}$. The permanent component $\ln(Z_{P,t}/Z_{P,t-1})$ follows a stationary AR(1) process with mean $\ln(\overline{\mu}_Z)$, persistence parameter ρ_{ZP}, and innovation $\epsilon_{ZP,t}$ with standard deviation σ_{ZP}. The transitory component $\ln(\nu_{Z,t})$ follows a stationary

[6] We do not discuss the micro-foundations of this type of financial frictions. Kiyotaki and Moore (1997) show that similar credit constraints arise in the context of costly contract enforcement. For instance, if the household fails to repay debt, the creditor can seize real estate but then recovers only a fraction $\psi_{B,t}$ of its value due to liquidation costs.

AR(1) process with mean 0, persistence parameter ρ_{ZT}, and innovation $\epsilon_{ZT,t}$ with standard deviation σ_{ZT}.

Physical capital accumulates according to

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\varphi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_I\right)^2\right] I_t,$$

where I_t denotes investment in new capital goods, μ_I is the steady-state growth rate of investment, δ is the depreciation parameter, and φ is an adjustment cost parameter.

The entrepreneur faces a flow budget constraint given by

$$C_{E,t} + P_{H,t}(H_{E,t} - H_{E,t-1}) + B_{E,t-1} + \frac{I_t}{Q_t} + W_t N_t \leq Y_t + \frac{B_{E,t}}{R_t},$$

where $B_{E,t}$ denotes the new entrepreneurial debt. Q_t resembles a process for investment-specific technology, but it can be interpreted more broadly as a disturbance to the process of transforming consumption into investment goods.[7] As with neutral technology, we assume that investment-specific technology features both permanent and transitory components: $Q_t = Q_{P,t}\nu_{Q,t}$. The permanent component $\ln(Q_{P,t}/Q_{P,t-1})$ follows a stationary AR(1) process with mean $\ln(\overline{\mu}_Q)$, persistence parameter ρ_{QP}, and innovation $\epsilon_{QP,t}$ with standard deviation σ_{QP}. The transitory component $\ln(\nu_{Q,t})$ follows a stationary AR(1) process with mean 0, persistence parameter ρ_{QT}, and innovation $\epsilon_{QT,t}$ with standard deviation σ_{QT}. In equilibrium, the relative price of investment is equal to the inverse of investment-specific technology: $P_{I,t} = 1/Q_t$.

Finally, the entrepreneur faces a borrowing constraint given by

$$B_{E,t} \leq \rho_B B_{E,t-1} + (1-\rho_B)\psi_{E,t} E_t P_{H,t+1} H_{E,t},$$

with $\psi_{E,t}$ a collateral shock reflecting shifts in credit availability for entrepreneurs.[8] We assume that $\ln(\psi_{E,t})$ follows a stationary AR(1) process with mean $\ln(\overline{\psi}_E)$, persistence parameter $\rho_{\psi E}$, and innovation $\epsilon_{\psi E,t}$ with standard deviation $\sigma_{\psi E}$.

[7] Greenwood, Hercowitz, and Krusell (1997), Fisher (2006), and Justiniano, Primiceri, and Tambalotti (2010) interpret Q_t as investment-specific technology, while Justiniano, Primiceri, and Tambalotti (2011) and Moura (2018) consider richer interpretations.

[8] Our specification of the borrowing constraint implies that business capital has no value as collateral. We motivate this restriction by our empirical strategy matching business capital to durable equipment and intellectual property products, with residential and commercial structures being proxied by real estate. Empirical evidence suggest that non-structure capital is highly firm-specific, reducing its value to outsiders and therefore its ability to serve as collateral. For instance, Ramey and Shapiro (2001) and Kermani and Ma (2020) estimate liquidation rates for producer equipment below 30%. Our approach amounts to calibrating this outside value to 0%.

2.3. **Market clearing.** Markets for the final good, labor, bonds, and real estate clear in equilibrium. In the market for the final good, output can be used for consumption and investment:

$$Y_t = C_t + I_t/Q_t,$$

where aggregate consumption verifies $C_t = C_{B,t} + C_{E,t} + C_{S,t}$. In the labor market, labor demand equals labor supply:

$$N_t = N_{B,t} + N_{S,t}.$$

In the bond market, saving by the patient household equals borrowing by the impatient household and the entrepreneur:

$$S_t = B_{B,t} + B_{E,t}.$$

In the RE market, exogenous supply equals total demand:

$$\overline{H} = H_{B,t} + H_{E,t} + H_{S,t}.$$

2.4. **Trends and shocks.** The economy inherits a stochastic trend from the permanent preference and technology shocks. Denoting $\Gamma_t = (Z_{P,t}Q_{P,t}^{(1-\phi)\alpha})^{1/[1-(1-\phi)\alpha]}$, standard arguments imply that the following variables are stationary along the balanced growth path of the model: Y_t/Γ_t, C_t/Γ_t, $C_{B,t}/\Gamma_t$, $C_{E,t}/\Gamma_t$, $C_{S,t}/\Gamma_t$, S_t/Γ_t, $B_{B,t}/\Gamma_t$, $B_{E,t}/\Gamma_t$, $I_t/(\Gamma_t Q_{P,t})$, $K_t/(\Gamma_t Q_{P,t})$, $P_{H,t}/\Gamma_t$, $P_{K,t}Q_{P,t}$, W_t/Γ_t, $\Lambda_{B,t}\Gamma_t/A_t$, $\Lambda_{E,t}\Gamma_t$, $\Lambda_{S,t}\Gamma_t/A_t$. It follows from these balanced-growth restrictions that $\mu_I = (\overline{\mu}_Z\overline{\mu}_Q)^{1/[1-(1-\phi)\alpha]}$. Online Appendix II presents the detrended system.

Finally, the model is driven by nine shocks: three preference shocks (discount rate, housing demand, labor supply), four technology shocks (permanent and transitory shocks to neutral and investment-specific technology), and two collateral shocks (to household and entrepreneur collateral). To justify likelihood-based estimation, we postulate that the innovations to these nine shocks (the ϵ's) are Gaussian.

3. Estimation

We stationarize the equilibrium system and linearize it around a steady state with binding credit constraints. We then fit the model to quarterly U.S. time series using standard Bayesian methods. This section describes the data, as well as the calibration and estimation steps.

3.1. **Data.** We estimate the model from seven observables. The first six correspond to the time series used by Liu, Wang, and Zha (2013) to fit a similar model: real per-capita consumption growth, real per-capita investment growth, the growth rate of the relative price of investment, per-capita hours worked, real per-capita business debt growth, and the growth rate of the relative price of real estate. Since our model features an additional borrowing constraint on households, we include real per-capita household debt growth as our seventh observable.[9]

Appendix A describes in detail the sources and the construction of each series. Here, we emphasize two points. First, as mentioned above, our definition of investment excludes both residential and commercial structures, which are implicitly absorbed in the RE variable H_t. This choice follows Liu, Wang, and Zha (2013) and Iacoviello (2015). Second, we need to take a stand on the definition of RE prices. The literature is diverse, with Iacoviello (2005, 2015) focusing on residential housing, Miao, Wang, and Zha (2020) focusing on commercial real estate, and Liu, Wang, and Zha (2013) and Davis, Huang, and Sapci (2022) focusing on residential land. Our benchmark choice is to define real estate as residential housing, but we also discuss results for commercial real estate and for land in Section 5.1.[10]

The sample runs from 1975Q1 to 2018Q2, two dates that correspond to the first and last available quarterly observations on residential land prices. Since we will be comparing the fit of the DSGE model to that of various BVARs, we condition all estimations on the first twenty observations (1975Q1-1979Q4). In practice, we use these observations to initialize the Kalman filter when estimating the DSGE model and we compute the likelihood based only on data ranging from 1980Q1 to 2018Q2.[11]

3.2. **Calibration and prior distributions.** We split the model parameters into two subsets. The first subset contains 11 parameters that we calibrate and keep fixed during estimation. The second subset contains 23 parameters that are estimated from the data.

Table 1 summarizes our calibration. We assume that the patient household discounts the future at the rate $\beta_S = 0.996$, which generates a steady-state annual real interest rate of 3 percent at the prior mean. We normalize total RE supply and average hours worked to unity: $\overline{H} = \overline{N} = 1$. We adjust the preference weight $\overline{\theta}_N$ accordingly. In the production function, we set $\alpha = 0.35$ to generate a labor share of 65 percent. We set the average loan-to-value ratios to $\overline{\psi}_B = \overline{\psi}_E = 0.70$, in the range of available estimates for credit limits

[9]As in Liu, Wang, and Zha (2013), our estimation exercise allows for two more shocks than observables. To alleviate identification concerns, we check that all estimated parameters are locally identified both at the prior mode and at the posterior mode using the Iskrev (2010) rank test. Furthermore, the two "excess" shocks arise because of the inclusion of both transitory and permanent components in neutral and investment-specific technology. This lowers identification concerns because permanent and transitory shocks trigger different dynamics in models with forward-looking agents.

[10]We also considered a factor structure in which the relative prices of residential housing, commercial real estate, and land would all be interpreted as noisy observations of a "true" underlying RE price. In theory, this factor model could extract useful information from the comovements between the three observables. In practice, we found that it essentially interpreted the price of residential housing as the common factor, due to the much higher variance of the prices of commercial real estate and land.

[11]See Lubik and Schorfheide (2006), Chang, Doh, and Schorfheide (2007), or Iacoviello (2015) for examples of DSGE model estimation based on conditional likelihood.

Table 1. Calibrated parameters

Parameter	Value	Description	Parameter	Value	Description
Preferences			*Financial frictions*		
β_S	0.996	Discount rate (patient)	$\overline{\psi}_B, \overline{\psi}_E$	0.70	Loan-to-value ratio
β_B, β_E	0.986	Discount rate (impatient)	*Normalizations*		
$\overline{\theta}_H$	0.07	Housing preference weight	$\overline{H}$	1.00	Real estate supply
Production and depreciation			$\overline{N}$	1.00	Labor supply
α	0.35	Non-labor share			
ϕ	0.10	Relative real estate share			
δ	0.03	Depreciation rate			

(Iacoviello and Neri, 2010; Liu, Wang, and Zha, 2013; Justiniano, Primiceri, and Tambalotti, 2019).

Finally, we select the values of five parameters — the discount rates of the impatient household β_B and the entrepreneur β_E, the depreciation rate δ, the relative production share of real estate ϕ, and the utility weight on housing $\overline{\theta}_H$ — by matching four targets from the data: an average consumption-to-output ratio of 0.78, an average residential real estate-to-output ratio of 2.28, an average commercial real estate-to-output ratio of 1.17, and an average debt-to-output ratio of 1.38.[12] This procedure yields $\beta_B = \beta_E = 0.986$, $\delta = 0.03$, $\phi = 0.10$, and $\overline{\theta}_H = 0.07$. The rate at which borrowers discount the future is close to the value estimated by Liu, Wang, and Zha (2013) and ensures that households and entrepreneurs are impatient enough for the collateral constraints to be binding around the steady state of the linearized model. The depreciation parameter is slightly above the standard quarterly value of 0.025; this can be rationalized by the fact that business capital in the model corresponds to equipment, which depreciates faster than structures. Table 2 shows that this calibration brings the steady-state properties of our model in line with the data.

Table 3 reports our prior assumptions about the estimated parameters, which are mostly standard. We use Gaussian distributions centered around 1 and 1.01 for the average growth rates of neutral and investment-specific technology, $\overline{\mu}_Z$ and $\overline{\mu}_Q$. Faster growth in the investment-producing technology is consistent with the behavior of the U.S. economy since the late 1970s, as discussed in Greenwood, Hercowitz, and Krusell (1997) and Moura (2021). We set the prior mean of γ, the degree of consumption habits, to 0.5, with a standard deviation of 0.15. We use a relatively flexible prior for the investment adjustment cost φ, with a prior mean of 4 and a standard deviation of 2. For ρ_B, which determines the smoothness of the borrowing constraints, we use a Beta prior with a mean of 0.7 and a standard deviation of 0.15. We use the same Beta$(0.7, 0.15)$ prior for most of the autoregressive parameters

[12] Our calibration procedure imposes $\beta_B = \beta_E$, effectively inferring four parameters from four moments. See the notes to Table 2 for a discussion of the data used to compute the targets.

Table 2. Steady-state properties at the prior mode

Variable	Interpretation	Model	Data
$4(R-1)$	Annual real interest rate	0.03	—
C/Y	Consumption share	0.78	0.78[a]
$P_H(H_B+H_S)/(4Y)$	Real estate wealth to output: Households	2.28	2.28[b]
$P_H H_E/(4Y)$	Real estate wealth to output: Entrepreneur	1.17	1.17[c]
$(B_B+B_E)/(4Y)$	Debt-to-output ratio	1.39	1.39[d]

Notes. Data are sample averages computed over the period 1975-2018. Model ratios are computed at the prior mode.

[a]: Consumption includes nondurable goods and services. Output is the sum of consumption and investment, defined as the sum of producer equipment, intellectual property products, and consumer durable goods. Source: BEA.

[b]: Household real estate wealth includes residential real estate assets owned by households and by the non-corporate business sector. Source: Federal Reserve Board.

[c]: Entrepreneur real estate wealth includes commercial real estate assets owned by the non-financial corporate business sector and by the non-corporate business sector. Source: Federal Reserve Board.

[d]: Debt includes total loans on the balance sheet of households and of the non-financial business sector. Source: Federal Reserve Board.

defining the forcing processes. For the autocorrelation of permanent neutral and investment-specific technology growth, we instead center the prior at zero since the processes are already expressed in log-differences.[13] Finally, we use Inverse Gamma priors for the standard deviations of all innovations, which have been scaled to be of roughly similar size to help mode finding.

3.3. **Posterior estimates.** Table 3 also reports the estimation results. We find a moderate degree of consumption habits ($\gamma = 0.45$) and a small size of investment adjustment costs ($\varphi = 0.17$), in line with the estimates reported by Liu, Wang, and Zha (2013). These authors argue that small investment adjustment costs are useful for the DSGE model to match the comovements between RE prices and investment in the data. On the other hand, we find substantial inertia in the borrowing constraints ($\rho_B = 0.92$), so that the financial friction constitutes the main propagation mechanism in the model.

Turning to the parameters defining the exogenous processes, we find high persistence in the discount rate shock $\ln(A_t/A_{t-1})$ and the labor supply shock $\theta_{N,t}$, with estimated autoregressive coefficients close to 0.99 for both. The housing demand shock is less persistent ($\rho_H = 0.96$) but more volatile. In addition, we find less persistence in the entrepreneur collateral shock than in the household collateral shock ($\rho_{\Psi E} = 0.72$ vs. $\rho_{\Psi B} = 0.97$). Since

[13]In practice, we use Beta$(0.5, 0.15)$ distributions for $\rho_{QP} + 0.5$ and $\rho_{ZP} + 0.5$.

Table 3. Estimation results

Parameter	Description	Prior distribution			Posterior distribution	
		Distribution	Mean	SD	Mode	[5%, 95%]
$\overline{\mu}_Q$	Investment technology growth	Normal	1.01	0.01	1.01	[1.01, 1.01]
$\overline{\mu}_Z$	Neutral technology growth	Normal	1.00	0.01	1.00	[1.00, 1.00]
γ	Consumption habits	Beta	0.50	0.15	0.45	[0.39, 0.50]
φ	Investment adjustment costs	Gamma	4.00	2.00	0.17	[0.14, 0.22]
ρ_B	Collateral constraint smoothness	Beta	0.70	0.15	0.92	[0.90, 0.95]
ρ_A	Discount rate shock	Beta	0.70	0.15	0.99	[0.99, 0.99]
$\rho_{QP} + 0.5$	Permanent investment technology shock	Beta	0.50	0.15	0.66	[0.58, 0.75]
$\rho_{ZP} + 0.5$	Permanent neutral technology shock	Beta	0.50	0.15	0.86	[0.70, 0.94]
$\rho_{\Psi B}$	Household collateral shock	Beta	0.70	0.15	0.97	[0.94, 0.99]
$\rho_{\Psi E}$	Entrepreneur collateral shock	Beta	0.70	0.15	0.72	[0.61, 0.79]
ρ_H	Housing demand shock	Beta	0.70	0.15	0.96	[0.94, 0.97]
ρ_N	Labor supply shock	Beta	0.70	0.15	0.99	[0.99, 0.99]
ρ_{QT}	Transitory investment technology shock	Beta	0.70	0.15	0.76	[0.47, 0.93]
ρ_{ZT}	Transitory neutral technology shock	Beta	0.70	0.15	0.76	[0.41, 0.86]
$0.0005\sigma_A$	SD: Discount rate shock	Inv. Gamma	1.00	2.00	0.26	[0.22, 0.34]
$0.005\sigma_{QP}$	SD: Permanent investment technology shock	Inv. Gamma	1.00	2.00	0.92	[0.84, 1.02]
$0.0005\sigma_{ZP}$	SD: Permanent neutral technology shock	Inv. Gamma	1.00	2.00	6.67	[5.64, 8.48]
$0.05\sigma_{\Psi B}$	SD: Household collateral shock	Inv. Gamma	1.00	2.00	1.15	[0.85, 1.80]
$0.05\sigma_{\Psi E}$	SD: Entrepreneur collateral shock	Inv. Gamma	1.00	2.00	2.31	[1.74, 3.52]
$0.1\sigma_H$	SD: Housing demand shock	Inv. Gamma	1.00	2.00	1.06	[0.91, 1.48]
$0.005\sigma_N$	SD: Labor supply shock	Inv. Gamma	1.00	2.00	1.40	[1.26, 1.55]
$0.00005\sigma_{QT}$	SD: Transitory investment technology shock	Inv. Gamma	1.00	2.00	0.50	[0.25, 1.71]
$0.005\sigma_{ZT}$	SD: Transitory neutral technology shock	Inv. Gamma	1.00	2.00	0.90	[0.67, 1.03]

Notes. The estimation sample is 1975Q1-2018Q2, with observations 1975Q1-1979Q4 used to initialize the Kalman filter. The posterior distribution is constructed from the random-walk Metropolis-Hastings algorithm with a single chain of 1,000,000 draws and a burn-in period of 800,000 draws. The acceptance rate is 29% and standard tests confirm convergence.

the collateral shocks fill the gap between the debt levels implied by the model and those observed in the data, one possible interpretation is that borrowing constraints provide a more accurate representation of the financial frictions facing entrepreneurs compared to those facing households. We substantiate this view below. Finally, the estimated standard deviations suggest that the data favor a specification of the model without transitory investment-specific technology shocks.

4. Model Evaluation

This section evaluates the empirical performance of the DSGE model. We start by comparing the model to various BVARs. In light of the poor relative fit of the DSGE model, we use posterior predictive checks to highlight several important discrepancies between the model and the data, with a focus on the RE dimension. Finally, we discuss the role of shocks in the DSGE model by looking at shock decompositions and IRFs.

4.1. **Comparison with BVARs.** As noted by Lubik and Schorfheide (2006), An and Schorfheide (2007), or Smets and Wouters (2003, 2007), it is possible to assess the fit of a DSGE model based on comparison to a more general reference model. VARs provide a natural benchmark for DSGE models since linearized economies deliver, at least approximately, restricted VAR representations for the vector of observables (Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson, 2007). Hence, our first step is to evaluate the DSGE model against different BVARs using marginal data densities.[14]

Sims (2003) highlights several difficulties related to the Bayesian comparison of DSGE and VAR models. First, there is uncertainty with respect to the appropriate order p of the VAR. We deal with this issue by considering four different VARs, with $p = 1$ to $p = 4$. Second, marginal data densities automatically penalize more complex models, so that appropriate BVAR shrinkage is crucial for the procedure not to systematically favor the DSGE model. We implement Bayesian shrinkage using the dummy-observation version of the Minnesota prior available in Dynare (2011).[15] Third, conditioning estimation on a training sample limits prior influence on model comparison outcomes. This is why, as discussed above, we condition the estimation of the DSGE model and all BVARs on the first twenty observations, corresponding to 1975Q1-1979Q4.

Table 4 reports the log marginal data densities for the DSGE model and the alternative BVARs. The BVAR marginal data densities are quite sensitive to the choice of hyperparameters like the lag order and the prior tightness: the data consistently favor models with more lags, and laxer priors ($\tau = 3$) are preferred to tighter priors ($\tau = 5$) for all specifications but the BVAR(4). The key finding is that all BVARs dominate the DSGE model, with advantages ranging from 90 to 240 log density points. Interpreted through standard metrics, the associated Bayes factors signal decisive empirical evidence against the model (Jeffreys,

[14]Del Negro, Schorfheide, Smets, and Wouters (2007) propose an alternative evaluation strategy: use the DSGE model to generate a prior distribution for the VAR coefficients and document how the fit of the BVAR changes as the restrictions implied by economic theory are relaxed. We do not pursue this strategy here.

[15]The standard Minnesota Prior shrinks the VAR model toward a multivariate random walk, so our estimated VARs include the data in log levels. This is consistent with the DSGE model, which implies cointegration between the log levels. Doan, Litterman, and Sims (1984) represent the Minnesota prior using dummy observations, characterized by a weight τ and a decay factor d. They allow for two additional sets of dummy observations: "sum-of-coefficients" or "own-persistence" dummy data providing additional shrinkage toward the random-walk specification (weight μ) and "initial-observation" or "co-persistence" dummy data instead shrinking the model toward cointegration (weight λ). The hyperparameters τ, d, μ, and λ control the tightness of the various components of the prior. In line with standard practice, for instance Lubik and Schorfheide (2006), we set $d = 0.5$, $\mu = 2$, $\lambda = 5$, and we consider $\tau = 3$ and $\tau = 5$.

Table 4. Log marginal data densities and Bayes factors

Model	Log marginal data density	Bayes factor relative to DSGE
DSGE	$3,530.69$	1.0
BVAR(1): $\tau = 3$	$3,622.18$	exp(91.5)
BVAR(2): $\tau = 3$	$3,746.95$	exp(216.3)
BVAR(3): $\tau = 3$	$3,767.90$	exp(237.2)
BVAR(4): $\tau = 3$	$3,769.85$	exp(239.2)
BVAR(1): $\tau = 5$	$3,618.99$	exp(88.3)
BVAR(2): $\tau = 5$	$3,720.73$	exp(190.0)
BVAR(3): $\tau = 5$	$3,760.22$	exp(229.5)
BVAR(4): $\tau = 5$	$3,775.34$	exp(244.7)

Notes. The estimation sample is 1975Q1-2018Q2, conditional on observations 1975Q1-1979Q4. Log marginal data densities are computed based on Geweke's (1999) Modified Harmonic Mean Estimator for the DSGE model and analytically for the BVARs. Bayes factors attribute equal prior probabilities to all models.

1961; Kass and Raftery, 1995). We conclude that the time series fit of the DSGE model is not good relative to BVARs, as even simple first-order specifications perform better.[16]

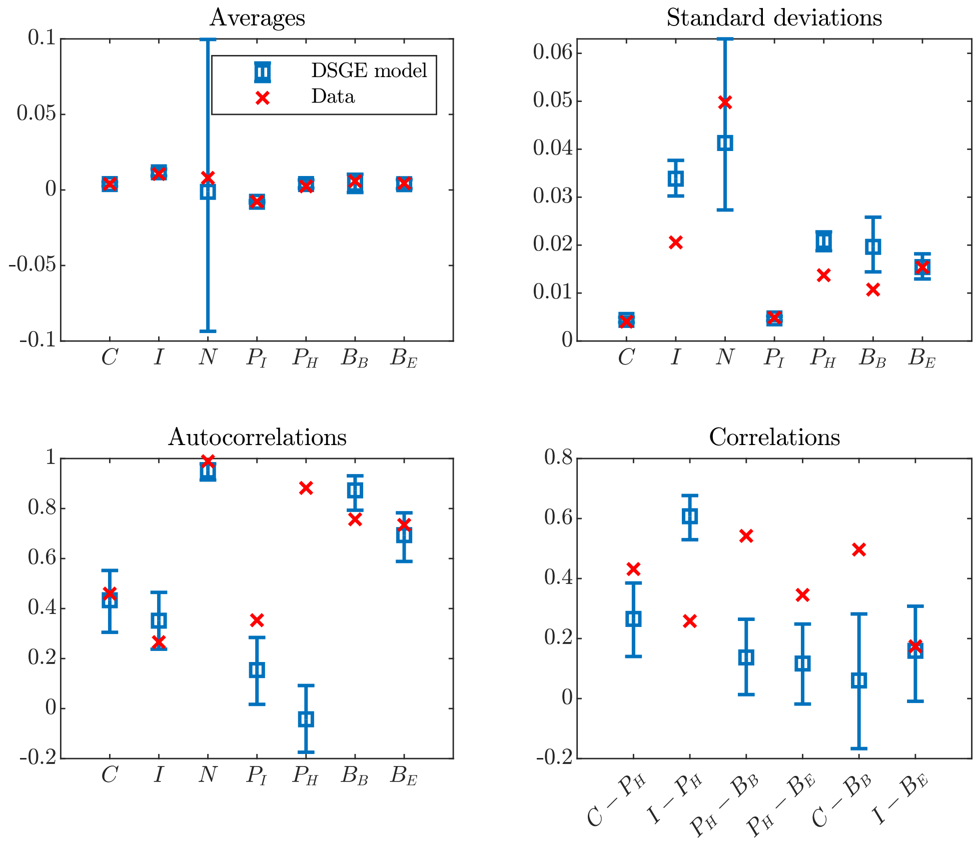
4.2. **DSGE model misspecification.** This decisive rejection suggests that the DSGE model suffers from important misspecifications limiting its ability to fit the data. We rely on posterior predictive checks to identify these deficiencies: Figure 1 plots the posterior predictive distributions of the sample means, standard deviations, first-order autocorrelations, and selected correlations of the observables. (Recall that we use log differences for consumption, investment, the relative price of investment, RE prices, and debt, and that we demean log hours worked.) Computations are based on 500 draws from the posterior distribution of DSGE model parameters: for each draw, we solve the model, simulate an artificial dataset of 153 observations (the size of our estimation sample), and compute sample moments. The figure also reports the corresponding data moments. An empirical moment lying well inside the posterior predictive distribution signals the ability of the DSGE model to reproduce the data, while an empirical moment far in the tails of the posterior predictive distribution signals a discrepancy.[17]

The top-left panel focuses on first moments. It shows that the DSGE model reproduces well the different average growth rates of the observables. In particular, due to investment-specific technological progress, there is faster growth in real investment than in real consumption, and

[16]BVARs do not systematically outperform DSGE models according to the marginal likelihood criterion. For instance, the DSGE model in Smets and Wouters (2007) does about as well as a BVAR(4) along this dimension.

[17]An and Schorfheide (2007) provide an introduction to posterior predictive checks, as well as further references from the Bayesian statistics literature. Schorfheide (2000), Chang, Doh, and Schorfheide (2007), and Leeper, Traum, and Walker (2017) are examples of previous applications to DSGE model evaluation.

FIGURE 1. Posterior predictive distributions: Sample moments



Notes. Boxplots present the mean, the 5th quantile, and the 95th quantile of posterior predictive distributions of sample moments, based on 500 draws. Crosses indicate data counterparts. Consumption (C), investment (I), the relative price of investment (P_I), RE prices (P_H), household debt (B_B), and business debt (B_E) are in log-differences, while hours worked (N) are in log-levels.

the relative price of investment has a downward trend. The posterior predictive distributions for the sample means are generally tight, reflecting the tight posterior distributions of $\overline{\mu}_Q$ and $\overline{\mu}_Z$ in Table 3. The exception is hours worked, whose strong persistence generates highly variable sample posterior moments. Overall, the performance of the DSGE model with respect to trends is satisfactory.

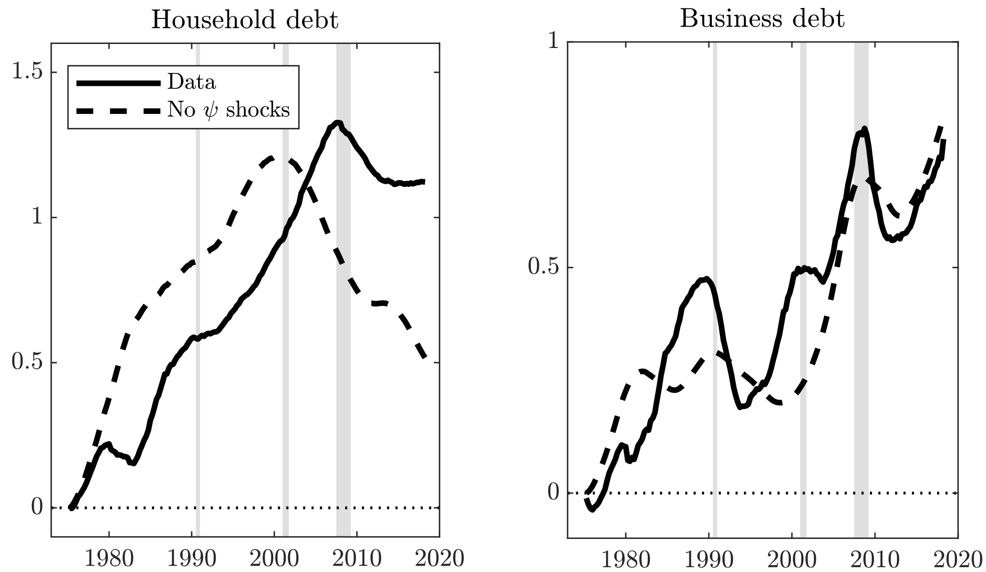
As shown in the top-right panel, the performance is less convincing when it comes to standard deviations. On the one hand, the DSGE model is able to reproduce the volatility of four variables: consumption growth, log hours worked, growth in the relative price of investment, and business debt growth. On the other, the empirical standard deviations of investment growth, RE price growth, and household debt growth are far below their posterior predictive distributions, signaling that the model significantly overestimates the volatility of these variables. This is the first symptom of an important discrepancy between the data and the DSGE model. Because a likelihood-based estimator automatically trades off the benefits and costs of matching different moments, it must be the case that worsening the fit with respect to these three standard deviations improves the model's ability to match other data properties — a clear sign of misspecification.

The bottom-left panel focuses on autocorrelations and provides further evidence of misspecification. For six observables, the performance of the DSGE model ranges from good (growth in consumption, investment, and business debt; log hours worked) to barely acceptable (growth in the relative price of investment and household debt). However, performance is abysmal for the seventh observable: RE price growth. In the data, quarterly changes in the log difference of RE prices exhibit a strong autocorrelation of 0.88, a pattern known as momentum in the housing literature (see Case and Shiller, 1989; Cutler, Poterba, and Summers, 1991; Guren, 2018). In contrast, the DSGE model yields a posterior distribution roughly centered at zero for the autocorrelation of RE price growth, with a 90% posterior credible set ranging from -0.17 to 0.09.

To some extent, this shortcoming of the DSGE model is not surprising. As noted by Guren (2018), autocorrelation in asset price changes is a theoretical puzzle because forward-looking models embed a strong arbitrage force that eliminates momentum. This property is well known in the housing literature: for instance, Glaeser, Gyourko, Morales, and Nathanson (2014) remark that "no reasonable parameter set" can make a frictionless model consistent with short-run momentum. Additional frictions can help, including search frictions (Head, Lloyd-Ellis, and Sun, 2014), gradual learning about market conditions (Anenberg, 2016), behavioral biases (Glaeser and Nathanson, 2017), belief heterogeneity (Burnside, Eichenbaum, and Rebelo, 2016), strategic complementarities between sellers (Guren, 2018), and natural expectations (Pancrazi and Pietrunti, 2019). On the other hand, we can conclude from our analysis that the main frictions present in quantitative DSGE models with housing, including consumption habits, investment adjustment costs, and collateral constraints, are not able to create momentum, leading to an important gap with the data. A contribution of our paper is to document this limitation of the existing DSGE-housing literature.

The forward-looking behavior of economic agents makes this misspecification especially worrying for the DSGE model. In the real world, households and firms expect RE prices to continue growing for several quarters when they observe an initial rise. This pattern shapes both their incentives to sell or buy real estate and their choices for consumption, investment, and labor supply and demand. In contrast, lack of momentum in the model implies that agents do not foresee future price movements and therefore behave differently. The presence of collateral constraints in the model amplifies this discrepancy: since financial frictions boost the feedback from real estate to aggregate demand, the inability to generate price momentum necessarily leads to misspecification in consumption and investment decisions.

Figure 2. Household and business debt: The role of collateral shocks



Notes. Solid lines represent the data in log levels. Dashed lines represent the counterfactual paths that would be obtained without collateral shocks ($\psi_{B,t} = \overline{\psi}_B$ and $\psi_{E,t} = \overline{\psi}_E$ for all periods). Shaded bands represent the NBER recession dates. Computations at the posterior mode.

To assess the performance of the DSGE model with respect to this collateral channel, the bottom-right panel examines various comovements between the components of aggregate demand (consumption, investment), RE prices, and business and household debt. The results highlight two important difficulties. First, while the model reproduces at least qualitatively the positive correlation between consumption growth and RE price growth, it is hard to argue that this good fit originates from the collateral constraint. Indeed, the model also significantly underestimates the relationship between growth in consumption and in household debt, as well as between growth in household debt and in RE prices. Second, the model vastly overestimates comovements between growth in investment and in RE prices. This is surprising because the fit is good for comovements between growth in RE prices and in business debt, and especially between growth in investment and in business debt. Again, this pattern suggests that the collateral constraint in the DSGE model is not the main source of comovements between the RE market and business investment.

As a further check, Figure 2 compares the observed levels of household and business debt to counterfactual paths obtained by omitting the contributions of the two collateral shifters $\psi_{B,t}$ and $\psi_{E,t}$. Since these shocks absorb all discrepancies between the data and the debt levels implied by the DSGE model conditional on RE prices, comparing actual and counterfactual paths provides another measure of fit for collateral constraints. The charts reveal uncomfortably wide and persistent gaps for household debt, which echo both the large estimated value for the persistence of the household collateral shock ($\rho_{\psi B} = 0.97$) and the model's inability to reproduce the correlation between household debt and RE prices (bottom-right panel in Figure 1). The performance is better for business debt, as smaller and less persistent gaps signal closer correspondence between the model-implied and actual debt levels. Nevertheless, the fit of the entrepreneur collateral constraint remains questionable.

To conclude, our posterior analysis reveals that the DSGE model with collateral constraints and real estate is misspecified along three important dimensions: it overestimates the cyclical variance of investment, RE prices, and household debt; it fails to generate momentum in RE prices; it cannot account for the cyclical comovements between aggregate demand, debt, and RE prices. Identifying these deficiencies is important both to understand the limitations of the current models and to guide the search for improvements.

4.3. **Economic implications.** We now look at variance decompositions, IRFs, and historical shock decompositions to study the economic implications of the DSGE model. As before, we focus on the RE dimension.

Table 5 presents the variance decomposition for the seven observables used in estimation, plus the growth rate of output. Starting with technology shocks, the data attribute a significant role to the two neutral shocks, which explain together about half of the variance of growth in output and in consumption, and 35 percent of the variance of investment growth. The permanent investment-specific shock drives the price of investment but has little effect on other variables. On the other hand, technology disturbances contribute little to fluctuations in hours or to growth in RE prices and in debt. The labor supply shock explains hours worked and has important effects on growth in output and consumption. The two collateral shocks explain between 75 and 90 percent of the variance of debt growth in the model, which highlights again the poor fit of the collateral constraints.

There is an interesting relationship between shocks to the discount rate and housing demand. On the one hand, the discount shock is the main source of fluctuations in RE prices, as it accounts alone for 70 percent of the variance of δp_H. In comparison, the housing demand shock explains only 15 percent. On the other hand, the housing demand shock has much stronger spillovers on aggregate quantities: for instance, it explains 20 and 40 percent of growth in output and investment, while the discount rate shock accounts for less than 10 percent of fluctuations in both variables.

This result runs counter to most of the literature integrating real estate into estimated DSGE models. For instance, Iacoviello and Neri (2010), Liu, Wang, and Zha (2013), and Iacoviello (2015) all find that direct shocks to housing preferences are the most important drivers of RE prices, and that discount rate shocks do not matter much. An exception is Miao, Wang, and Zha (2020), who identify discount rate shocks as the main drivers of RE prices in a setup that abstracts from housing demand shocks. Below, we argue that our estimation exercise favors the discount rate shock because the housing demand shock

TABLE 5. Variance decomposition

Innovation	Y	C	I	N	P_I	P_H	B_B	B_E
Discount (ϵ_A)	7	2	9	4	0	70	1	6
Housing demand (ϵ_H)	21	7	40	5	0	15	21	2
Labor supply (ϵ_N)	21	37	9	89	0	5	0	0
Household collateral ($\epsilon_{\Psi B}$)	0	0	1	0	0	1	76	0
Entrepreneur collateral ($\epsilon_{\Psi E}$)	2	1	3	1	0	0	0	91
Permanent IS technology (ϵ_{QP})	1	2	5	0	100	0	0	0
Permanent neutral technology (ϵ_{ZP})	15	37	5	0	0	3	0	0
Transitory IS technology (ϵ_{QT})	0	0	0	0	0	0	0	0
Transitory neutral technology (ϵ_{ZT})	33	15	29	1	0	5	0	0

Notes. Figures are in percent and correspond to the mean of 500 draws from the posterior distribution of the variance decomposition. Output (Y), consumption (C), investment (I), the relative price of investment (P_I), the real estate price (P_H), household debt (B_B), and business debt (B_E) are in log-differences, while hours worked (N) are in log-levels.

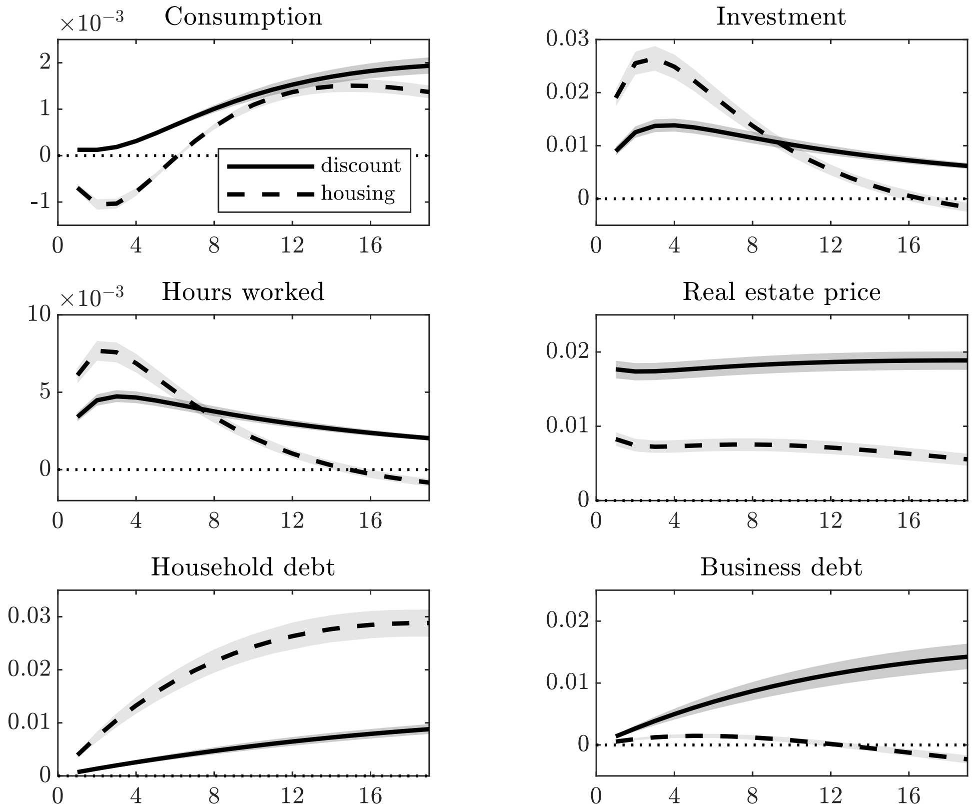
generates counterfactual comovements between key variables like consumption, investment, RE prices, and debt.

This result also corroborates the failure of collateral constraints to propagate movements in RE prices to aggregate quantities in the model. Indeed, if the constraints were the main transmission channel for RE shocks, we would expect the disturbance with the largest effect on prices to also have the strongest equilibrium impact on debt and quantities. The fact that the discount rate shock has limited impact on debt and quantities in spite of its importance for asset prices signals weak propagation from collateral constraints.

Figure 3 presents the estimated responses of the main variables to the discount and housing demand shocks. A positive discount rate shock creates an economy-wide expansion characterized by comovements between consumption, investment, hours worked, RE prices, and debt levels. Due to the specification of the process for the log difference of A_t, a positive discount shock makes households more patient in the short run. This has two effects on the economy. First, households attach more value to assets that pay off in the future, in particular real estate. Hence, demand for real estate surges. Second, households substitute leisure across time to supply more labor. Higher hours worked boost the marginal product of the other inputs, and entrepreneurs react by demanding more real estate as well. This rise in demand creates upward pressure on RE prices, which increase persistently after the shock. As collateral constraints are relaxed, households and entrepreneurs issue additional debt, which supports aggregate demand further. Finally, the additional units of output are invested rather than consumed, allowing for higher future production.

After a positive housing demand shock, households give more value to housing services, so that they cut consumption and increase labor supply to free resources for RE accumulation.

Figure 3. Impulse-response functions to positive shocks to the discount rate and housing demand

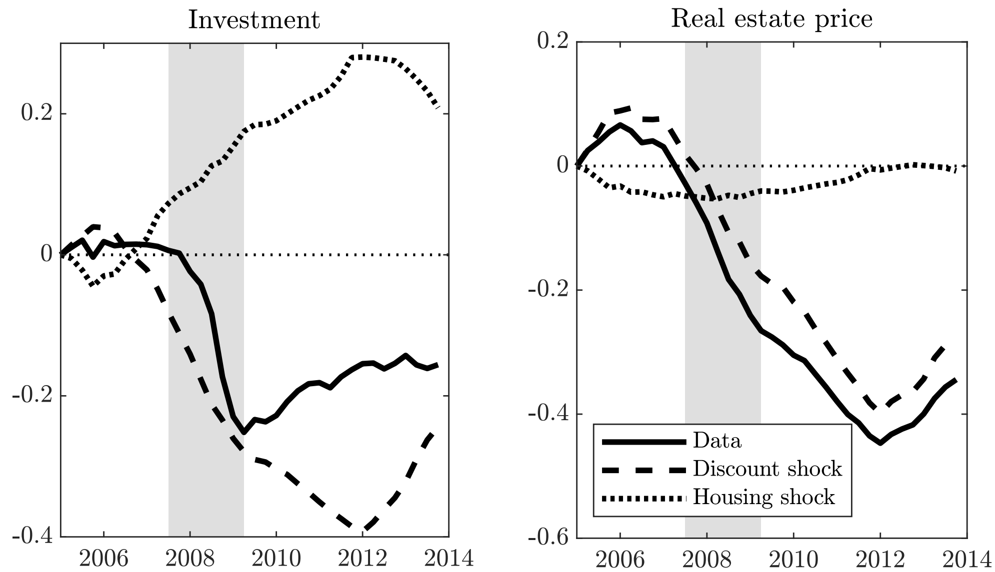


Notes. Solid lines represent posterior mean responses for variables in log-levels, while dashed lines represent 68% posterior confidence bands. Computations based on 500 draws.

Higher household demand puts upward pressures on RE prices, which relaxes the borrowing constraints. Entrepreneurs face a trade off. On the one hand, non-labor inputs have higher marginal products due to the positive response of hours worked, so that entrepreneur demand for real estate increases. On the other, the rise in RE prices weighs on entrepreneur demand. The responses indicate that the negative effect dominates: entrepreneur debt barely responds to the shock, signaling that entrepreneurs sell real estate to households to benefit from higher prices. The persistent increase in investment suggests that entrepreneurs turn to the cheaper inputs and favor capital accumulation instead.

Comparing these responses explains why the estimated DSGE model favors the discount rate shock over the housing demand shock. In the data, consumption and investment growth are positively correlated, and business debt features strong positive comovements with household debt and RE prices. The discount rate shock produces similar dynamics in the model, while the housing demand shock does not. Allowing collateral constraints for both business

FIGURE 4. Accounting for the 2008 financial crisis



Notes. Solid lines represent the data in log levels. Dashed lines represent counterfactual paths conditional on discount rate shocks only. Dotted lines represent counterfactual paths conditional on housing demand shocks only. The shaded band represents the NBER recession dates. Computations at the posterior mode.

and households in the model drives this finding: in particular, the reallocation of real estate from entrepreneurs to households after a housing demand shock limits the shock's ability to generate the correct comovements. Thus, models in which a single group of agents faces a collateral constraint, whether households (Iacoviello and Neri, 2010) or entrepreneurs (Liu, Wang, and Zha, 2013), underestimate this substitution effect and, as a result, attribute a larger role to household demand shocks.

The counterfactual dynamics triggered by housing demand shocks can also be seen from Figure 4, which compares the actual paths of business investment and RE prices during the financial crisis and its aftermath to the counterfactual trajectories that would have happened if only discount rate shocks or housing demand shocks had occurred in history. Discount rate shocks account well for the sharp contractions in investment and RE prices. On the other hand, housing demand shocks do not generate the correct comovements: they trigger a mild fall in RE prices and a sustained rise in investment.

We conclude this section with a word of caution. The poor empirical performance of the DSGE model means that these IRFs and shock decompositions should be taken with a grain of salt. The model clearly lacks important features to reproduce the data, so that it is hazardous to attach too much confidence to its quantitative implications.

5. Robustness

This section documents the robustness of our main findings. First, we estimate the DSGE model using alternative measures of RE prices considered in the literature. Second, we use a shorter sample that excludes the 2008 crisis and its aftermath, to focus on "normal times." Third, we evaluate the model at business-cycle frequencies, as isolated by the Hodrick-Prescott filter. All checks confirm the limitations of the DSGE setup.

5.1. **Alternative real estate prices.** The DSGE-housing literature considers various definitions of RE prices. Following Iacoviello (2005, 2015) and Iacoviello and Neri (2010), our benchmark estimates rely on the price of residential housing. Therefore, our first robustness check is to confirm that our findings still hold when the model is estimated using alternative series: the price of commercial real estate as in Miao, Wang, and Zha (2020) and the price of residential land as in Liu, Wang, and Zha (2013) and Davis, Huang, and Sapci (2022). Appendix A provides the data sources. Both the observed sample (1975Q1-2018Q2) and the empirical approach (the Bayesian framework, using the first 20 observations as training sample) are the same as in our benchmark analysis.

Table 6 reports the results. To save space, we focus on selected moments of interest: some standard deviations, the autocorrelation of RE price growth, and the correlations related to the collateral channel. The statistics highlight that the two estimated models suffer from the same empirical shortcomings as our benchmark DSGE model: the volatilities of investment growth, RE price growth, and household debt growth are significantly overestimated; there is no momentum in RE prices; the models fail to reproduce most comovements between the components of private aggregate demand (consumption and investment), household and business debt, and RE prices. The discrepancies between theory and the data are significant: empirical moments often fall out of 90% posterior confidence bands, a pattern signaling severe model misspecification.

This exercise confirms that the empirical deficiencies of the DSGE models originate from their common structure, rather than from the data chosen for estimation. The next exercise reinforces this insight by evaluating the model on a different sample.

5.2. **Excluding the financial crisis.** Our baseline estimation sample runs from 1975 to 2018 and arguably merges periods with different macroeconomic characteristics. For instance, it is common to distinguish between a period of high aggregate volatility (up to 1985), a period of stability (1986-2006), and the financial crisis and its aftermath (2007 and after).[18] Our second robustness check is to evaluate the DSGE model on the 1975-2006 sample that excludes the financial crisis, in order to assess whether the empirical performance

[18]For instance, Smets and Wouters (2007) find important differences in shock volatilities before and after 1985 in an estimated DSGE model, while Guerron-Quintana and Jinnai (2019) uncover level shifts in GDP and its components after 2008.

Table 6. Robustness checks: Alternative real estate price series

Moments	Commercial RE price		Residential land price	
	Data	Model	Data	Model
A. Standard deviations				
Investment	0.021	[0.035, 0.044]	0.021	[0.032, 0.042]
RE prices	0.030	[0.040, 0.048]	0.032	[0.040, 0.048]
Household debt	0.011	[0.016, 0.030]	0.011	[0.014, 0.024]
Business debt	0.015	[0.013, 0.018]	0.015	[0.013, 0.017]
B. Autocorrelation				
RE prices	0.485	[−0.164, 0.010]	0.840	[−0.170, 0.108]
C. Correlations				
Consumption - RE prices	0.218	[0.035, 0.295]	0.453	[0.063, 0.310]
Investment - RE prices	0.106	[0.513, 0.665]	0.264	[0.495, 0.655]
RE prices - Household debt	0.207	[−0.055, 0.219]	0.547	[−0.040, 0.212]
RE prices - Business debt	0.432	[−0.021, 0.224]	0.321	[−0.030, 0.218]
Consumption - Household debt	0.497	[−0.189, 0.272]	0.497	[−0.179, 0.281]
Investment - Business debt	0.174	[−0.089, 0.223]	0.174	[−0.113, 0.239]

Notes. The observed sample is 1975Q1-2018Q2 for each estimation exercise, with observations 1975Q1-1979Q4 used to initialize the Kalman filter. Each posterior distribution is constructed from the random-walk Metropolis-Hastings algorithm with a single chain of 1,000,000 draws and a burn-in period of 800,000 draws. The acceptance rates are 28% and 30% respectively, and standard tests confirm convergence. For each exercise, column 'Data' reports sample moments and column 'Model' reports the 5th and 95th quantiles of posterior credible distributions, based on 500 draws. All variables are in log-differences.

improves when we focus on "normal times." (Results are similar when we focus instead on the shorter Great Moderation subsample, 1985-2006.) Here, we revert to our benchmark definition of real estate as residential housing.

The results are in column "Shorter estimation sample" in Table 7, which focuses on the same moments as before. Interestingly, comparison with Section 4.2 indicates that removing the financial crisis from the sample does not affect the empirical regularities of the data: the standard deviations, the persistence of RE price growth, and the comovements between the variables are all close to their full-sample counterparts. This means that full-sample moments reflect standard business cycles rather than special crisis dynamics. Therefore, the poor fit of the DSGE model over the shorter sample comes at no surprise, and confirms that the setup cannot reproduce basic features of the data.

5.3. **Business-cycle properties.** Finally, our third robustness check follows Iacoviello and Neri (2010) in evaluating the fit of the DSGE model at business-cycle frequencies, as isolated by the HP filter with smoothing parameter 1,600. Looking at detrended time series provides a different perspective on both the data and the model, and allows us to verify whether the

Table 7. Robustness checks: Alternative estimation sample and HP-filtered moments

Moments	Shorter estimation sample		HP-filtered moments	
	Data	Model	Data	Model
A. Standard deviations				
Investment	0.021	[0.036, 0.046]	0.036	[0.046, 0.070]
RE prices	0.014	[0.015, 0.019]	0.031	[0.021, 0.030]
Household debt	0.011	[0.013, 0.022]	0.019	[0.027, 0.051]
Business debt	0.015	[0.013, 0.018]	0.036	[0.024, 0.040]
B. Autocorrelation				
RE prices	0.883	[−0.211, 0.098]	0.966	[0.557, 0.771]
C. Correlations				
Consumption - RE prices	0.432	[0.435, 0.638]	0.541	[−0.061, 0.473]
Investment - RE prices	0.258	[0.507, 0.691]	0.493	[0.449, 0.741]
RE prices - Household debt	0.542	[−0.054, 0.260]	0.622	[−0.176, 0.422]
RE prices - Business debt	0.345	[−0.018, 0.286]	0.245	[−0.200, 0.384]
Consumption - Household debt	0.497	[−0.185, 0.336]	0.571	[−0.248, 0.440]
Investment - Business debt	0.174	[−0.014, 0.344]	0.219	[−0.057, 0.525]

Notes. The estimation samples are 1975Q1-2006Q4 and 1975Q1-2018Q2 respectively, with observations 1975Q1-1979Q4 used to initialize the Kalman filter. Real estate corresponds to residential housing. Each posterior distribution is constructed from the random-walk Metropolis-Hastings algorithm with a single chain of 1,000,000 draws and a burn-in period of 800,000 draws. The acceptance rates are 24% and 29% respectively, and standard tests confirm convergence. For each exercise, column 'Data' reports sample moments and column 'Model' reports the 5th and 95th quantiles of posterior credible distributions, based on 500 draws. All variables are in log-differences.

limitations of the setup are apparent from filtered data. We proceed as follows. Using the model estimated in Section 3, we draw 500 parameters from the posterior distribution and simulate for each draw an artificial dataset of 153 observations, the size of our estimation sample. Then, we cumulate the variables expressed in log-differences to obtain artificial series in level, to which we apply the HP filter.

Column "HP-filtered moments" in Table 7 compares these model-implied moments with data moments, obtained by filtering the original variables. Overall, the results identify the same shortcomings of the DSGE model as found by looking at growth rates. Thus, we conclude that our findings are robust with respect to the approach used to detrend the model and the data.

6. Conclusion

The late-2000s boom-bust cycle in U.S. house prices and the subsequent recession led macroeconomists to incorporate the real estate market in quantitative DSGE models. Such models have been used to understand the drivers of house price fluctuations and the spillovers between the real estate sector and the rest of the economy. However, evidence regarding their ability to fit the data remains scarce.

In this paper, we assess the empirical performance of a benchmark DSGE model with real estate and collateral constraints. We estimate the model from U.S. data using Bayesian methods and evaluate its fit along various dimensions. Our main result is negative: the DSGE model does not perform well. It is strongly rejected when tested against unrestricted Bayesian VARs and it is not able to reproduce key cyclical properties of the data, in particular the persistence of RE prices and various comovements between aggregate demand, RE prices, and debt. Performance does not improve with alternative definitions of RE prices, estimation samples, or detrending approaches. Overall, our findings raise doubts about the specification of DSGE models with real estate and collateral constraints.

Our results demonstrate that more research is needed to improve on the current setup. Our analysis also identifies the dimensions deserving special attention: sources of short-term momentum in RE prices and collateral constraints more in line with aggregate data. Explicitly representing financial intermediation or taking into account nominal frictions might also be useful. As we discuss in the paper, an emerging literature is starting to consider these issues, but how and when a new benchmark DSGE model with real estate and collateral constraints will emerge remains an open question. Until a better setup is found, policy recommendations for the RE market derived from DSGE models will remain questionable.

References

Adjemian, S., H. Bastani, M. Juillard, F. Karamé, J. Maih, F. Mihoubi, W. Mutschler, G. Perendia, J. Pfeifer, M. Ratto, and S. Villemot (2011): "Dynare: Reference Manual Version 4," Dynare Working Papers 1, CEPREMAP.

Alpanda, S., and S. Zubairy (2017): "Addressing household indebtedness: Monetary, fiscal or macroprudential policy?," *European Economic Review*, 92(C), 47–73.

An, S., and F. Schorfheide (2007): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 26(2-4), 113–172.

Anenberg, E. (2016): "Information Frictions and Housing Market Dynamics," *International Economic Review*, 57(4), 1449–1479.

Burnside, C., M. Eichenbaum, and S. Rebelo (2016): "Understanding Booms and Busts in Housing Markets," *Journal of Political Economy*, 124(4), 1088–1147.

Case, K. E., and R. J. Shiller (1989): "The Efficiency of the Market for Single-Family Homes," *American Economic Review*, 79(1), 125–137.

Catherine, S., T. Chaney, Z. Huang, D. Sraer, and D. Thesmar (2022): "Quantifying Reduced-Form Evidence on Collateral Constraints," *Journal of Finance*, 77(4), 2143–2181.

Chaney, T., D. Sraer, and D. Thesmar (2012): "The Collateral Channel: How Real Estate Shocks Affect Corporate Investment," *American Economic Review*, 102(6), 2381–2409.

Chang, Y., T. Doh, and F. Schorfheide (2007): "Non-stationary Hours in a DSGE Model," *Journal of Money, Credit and Banking*, 39(6), 1357–1373.

Cutler, D. M., J. M. Poterba, and L. H. Summers (1991): "Speculative Dynamics," *Review of Economic Studies*, 58(3), 529–546.

Davis, J. S., K. X. Huang, and A. Sapci (2022): "Land Price Dynamics and Macroeconomic Fluctuations with Imperfect Substitution in Real Estate Markets," *Journal of Economic Dynamics and Control*, 134(C).

Davis, M. A., and J. Heathcote (2005): "Housing And The Business Cycle," *International Economic Review*, 46(3), 751–784.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2007): "On the Fit of New Keynesian Models," *Journal of Business & Economic Statistics*, 25, 123–143.

Doan, T., R. Litterman, and C. Sims (1984): "Forecasting and Conditional Projection using Realistic Prior Distributions," *Econometric Reviews*, 3(1), 1–100.

Fernández-Villaverde, J., J. F. Rubio-Ramírez, T. J. Sargent, and M. W. Watson (2007): "ABCs (and Ds) of Understanding VARs," *American Economic Review*, 97(3), 1021–1026.

Fisher, J. D. M. (2006): "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks," *Journal of Political Economy*, 114(3), 413–451.

Garriga, C., F. E. Kydland, and R. Sustek (2017): "Mortgages and Monetary Policy," *Review of Financial Studies*, 30(10), 3337–3375.

——— (2021): "MoNK: Mortgages in a New-Keynesian Model," *Journal of Economic Dynamics and Control*, 123(C).

Geweke, J. (1999): "Using Simulation Methods for Bayesian Econometric Models: Inference, Development, and Communication," *Econometric Reviews*, 18(1), 1–73.

Glaeser, E. L., J. Gyourko, E. Morales, and C. G. Nathanson (2014): "Housing Dynamics: An Urban Approach," *Journal of Urban Economics*, 81(C), 45–56.

Glaeser, E. L., and C. G. Nathanson (2017): "An Extrapolative Model of House Price Dynamics," *Journal of Financial Economics*, 126(1), 147–170.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change," *American Economic Review*, 87(3), 342–362.

Guerron-Quintana, P. A., and R. Jinnai (2019): "Financial Frictions, Trends, and the Great Recession," *Quantitative Economics*, 10(2), 735–773.

Guren, A. M. (2018): "House Price Momentum and Strategic Complementarity," *Journal of Political Economy*, 126(3), 1172–1218.

Hansen, G. D. (1985): "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16(3), 309–327.

Head, A., H. Lloyd-Ellis, and H. Sun (2014): "Search, Liquidity, and the Dynamics of House Prices and Construction," *American Economic Review*, 104(4), 1172–1210.

Iacoviello, M. (2005): "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 95(3), 739–764.

——— (2015): "Financial Business Cycles," *Review of Economic Dynamics*, 18(1), 140–164.

Iacoviello, M., and S. Neri (2010): "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, 2(2), 125–164.

Iskrev, N. (2010): "Local Identification in DSGE Models," *Journal of Monetary Economics*, 57(2), 189–202.

Jeffreys, H. (1961): *Theory of Probability*. Oxford, Oxford, England, third edn.

Justiniano, A., G. Primiceri, and A. Tambalotti (2010): "Investment shocks and business cycles," *Journal of Monetary Economics*, 57(2), 132–145.

——— (2011): "Investment Shocks and the Relative Price of Investment," *Review of Economic Dynamics*, 14(1), 101–121.

——— (2015): "Household Leveraging and Deleveraging," *Review of Economic Dynamics*, 18(1), 3–20.

——— (2016): "A Simple Model of Subprime Borrowers and Credit Growth," *American Economic Review*, 106(5), 543–547.

——— (2019): "Credit Supply and the Housing Boom," *Journal of Political Economy*, 127(3), 1317–1350.

Kass, R. E., and A. E. Raftery (1995): "Bayes Factors," *Journal of the American Statistical Association*, 90(430), 773–795.

Kermani, A., and Y. Ma (2020): "Asset Specificity of Non-Financial Firms," NBER Working Papers 27642, National Bureau of Economic Research, Inc.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

Kydland, F. E., P. Rupert, and R. Sustek (2016): "Housing Dynamics Over The Business Cycle," *International Economic Review*, 57(4), 1149–1177.

Leeper, E. M., N. Traum, and T. B. Walker (2017): "Clearing Up the Fiscal Multiplier Morass," *American Economic Review*, 107(8), 2409–2454.

Liu, Z., P. Wang, and T. Zha (2013): "Land-Price Dynamics and Macroeconomic Fluctuations," *Econometrica*, 81(3), 1147–1184.

Lubik, T., and F. Schorfheide (2006): "A Bayesian Look at New Open Economy Macroeconomics," in *NBER Macroeconomics Annual 2005, Volume 20*, NBER Chapters, pp. 313–382. National Bureau of Economic Research, Inc.

Mian, A., K. Rao, and A. Sufi (2013): "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics*, 128(4), 1687–1726.

Mian, A., and A. Sufi (2011): "House Prices, Home Equity-Based Borrowing, and the US Household Leverage Crisis," *American Economic Review*, 101(5), 2132–2156.

——— (2014): "What Explains the 2007–2009 Drop in Employment?," *Econometrica*, 82, 2197–2223.

Miao, J., P. Wang, and T. Zha (2020): "Discount Shock, Price–Rent Dynamics, And The Business Cycle," *International Economic Review*, 61(3), 1229–1252.

Mishkin, F. S. (2011): "Over the Cliff: From the Subprime to the Global Financial Crisis," *Journal of Economic Perspectives*, 25(1), 49–70.

Moura, A. (2018): "Investment Shocks, Sticky Prices, and the Endogenous Relative Price of Investment," *Review of Economic Dynamics*, 27, 48–63.

——— (2021): "Trend Breaks and the Long-Run Implications of Investment-Specific Technological Progress," MPRA Paper 112350, University Library of Munich, Germany.

Pancrazi, R., and M. Pietrunti (2019): "Natural expectations and home equity extraction," *Journal of Housing Economics*, 46(C).

Ramey, V. A., and M. D. Shapiro (2001): "Displaced Capital: A Study of Aerospace Plant Closings," *Journal of Political Economy*, 109(5), 958–992.

Schorfheide, F. (2000): "Loss Function-Based Evaluation of DSGE Models," *Journal of Applied Econometrics*, 15(6), 645–670.

Sims, C. A. (2003): "Probability Models for Monetary Policy Decisions," Manuscript, Princeton University.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1(5), 1123–1175.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Appendix A. Data

This appendix describes the sources and the construction of the observables used in estimation.

Population. U.S. population (BEA, NIPA Table 2.1, line 40).

Nominal consumption. Personal consumption expenditures on nondurable goods and services (BEA, NIPA Table 1.1.5, lines 5 and 6).

Nominal investment. Sum of personal consumption expenditures on durable goods, investment in equipment, and investment in intellectual property products (BEA, NIPA Table 1.1.5, lines 4, 11, and 12).

Consumption price. Chain-linked price index for personal consumption expenditures on nondurable goods and services (BEA, NIPA Table 1.1.3 for the quantities and Table 1.1.4 for the prices).

Investment price. Chain-linked price index for personal consumption expenditures on durable goods, investment in equipment, and investment in intellectual property products (BEA, NIPA Table 1.1.3 for the quantities and Table 1.1.4 for the prices).

Consumption per capita. (Model variable: C_t.) Nominal consumption divided by the consumption price and by population.

Investment per capita. (Model variable: I_t.) Nominal investment divided by the investment price and by population.

Relative price of investment. (Model variable: $1/Q_t$.) Investment price divided by the consumption price.

Output per capita. (Model variable: Y_t.) Computed as $Y_t = C_t + I_t/Q_t$. Not used in estimation, but useful to compute sample averages for the calibration.

Hours worked per capita. (Model variable: N_t.) Hours worked for all employed persons in the nonfarm business sector (BLS, series HOANBS in FRED), divided by population.

Real estate price. (Model variable: $P_{H,t}$.) Residential RE price: S&P CoreLogic Case/Shiller price index for U.S. homes (available from Robert Shiller's webpage, `http://www.econ.yale.edu/~shiller/data/Fig3-1.xls`). Commercial RE price: commercial real estate price index published by the Federal Reserve Board (series BOGZ1FL075035503Q in FRED). Land price: Davis-Heathcote nominal series for residential land prices, based on the S&P CoreLogic Case/Shiller data (available online, `https://www.aei.org/historical-land-price-indicators/`). Converted to real terms using the consumption price.

Household debt per capita. (Model variable: $B_{B,t}$.) Loan liabilities on the balance sheet of households and nonprofit organizations (Federal Reserve Board, Financial Accounts of the U.S., Table L101, series FL154123005). Converted to real, per-capita terms using the consumption price and population.

Entrepreneur debt per capita. (Model variable: $B_{E,t}$.) Loan liabilities on the balance sheet of the non-financial business sector (Federal Reserve Board, Financial Accounts of the U.S., Table L102, series FL144123005). Converted to real, per-capita terms using the consumption price and population.

Online Appendix
Not for Publication

Online Appendix I. Equilibrium Conditions

This appendix reports the optimality conditions for the DSGE model. The equations below correspond to a more general setup in which the disutility of work takes the form $\theta_{N,t}N_t^{1+\kappa}/(1+\kappa)$ for both types of households, with $\kappa \geq 0$.

I.1. Patient household.

$$
\begin{aligned}
\Lambda_{S,t} &= A_t\left(\frac{1}{C_{S,t}-\gamma C_{S,t-1}} - E_t\frac{\beta_S\gamma A_{t+1}/A_t}{C_{S,t+1}-\gamma C_{S,t}}\right),\\
P_{H,t}\Lambda_{S,t} &= \frac{A_t\theta_{H,t}}{H_{S,t}} + \beta_S E_t P_{H,t+1}\Lambda_{S,t+1},\\
W_t\Lambda_{S,t} &= A_t\theta_{N,t}N_{S,t}^{\kappa},\\
\Lambda_{S,t} &= \beta_S R_t E_t\Lambda_{S,t+1}.
\end{aligned}
$$

I.2. Impatient household.

$$
\begin{aligned}
W_tN_{B,t} + \frac{B_{B,t}}{R_t} &= C_{B,t} + P_{H,t}(H_{B,t}-H_{B,t-1}) + B_{B,t-1}\\
B_{B,t} &= \rho_B B_{B,t-1} + (1-\rho_B)\psi_{B,t}E_tP_{H,t+1}H_{B,t}\\
\Lambda_{B,t} &= A_t\left(\frac{1}{C_{B,t}-\gamma C_{B,t-1}} - E_t\frac{\beta_B\gamma A_{t+1}/A_t}{C_{B,t+1}-\gamma C_{B,t}}\right),\\
P_{H,t}\Lambda_{B,t} &= \frac{A_t\theta_{H,t}}{H_{B,t}} + \beta_B E_t P_{H,t+1}\Lambda_{B,t+1} + (1-\rho_B)\psi_{B,t}\Lambda_{B,t}\Omega_{B,t}E_tP_{H,t+1},\\
W_t\Lambda_{B,t} &= A_t\theta_{N,t}N_{B,t}^{\kappa},\\
\Lambda_{B,t}\left(\frac{1}{R_t} - \Omega_{B,t}\right) &= \beta_B E_t\Lambda_{B,t+1}(1-\rho_B\Omega_{B,t+1}).
\end{aligned}
$$

I.3. Entrepreneur.

$$
\begin{aligned}
Y_t &= Z_t\left(H_{E,t-1}^{\phi}K_{t-1}^{1-\phi}\right)^{\alpha}N_t^{1-\alpha}\\
Y_t + \frac{B_{E,t}}{R_t} &= C_{E,t} + P_{H,t}(H_{E,t}-H_{E,t-1}) + B_{E,t-1} + \frac{I_t}{Q_t} + W_tN_t\\
K_t &= (1-\delta)K_{t-1} + \left[1 - \frac{\varphi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_I\right)^2\right]I_t\\
B_{E,t} &= \rho_B B_{E,t-1} + (1-\rho_B)\psi_{E,t}E_tP_{H,t+1}H_{E,t}\\
\Lambda_{E,t} &= \frac{1}{C_{E,t}-\gamma C_{E,t-1}} - E_t\frac{\beta_E\gamma}{C_{E,t+1}-\gamma C_{E,t}},
\end{aligned}
$$

$$
\begin{aligned}
P_{H,t}\Lambda_{E,t} &= \beta_E E_t \Lambda_{E,t+1}\left(P_{H,t+1} + \phi\alpha \frac{Y_{t+1}}{H_{E,t}}\right) + (1-\rho_B)\psi_{E,t}\Lambda_{E,t}\Omega_{E,t}E_t P_{H,t+1},\\
W_t N_t &= (1-\alpha)Y_t,\\
P_{K,t}\Lambda_{E,t} &= \beta_E E_t \Lambda_{E,t+1}\left[(1-\delta)P_{K,t+1} + (1-\phi)\alpha\frac{Y_{t+1}}{K_t}\right],\\
\frac{1}{Q_t} &= P_{K,t}\left[1 - \frac{\varphi}{2}\left(\frac{I_t}{I_{t-1}} - \mu_I\right)^2 - \varphi\left(\frac{I_t}{I_{t-1}} - \mu_I\right)\frac{I_t}{I_{t-1}}\right]\\
&\quad + \beta_E \varphi E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t}} P_{K,t+1}\left(\frac{I_{t+1}}{I_t} - \mu_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2,\\
\Lambda_{E,t}\left(\frac{1}{R_t} - \Omega_{E,t}\right) &= \beta_E E_t \Lambda_{E,t+1}(1 - \rho_B \Omega_{E,t+1}).
\end{aligned}
$$

Online Appendix II. Detrending and Stationary System

The economy inherits stochastic trends from shocks to preferences and technology. Defining $g_{A,t} \equiv A_t/A_{t-1}$, $g_{Z,t} \equiv Z_{P,t}/Z_{P,t-1}$, $g_{Q,t} \equiv Q_{P,t}/Q_{P,t-1}$, and $g_{\Gamma,t} \equiv \Gamma_t/\Gamma_{t-1}$, we obtain the following stationary representation of equilibrium dynamics.

II.1. Patient household.

$$\tilde{\Lambda}_{S,t} = \frac{1}{\tilde{C}_{S,t} - \gamma \tilde{C}_{S,t-1}/g_{\Gamma,t}} - E_t \frac{\beta_S \gamma g_{A,t+1}}{\tilde{C}_{S,t+1} g_{\Gamma,t+1} - \gamma \tilde{C}_{S,t}}, \tag{1}$$

$$\tilde{P}_{H,t}\tilde{\Lambda}_{S,t} = \frac{\theta_{H,t}}{H_{S,t}} + \beta_S E_t \tilde{P}_{H,t+1}\tilde{\Lambda}_{S,t+1} g_{A,t+1}, \tag{2}$$

$$\tilde{W}_t \tilde{\Lambda}_{S,t} = \theta_{N,t} N_{S,t}^{\kappa}, \tag{3}$$

$$\tilde{\Lambda}_{S,t} = \beta_S R_t E_t \tilde{\Lambda}_{S,t+1} \frac{g_{A,t+1}}{g_{\Gamma,t+1}}. \tag{4}$$

II.2. Impatient household.

$$\tilde{W}_t N_{B,t} + \frac{\tilde{B}_{B,t}}{R_t} = \tilde{C}_{B,t} + \tilde{P}_{H,t}(H_{B,t} - H_{B,t-1}) + \frac{\tilde{B}_{B,t-1}}{g_{\Gamma,t}}, \tag{5}$$

$$\tilde{B}_{B,t} = \rho_B \frac{\tilde{B}_{B,t-1}}{g_{\Gamma,t}} + (1-\rho_B)\psi_{B,t} E_t \tilde{P}_{H,t+1} g_{\Gamma,t+1} H_{B,t}, \tag{6}$$

$$\tilde{\Lambda}_{B,t} = \frac{1}{\tilde{C}_{B,t} - \gamma \tilde{C}_{B,t-1}/g_{\Gamma,t}} - E_t \frac{\beta_B \gamma g_{A,t+1}}{\tilde{C}_{B,t+1} g_{\Gamma,t+1} - \gamma \tilde{C}_{B,t}}, \tag{7}$$

$$
\begin{aligned}
\tilde{P}_{H,t}\tilde{\Lambda}_{B,t} &= \frac{\theta_{H,t}}{H_{B,t}} + \beta_B E_t \tilde{P}_{H,t+1}\tilde{\Lambda}_{B,t+1} g_{A,t+1}\\
&\quad + (1-\rho_B)\psi_{B,t}\tilde{\Lambda}_{B,t}\Omega_{B,t} E_t \tilde{P}_{H,t+1} g_{\Gamma,t+1},
\end{aligned} \tag{8}
$$

$$\tilde{W}_t \tilde{\Lambda}_{B,t} = \theta_{N,t} N_{B,t}^{\kappa}, \tag{9}$$

$$\tilde{\Lambda}_{B,t}\left(\frac{1}{R_t}-\Omega_{B,t}\right)=\beta_B E_t\frac{\tilde{\Lambda}_{B,t+1}}{g_{\Gamma,t+1}}g_{A,t+1}(1-\rho_B\Omega_{B,t+1}). \tag{10}$$

II.3. **Entrepreneur.**

$$\frac{\tilde{Y}_t}{\nu_{Z,t}}=(g_{Z,t}g_{Q,t})^{\frac{-(1-\phi)\alpha}{1-(1-\phi)\alpha}}\left(H_{E,t-1}^{\phi}\tilde{K}_{t-1}^{1-\phi}\right)^{\alpha}N_t^{1-\alpha}, \tag{11}$$

$$\tilde{Y}_t+\frac{\tilde{B}_{E,t}}{R_t}=\tilde{C}_{E,t}+\tilde{P}_{H,t}(H_{E,t}-H_{E,t-1})+\frac{\tilde{B}_{E,t-1}}{g_{\Gamma,t}}+\frac{\tilde{I}_t}{\nu_{Q,t}}+\tilde{W}_tN_t, \tag{12}$$

$$\tilde{K}_t=(1-\delta)\frac{\tilde{K}_{t-1}}{g_{\Gamma,t}g_{Q,t}}+\left[1-\frac{\varphi}{2}\left(\frac{\tilde{I}_tg_{\Gamma,t}g_{Q,t}}{\tilde{I}_{t-1}}-\mu_I\right)^2\right]\tilde{I}_t, \tag{13}$$

$$\tilde{B}_{E,t}=\rho_B\frac{\tilde{B}_{E,t-1}}{g_{\Gamma,t}}+(1-\rho_B)\psi_{E,t}E_t\tilde{P}_{H,t+1}g_{\Gamma,t+1}H_{E,t}, \tag{14}$$

$$\tilde{\Lambda}_{E,t}=\frac{1}{\tilde{C}_{E,t}-\gamma\tilde{C}_{E,t-1}/g_{\Gamma,t}}-E_t\frac{\beta_E\gamma}{\tilde{C}_{E,t+1}g_{\Gamma,t+1}-\gamma\tilde{C}_{E,t}}, \tag{15}$$

$$\begin{aligned}\tilde{P}_{H,t}\tilde{\Lambda}_{E,t}&=\beta_EE_t\tilde{\Lambda}_{E,t+1}\left(\tilde{P}_{H,t+1}+\phi\alpha\frac{\tilde{Y}_{t+1}}{H_{E,t}}\right)\\&\quad+(1-\rho_B)\psi_{E,t}\tilde{\Lambda}_{E,t}\Omega_{E,t}E_t\tilde{P}_{H,t+1}g_{\Gamma,t+1},\end{aligned} \tag{16}$$

$$\tilde{W}_tN_t=(1-\alpha)\tilde{Y}_t, \tag{17}$$

$$\tilde{P}_{K,t}\tilde{\Lambda}_{E,t}=\beta_EE_t\tilde{\Lambda}_{E,t+1}\left((1-\delta)\frac{\tilde{P}_{K,t+1}}{g_{\Gamma,t+1}g_{Q,t+1}}+(1-\phi)\alpha\frac{\tilde{Y}_{t+1}}{\tilde{K}_t}\right), \tag{18}$$

$$\begin{aligned}\frac{1}{\nu_{Q,t}}&=\tilde{P}_{K,t}\left[1-\frac{\varphi}{2}\left(\frac{\tilde{I}_tg_{\Gamma,t}g_{Q,t}}{\tilde{I}_{t-1}}-\mu_I\right)^2-\varphi\left(\frac{\tilde{I}_tg_{\Gamma,t}g_{Q,t}}{\tilde{I}_{t-1}}-\mu_I\right)\frac{\tilde{I}_tg_{\Gamma,t}g_{Q,t}}{\tilde{I}_{t-1}}\right]\\&\quad+\beta_E\varphi E_t\frac{\tilde{\Lambda}_{E,t+1}}{\tilde{\Lambda}_{E,t}}\frac{\tilde{P}_{K,t+1}}{g_{\Gamma,t+1}g_{Q,t+1}}\left(\frac{\tilde{I}_{t+1}g_{\Gamma,t+1}g_{Q,t+1}}{\tilde{I}_t}-\mu_I\right)\\&\quad\times\left(\frac{\tilde{I}_{t+1}g_{\Gamma,t+1}g_{Q,t+1}}{\tilde{I}_t}\right)^2,\end{aligned} \tag{19}$$

$$\tilde{\Lambda}_{E,t}\left(\frac{1}{R_t}-\Omega_{E,t}\right)=\beta_EE_t\frac{\tilde{\Lambda}_{E,t+1}}{g_{\Gamma,t+1}}(1-\rho_B\Omega_{E,t+1}). \tag{20}$$

II.4. **Market clearing.**

$$\tilde{Y}_t=\tilde{C}_t+\frac{\tilde{I}_t}{\nu_{Q,t}}, \tag{21}$$

$$\tilde{C}_t=\tilde{C}_{B,t}+\tilde{C}_{E,t}+\tilde{C}_{S,t}, \tag{22}$$

$$N_t = N_{B,t} + N_{S,t}, \tag{23}$$

$$\tilde{S}_t = \tilde{B}_{B,t} + \tilde{B}_{E,t}, \tag{24}$$

$$\overline{H} = H_{B,t} + H_{E,t} + H_{S,t}. \tag{25}$$

II.5. **Shocks.**

$$\ln(g_{A,t}) = \rho_A \ln(g_{A,t-1}) + u_{A,t}, \tag{26}$$

$$\ln(\theta_{H,t}) = (1-\rho_H)\ln(\overline{\theta}_H) + \rho_H \ln(\theta_{H,t-1}) + u_{H,t}, \tag{27}$$

$$\ln(\theta_{N,t}) = (1-\rho_N)\ln(\overline{\theta}_N) + \rho_N \ln(\theta_{N,t-1}) + u_{N,t}, \tag{28}$$

$$\ln(\psi_{B,t}) = (1-\rho_{\psi B})\ln(\overline{\psi}_B) + \rho_{\psi_B} \ln(\psi_{B,t-1}) + u_{\psi B,t}, \tag{29}$$

$$\ln(\psi_{E,t}) = (1-\rho_{\psi E})\ln(\overline{\psi}_E) + \rho_{\psi_E} \ln(\psi_{E,t-1}) + u_{\psi E,t}, \tag{30}$$

$$\ln(g_{Z,t}) = (1-\rho_{ZP})\ln(\overline{\mu}_Z) + \rho_{ZP} \ln(g_{Z,t-1}) + u_{ZP,t}, \tag{31}$$

$$\ln(\nu_{Z,t}) = \rho_{ZT} \ln(\nu_{Z,t-1}) + u_{ZT,t}, \tag{32}$$

$$\ln(g_{Q,t}) = (1-\rho_{QP})\ln(\overline{\mu}_Q) + \rho_{QP} \ln(g_{Q,t-1}) + u_{QP,t}, \tag{33}$$

$$\ln(\nu_{Q,t}) = \rho_{QT} \ln(\nu_{Q,t-1}) + u_{QT,t}, \tag{34}$$

$$\ln(g_{\Gamma,t}) = \frac{\ln(g_{Z,t}) + (1-\phi)\alpha \ln(g_{Q,t})}{1-(1-\phi)\alpha}. \tag{35}$$

Online Appendix III. Steady State

This section characterizes the steady state and shows that $\kappa = 0$ ensures it has a closed form expression.

We start by solving equations (31), (33), (35), (4), (10), (20), (1), (7), (15), (2), (8), (16), (13), (17), (19), (18), (21), (6), (5), (9), (14), (12), and (3). We obtain:

$$\begin{aligned}
g_Z &= \overline{\mu}_Z, \\
g_Q &= \overline{\mu}_Q, \\
g_\Gamma &= \left(\overline{\mu}_Z \overline{\mu}_Q^{(1-\phi)\alpha}\right)^{\frac{1}{1-(1-\phi)\alpha}}, \\
R &= g_\Gamma / \beta_S, \\
\Omega_B &= \frac{1/R - \beta_B/g_\Gamma}{1-\beta_B\rho_B/g_\Gamma}, \\
\Omega_E &= \frac{1/R - \beta_E/g_\Gamma}{1-\beta_E\rho_B/g_\Gamma}, \\
\tilde{\Lambda}_S \tilde{C}_S &= \frac{g_\Gamma - \beta_S\gamma}{g_\Gamma - \gamma}, \\
\tilde{\Lambda}_B \tilde{C}_B &= \frac{g_\Gamma - \beta_B\gamma}{g_\Gamma - \gamma},
\end{aligned}$$

$$\begin{aligned}
\tilde{\Lambda}_E \tilde{C}_E &= \frac{g_\Gamma - \beta_E \gamma}{g_\Gamma - \gamma}, \\
\tilde{P}_H H_S \tilde{\Lambda}_S &= \frac{\overline{\theta}_H}{1 - \beta_S}, \\
\tilde{P}_H H_B \tilde{\Lambda}_B &= \frac{\overline{\theta}_H}{1 - \beta_B - (1 - \rho_B)\overline{\psi}_B \Omega_B g_\Gamma}, \\
\frac{\tilde{P}_H H_E}{\tilde{Y}} &= \frac{\beta_E \phi \alpha}{1 - \beta_E - (1 - \rho_B)\overline{\psi}_E \Omega_E g_\Gamma}, \\
\frac{\tilde{I}}{\tilde{K}} &= 1 - \frac{1 - \delta}{g_\Gamma g_Q}, \\
\frac{\tilde{W} N}{\tilde{Y}} &= 1 - \alpha, \\
\tilde{P}_K &= 1, \\
\frac{\tilde{Y}}{\tilde{K}} &= \frac{1/\beta_E - (1 - \delta)/(g_\Gamma g_Q)}{\alpha(1 - \phi)}, \\
\frac{\tilde{C}}{\tilde{Y}} &= 1 - \frac{\tilde{I}/\tilde{K}}{\tilde{Y}/\tilde{K}}, \\
\tilde{\Lambda}_B \tilde{B}_B &= \frac{(1 - \rho_B)\overline{\psi}_B g_\Gamma}{1 - \rho_B / g_\Gamma} \tilde{P}_H H_B \tilde{\Lambda}_B, \\
\tilde{W} N_B \tilde{\Lambda}_B &= \tilde{\Lambda}_B \tilde{C}_B + (1/g_\Gamma - 1/R) \tilde{\Lambda}_B \tilde{B}_B, \\
\overline{\theta}_N N_B^{1+\kappa} &= \tilde{W} N_B \tilde{\Lambda}_B, \\
\frac{\tilde{B}_E}{\tilde{Y}} &= g_\Gamma \frac{(1 - \rho_B)\overline{\psi}_E}{1 - \rho_B / g_\Gamma} \frac{\tilde{P}_H H_E}{\tilde{Y}}, \\
\frac{\tilde{C}_E}{\tilde{Y}} &= 1 - \frac{\tilde{I}/\tilde{K}}{\tilde{Y}/\tilde{K}} - \frac{\tilde{W} N}{\tilde{Y}} - \frac{\tilde{B}_E}{\tilde{Y}} \left(\frac{1}{g_\Gamma} - \frac{1}{R} \right), \\
\frac{\tilde{\Lambda}_S}{\tilde{\Lambda}_B} &= \left(\frac{N_S}{N_B} \right)^\kappa.
\end{aligned}$$

The last equation implies

$$\frac{\tilde{C}_B}{\tilde{C}_S} = \frac{\tilde{C}_B}{\tilde{C}_S} \frac{\tilde{\Lambda}_B}{\tilde{\Lambda}_S} \left(\frac{N_S}{N_B} \right)^\kappa.$$

Then, equation (22) yields

$$\frac{\tilde{C}_B}{\tilde{Y}} = \frac{\tilde{C}}{\tilde{Y}} - \frac{\tilde{C}_E}{\tilde{Y}} - \frac{\tilde{C}_S}{\tilde{C}_B} \frac{\tilde{C}_B}{\tilde{Y}} = \frac{\frac{\tilde{C}}{\tilde{Y}} - \frac{\tilde{C}_E}{\tilde{Y}}}{1 + \frac{\tilde{C}_S}{\tilde{C}_B} \frac{\tilde{\Lambda}_S}{\tilde{\Lambda}_B} \left(\frac{N_B}{N_S} \right)^\kappa}.$$

Similarly, we have

$$\tilde{W} N_B \tilde{\Lambda}_B = \frac{\tilde{W} N}{\tilde{Y}} \, \frac{N_B}{N} \, \tilde{\lambda}_B \tilde{C}_B \, \frac{\tilde{Y}}{\tilde{C}_B},$$

and we can use equation (23) to get

$$\frac{N_B}{N} \left[1 + \underbrace{\frac{\tilde{C}_S}{\tilde{C}_B} \frac{\tilde{\Lambda}_S}{\tilde{\Lambda}_B}}_{\equiv \gamma_1} \left(\frac{N_B}{N - N_B} \right)^{\kappa} \right] = \underbrace{\frac{\tilde{W} N_B \tilde{\Lambda}_B \left(\frac{\tilde{C}}{\tilde{Y}} - \frac{\tilde{C}_E}{\tilde{Y}} \right)}{\frac{\tilde{W} N}{\tilde{Y}} \, \tilde{\Lambda}_B \tilde{C}_B}}_{\equiv \gamma_2}.$$

Defining $x \equiv N_B / N$, the above equation becomes

$$x \left[1 + \gamma_1 \left(\frac{1}{x} - 1 \right)^{-\kappa} \right] = \gamma_2,$$

with γ_1 and γ_2 known. A closed-form solution for x exists only for $\kappa = 0$ or $\kappa = 1$. As in Liu, Wang, and Zha (2013), we assume that $\kappa = 0$, so that households experience a linear disutility of work. This allows us to solve for x:

$$x = \frac{N_B}{N} = \frac{\tilde{W} N_B \tilde{\Lambda}_B \, \left(\frac{\tilde{C}}{\tilde{Y}} - \frac{\tilde{C}_E}{\tilde{Y}} \right)}{\frac{\tilde{W} N}{\tilde{Y}} \, \left(\tilde{\Lambda}_B \tilde{C}_B + \tilde{\Lambda}_S \tilde{C}_S \right)}.$$

Then, we immediately obtain

$$\begin{aligned}
N_S &= N - N_B, \\
\frac{\tilde{\Lambda}_S}{\tilde{\Lambda}_B} &= 1, \\
\frac{H_B}{H_S} &= \frac{\tilde{P}_H \tilde{\Lambda}_B H_B}{\tilde{P}_H \tilde{\Lambda}_S H_S}, \\
\frac{\tilde{C}_B}{\tilde{C}_S} &= \frac{\tilde{\Lambda}_B \tilde{C}_B}{\tilde{\Lambda}_S \tilde{C}_S}, \\
\frac{\tilde{C}_B}{\tilde{Y}} &= \frac{\tilde{C}/\tilde{Y} - \tilde{C}_E/\tilde{Y}}{1 + \tilde{C}_S/\tilde{C}_B}, \\
\frac{\tilde{C}_S}{\tilde{Y}} &= \frac{\tilde{C}}{\tilde{Y}} - \frac{\tilde{C}_E}{\tilde{Y}} - \frac{\tilde{C}_B}{\tilde{Y}}, \\
\frac{\tilde{P}_H H_S}{\tilde{Y}} &= \frac{\tilde{P}_H \tilde{\Lambda}_S H_S}{\tilde{\Lambda}_S \tilde{C}_S} \, \frac{\tilde{C}_S}{\tilde{Y}}, \\
\frac{\tilde{P}_H H_B}{\tilde{Y}} &= \frac{\tilde{P}_H \tilde{\Lambda}_B H_B}{\tilde{\Lambda}_B \tilde{C}_B} \, \frac{\tilde{C}_B}{\tilde{Y}}.
\end{aligned}$$

Equation (25) then gives

$$\frac{\tilde{P}_H}{\tilde{Y}} = \left(\frac{\tilde{P}_H H_B}{\tilde{Y}} + \frac{\tilde{P}_H H_S}{\tilde{Y}} + \frac{\tilde{P}_H H_E}{\tilde{Y}} \right) / \overline{H},$$

from which we get

$$H_E = \left(\frac{\tilde{P}_H H_E}{\tilde{Y}} \right) / \left(\frac{\tilde{P}_H}{\tilde{Y}} \right),$$

$$H_S = \left(\frac{\tilde{P}_H H_S}{\tilde{Y}} \right) / \left(\frac{\tilde{P}_H}{\tilde{Y}} \right),$$

$$H_B = \left(\frac{\tilde{P}_H H_B}{\tilde{Y}} \right) / \left(\frac{\tilde{P}_H}{\tilde{Y}} \right).$$

Finally, equation (11) gives

$$\tilde{Y} = \left[(g_Z g_Q)^{\frac{-(1-\phi)\alpha}{1-(1-\phi)\alpha}} H_E^{\phi\alpha} \left(\frac{\tilde{K}}{\tilde{Y}} \right)^{(1-\phi)\alpha} N^{1-\alpha} \right]^{\frac{1}{1-(1-\phi)\alpha}},$$

from which we deduce all variables.

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