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COMMUTERS, RESIDENTS AND JOB COMPETITION IN LUXEMBOURG

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Commuters, Residents and Job Competition in Luxembourg*

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April 20, 2007

Abstract

Despite strong employment growth, unemployment is rising in Luxembourg. Stronger job competition from non-residents could explain this 'unemployment paradox'. We construct a small theoretical search unemployment model of the Luxembourg labor market, with job competition between residents and cross-border commuters. We show both analytically and numerically that job competition alone cannot explain the unemployment paradox and we stress the role of the labor market institutions.

Keywords: Job competition, Commuters, Unemployment

JEL classification: J61, J64, R23

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Résumé non-technique

Ces dernières années, malgré une croissance continue de l'emploi, le taux de chômage luxembourgeois a augmenté de façon tendancielle, passant de 2% en 2000 à presque 5% actuellement. Ce phénomène *a priori* contradictoire a pour toile de fond le fait que sur les quelque 300 000 emplois existants au Luxembourg, environ 40% sont occupés par des frontaliers. Ce pourcentage est de surcroît encore en augmentation, puisque les entrées de travailleurs frontaliers représentent actuellement près de 70% des nouvelles créations nettes d'emplois. Ces observations soulèvent au moins deux questions. L'arrivée massive de travailleurs frontaliers peut-elle expliquer à elle seule la détérioration du chômage des résidents? Et quelles politiques économiques faudrait-il mettre en oeuvre pour réduire le chômage tout en préservant le fort taux de création d'emplois?

Pour tenter de répondre à ces questions, nous proposons dans ce papier un modèle théorique du marché du travail basé sur le modèle d'appariement de Pissarides. L'utilisation d'une fonction d'appariement ("*matching function*") entre demandeurs et offreurs d'emplois est une façon simple de prendre en compte l'existence de frictions, les difficultés d'appariement et les problèmes de coordinations liés au déficit d'information. Cette représentation du marché du travail est courante dans la littérature économique et particulièrement adaptée à la simulation de diverses politiques économiques. Une distinction entre résidents et frontaliers, et entre emplois privés et emplois publics est de plus introduite pour mieux faire coller le modèle à la réalité du marché du travail luxembourgeois. En outre, le modèle est calibré sur données luxembourgeoises.

Nos principaux résultats sont les suivants. Premièrement, une augmentation du nombre de frontaliers ne peut à elle seule expliquer la hausse du chômage au Luxembourg. Deuxièmement, une augmentation du nombre de frontaliers combinée à une augmentation de la population active peut expliquer la hausse conjointe de l'emploi et du chômage, ainsi que l'augmentation de la part des frontaliers dans l'emploi total. Troisièmement, une augmentation du volume de l'emploi public ou l'introduction de politiques protectionnistes n'est pas efficace pour réduire le chômage. Quatrièmement, une diminution du montant des allocations de chômage réduit le chômage mais a des effets ambigus sur le bien-être de la population domestique. Enfin, le meilleur moyen de stimuler l'emploi et les revenus tout en diminuant le chômage est une augmentation de la productivité globale de l'économie.

Ce papier peut se concevoir comme une première tentative de modélisation et de compréhension du marché du travail luxembourgeois, et est donc incomplet sous bien des aspects. Plus précisément, au moins deux extensions devraient être considérées à l'occasion de travaux futurs. Premièrement, nous abordons dans notre modèle la problématique de l'immigration de manière totalement simplifiée, *via* un choc exogène sur la population active. Or cette problématique est importante et loin d'être neutre pour le marché du travail, comme l'illustrent nos simulations. Rendre endogène la décision d'immigration, tout en gardant également endogène le nombre de travailleurs transfrontaliers, rendrait certainement le modèle plus "élégant" et permettrait probablement d'avoir une compréhension plus fine des flux de main-d'oeuvre sur le marché du travail. Deuxièmement, une autre explication potentielle à la hausse du chômage est l'inadéquation en-

tre qualifications demandées et qualifications offertes. Etudier cela nécessiterait dès lors l'introduction d'au minimum deux types d'emplois (qualifiés et peu qualifiés) et deux types de travailleurs (qualifiés et peu qualifiés).

1 Introduction

Unemployment is currently rising in Luxembourg despite strong employment growth. One possible explanation for this 'unemployment paradox' is that employers in Luxembourg prefer to recruit cross-border residents. The share of cross-border commuters in domestic employment is rising and about 66% of new jobs are currently filled by non-residents (see BCL (2006a,b) for an in depth exposition). In other words, the employment dynamics in Luxembourg seems mainly to benefit non-residents.

Why do Luxembourg residents encounter difficulties to enter the employment? In a recent report, the OECD (2006) mainly stresses three potential reasons[1]. Firstly, the relative generosity of replacement incomes (high unemployment benefits and social assistance) raises the reservation wage of residents, putting them at their disadvantage (with respect to cross-border residents) in the labor market. Secondly, the public sector offers attractive working conditions (protected employment, less competition from non-residents, high wages,...) and is practically limited to residents, giving the unemployed in Luxembourg weaker incentives to search for a private sector job. Finally, the OECD argues that the public employment service agency[2] is not efficient, which results in a matching problem between employment supply and demand.

In this paper, we construct a small theoretical model of the Luxembourg labor market and we use it to understand the competition between residents and cross-border commuters and the role of labor market institutions to explain the rise in Luxembourg unemployment[3]. Our model relates to the search-matching literature (see Mortensen and Pissarides (1999) and Pissarides (2000) for an overview) which explains unemployment by frictions in the labor market. It is worth noting that demand plays no role in the standard search-matching model: labor supply creates its own demand and employment level is therefore "supply-side" determined. This is a plausible assumption since our search-matching model is a long-run model (long-run output is generally interpreted as a supply-side process)[4]. Dolado, Felgueroso, and Jimeno (2000), Albrecht and Vroman (2002) and Gautier (2002), extend the matching model by introducing competition for low-skilled jobs between low- and high-skilled workers. Here we follow the same idea but introduce competition between residents and cross-border commuters for jobs in the private sector. More precisely, we have two types of workers (residents and cross-border commuters) and two types of jobs (private and public). Commuters may compete with residents for private jobs but not for public jobs[5]. The number of (potential) cross-border commuters is

[1] Other reasons are also mentioned as for instance human capital development. However, these are beyond the scope of this paper.

[2] ADEM: Administration de l'emploi.

[3] In our model, we do not distinguish between national and non-national residents and we assume they all are national. This is admittedly a very rough simplification since in 2001, 37% of residents were non-nationals (source: STATEC). Adding a third type of workers would not change our main results (see the discussion in section 6).

[4] In the short run, demand may however be important to explain employment fluctuations. See for instance Carlsson, Eriksson, and Gottfries (2004) for evidence based on Swedish data.

[5] See section 2 for evidence.

endogenous and depends on the relative unemployment and working conditions.

We first derive some analytical properties of the model. It is then calibrated on Luxembourg data and simulated. Our main findings are: (i) under certain conditions, increasing competition from cross-border commuters generates positive externalities on vacancy openings and decreases Luxembourg unemployment; job competition (between residents and non-residents) alone is therefore unable to explain the unemployment paradox; (ii) higher non-resident labor supply (more cross-border commuters) *combined with* a domestic labor supply shock (for instance an increase in active population) can explain both the unemployment paradox and the higher share of cross-border commuters, (iii) reducing unemployment benefits lowers unemployment but the welfare effect is ambiguous (iv) more public jobs do not solve the unemployment problem; and (v) the best way to boost the economy is to increase productivity.

In the next section, we present some facts about the Luxembourg labor market. These stylized facts will be used to justify and calibrate our model. Section 3 presents the model and some analytical results. Sections 4 and 5 respectively give the calibration and the results of some numeric simulations. Section 6 sums up and discusses the results and section 7 concludes.

2 Some stylized facts

In this section, we illustrate and comment some of the recent developments on the Luxembourg labor market.

Rise in employment and unemployment

Figure 1 illustrates some recent labor market developments. In the second half of the 90's, strong employment growth sharply reduced the unemployment level to 2.5% - 3.5%[6]. From 2001 until 2003, lower economic growth associated with stock market declines (bursting stock market bubble) reduced employment growth and triggered a rise in unemployment. Since 2004, the economy - and the stock market - have recovered but this inverse relationship between employment growth and unemployment seems broken. Despite strong employment growth, unemployment is still rising in Luxembourg[7].

Cross-border commuters and residents

One possible explanation is that employers mainly recruit non-residents. Situated in the middle of the *Grande Région*[8], Luxembourg has a labor catchment area of about 200 000 unemployed job seekers. The increased freedom of labor

[6]Depending on the definition we use: the narrow definition of unemployment excludes people participating in labor market activation measures organized by the public employment service.

[7]The recent increase in unemployment partly results from an administrative reform (disabled job-seekers registered as unemployed). However, regular unemployment also increase. Moreover, the rise in active population (+10 000 since 2004) could also explain the deterioration in unemployment. But this cannot be the sole explanation since employment increased by 30 000 during the same period.

[8]The *Grande Région* comprises Luxembourg, Lorraine, Rheinland-Pfalz, Saarland and Wallonie.

Sources: IGSS (Inspection Générale de la Sécurite Sociale) and STATEC.

Figure 1: Employment and unemployment dynamics

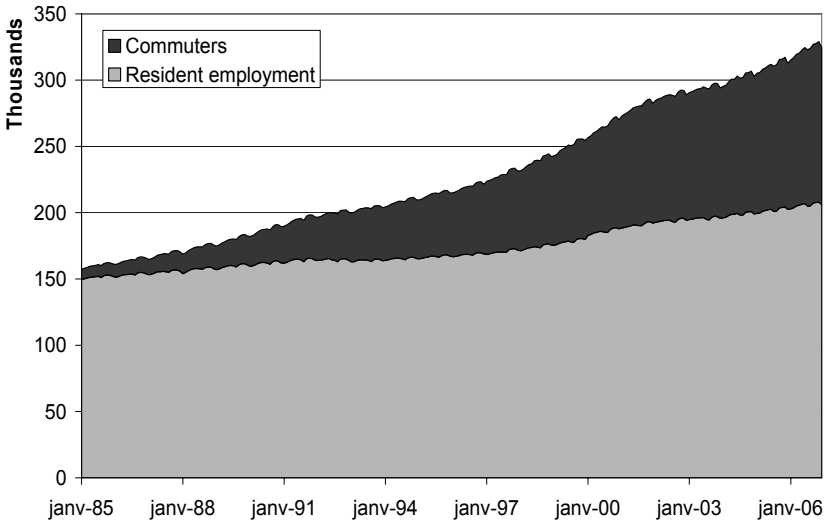
circulation also means higher competition for jobs between national and cross-border residents. Figure 2 shows that most employment growth reflects the employment of cross-border commuters: the share of cross-border commuters in domestic employment rose from 5% in 1985 to 38% in 2006. This share is still increasing since 66% of job openings are currently filled by cross-border workers - see BCL (2006a,b). Why do employers prefer to recruit cross-border commuters rather than residents? There are two main answers: firms *do not find* resident workers and/or firms *voluntarily prefer* recruiting cross-border workers. A skill mismatch problem (required qualifications are not available among national workers)[9] or a lack of interest to work could explain the first case. A higher reservation wage for residents than for cross-border commuters could explain the second case. These two possible answers are further explained in the next two paragraphs.

Public vs. private sector

Over the last decade, public employment has increased from 47 000 to 72 000, although its share in total domestic employment has remained stable around 24% (see figure 3)[10]. The share of public sector employment is in line with other major industrial countries. However, the public sector appears especially attractive for national residents: employment is protected, wages are high, and the *Letzebuergesch* language requirement severely hampers competition from non-residents. Figure 4 shows that in 2005, the average yearly gross wage in the public sector was € 51 500, while it was € 47 270 in the private sector, *i.e.* almost

[9] A skill mismatch problem is certainly not the whole story, since most of the low-skilled jobs are also filled by cross-border workers, and we assume in the paper that all workers have the same skill/productivity. However, future researches along the skill dimension would be definitively be interesting (see the Conclusion section).

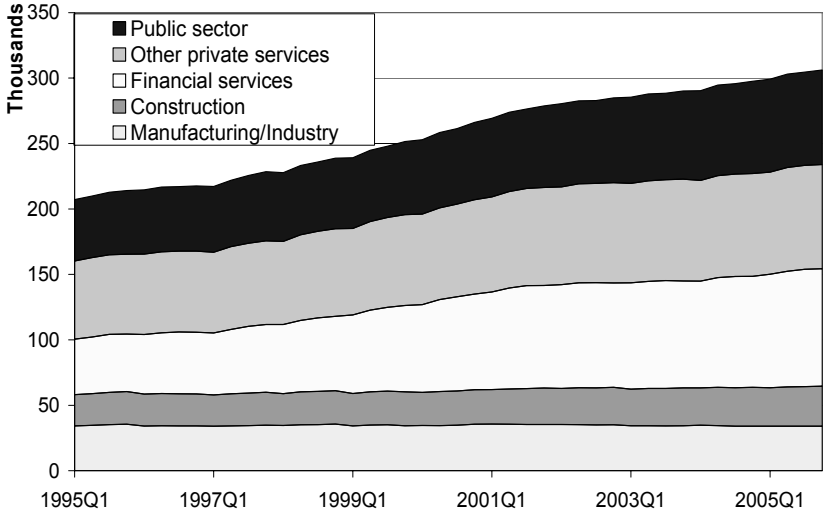
[10] A large definition of public employment is used (overall general government employment), which includes civil servants but also workers from other parts of the public sector (health care, education,...).



Sources: IGSS (Inspection Générale de la Sécurite Sociale) and STATEC.

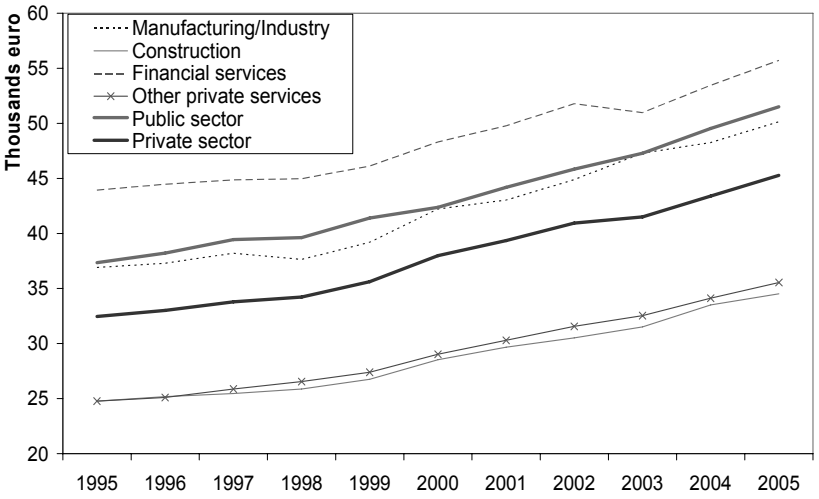
Figure 2: Cross-border commuters and resident employees

a 10% gap. According to OECD (2006), the difference is particularly important at the entry level and for low- and intermediate-skilled workers. The general attractiveness of the public sector explains why nationals have less interest in seeking private sector jobs and prefer public sector jobs. This may pose a problem since most of new vacancies are supplied by the private sector: over the last 10 years, 75 000 new jobs were created in the private sector while only 25 000 were created in the public sector (see figure 3).



Sources: IGSS (Inspection Générale de la Sécurite Sociale) and STATEC.

Figure 3: Public vs. private sector employment



Sources: IGSS (Inspection Générale de la Sécurite Sociale) and STATEC.

Figure 4: Public vs. private wages (yearly gross wages)

Unemployment benefits

Unemployment benefits in Luxembourg are available for up to one year after job loss and the gross replacement rate is 80% of the last salary. It is worth noting that benefits cease if the unemployed refuses a suitable job offer without any justification. Once this period has elapsed, unemployed persons have access to the RMG (Revenu Minimum Garanti), which is kept in line with the Social Minimum Wage. The RMG is open-ended, provided the unemployed is willing to consider all possibilities to improve his situation. Comparing with neighboring countries, unemployment benefits and social assistance are generous in Luxembourg. Table 1 displays the net replacement rates (during the initial phase of unemployment) in Luxembourg, Belgium, France and Germany, for different family types. On average, the replacement rate is 15 percentage points higher in Luxembourg. The OECD (2006) points out that the exclusion rule (benefits cease once the unemployed refuses a job without justification) may seem harsh by international standards. However, the OECD argues that such stringent conditions make them exceptional and render the system *de facto* generous[11]. Globally, high social security provisions (unemployment benefits and social assistance) increase the reservation wage of residents and put them at a disadvantage with respect to cross-border commuters (who tend to be less expensive for firms)[12].

[11]It is however worth noting that this OECD observation is stated without any objective justifications.

[12]The government proposed very recently to decrease the generosity of the system, especially for young unemployed people (draft law 5611).

	2/3 of APW		APW		3/2 of APW	
	no children	2 children	no children	2 children	no children	2 children
Belgium	79	80	66	66	49	53
France	82	89	75	79	71	71
Germany	72	86	69	81	69	76
Luxembourg	**85**	**91**	**86**	**90**	**86**	**91**

Source: OECD (2006), table 3.1, page 74. APW: average production worker wage. For each family type, the replacement ratio is a simple average of three sub-categories: single person, one-earner married couple and two-earner married couple.

Table 1: Net replacement ratio for different family types, initial phase of unemployment

3 Model

We assume a home (Luxembourg) region h concentrated at a single point and a foreign region (neighboring countries) f spread around the home region. Production is located in the home region and labor is the only input in production. There are two productive sectors. The public sector c only uses home labor [13] while the private sector o uses both home and foreign (cross-border commuters) labor [14]. Heterogeneity in foreign labor is reflected by differences in location, *i.e.* by differences in the distance d from the home region[15]. Foreign labor is mobile but the cost of commuting (transport cost) is proportional to the distance d. Figure 5 illustrates this spatial representation[16].

3.1 Labor market flows

Figure 6 shows the different labor market flows. Let V^o be the stock of private sector vacancies. Both home unemployed U^h and foreign unemployed U^f may apply for private sector vacancies[17]. Home unemployed search intensity s is endogenous, while the foreign unemployed search intensity is normalized to 1. Vacancies and workers meet according to a constant returns to scale matching function:

$$M^o = \mathcal{M}^o\left(V^o, sU^h + U^f\right). \tag{1}$$

Total private employment is composed of foreign workers N^{of} and home workers N^{oh}. It is worth noting that we allow for on-the-job search: home workers

[13] This home labor constraint can be directly enforced by law, or indirectly through the requirement of specific knowledge (as the knowledge of the home language(s)). See section 2 for evidence in Luxembourg.

[14] We use the superscript o (open) for private and the superscript c (closed) for public.

[15] Generalizing the model to also consider transport costs for home labor would not change anything, as long as we assume that all home unemployed search for a job.

[16] If the commuting cost is too high, we do not allow instead for migration. This could however be an interesting extension (see the Conclusion section).

[17] We assume that employed in the foreign region do not search for a job in the home region. See section 6 for a discussion.

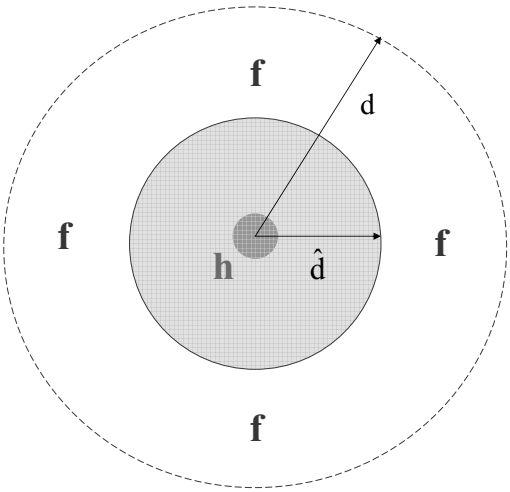


Figure 5: Spatial representation of the economy

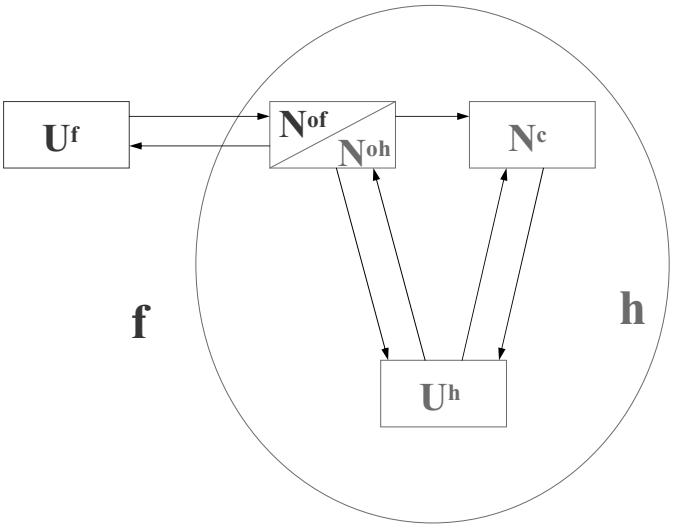


Figure 6: Labor market flows

continue searching for a more rewarding public sector job[18].

Let V_t^c be the stock of public sector vacancies. Only home labor may apply for public sector vacancies and all workers have the same exogenous search intensity[19]. Vacancies and workers meet according to a constant returns to scale matching function:

$$M^c = \mathcal{M}^c\left(V^c, U^h + N^{oh}\right). \tag{2}$$

Total public sector employment $\bar{N}^c$ is assumed exogenous.

The probabilities at which private and public sector vacancies are filled are:

$$q^o = \frac{M^o}{V^o} \qquad \text{and} \qquad q^c = \frac{M^c}{V^c}. \tag{3}$$

The probabilities at which workers find private and public sector jobs are:

$$p^o = \frac{M^o}{sU^h + U^f} \qquad \text{and} \qquad p^c = \frac{M^c}{U^h + N^{oh}}. \tag{4}$$

Home active population *pop* is exogenous:

$$pop = U^h + N^{oh} + \bar{N}^c. \tag{5}$$

The private sector job separation rate ψ^o and the public sector job separation rate ψ^c are exogenous and follow a Poisson process, and the steady state conditions for each employment state (inflow must be equal to outflow) are:

$$\psi^o N^{of} = p^o U^f, \tag{6}$$

$$\left(\psi^o + p^c\right) N^{oh} = p^o s U^h, \tag{7}$$

$$\psi^c \bar{N}^c = p^c\left(U^h + N^{oh}\right). \tag{8}$$

3.2 Foreign matches

We attach a discounted expected return W^x to every worker in state x and J^x to every firm in state x. r is the discount rate and is common to all agents. We introduce the following notations: the discounted expected return of a non-resident cross-border unemployed located at a distance d from the home region and of an employed cross-border commuter living at a distance d from the home region are respectively given by $rW^{uf}(d)$ and $rW^{of}(d)$. The discounted expected return of opening a vacancy for a private firm and the discounted expected return of employing a cross-border commuter living at a distance d for a private firm are respectively given by rJ^{vo} and $rJ^{of}(d)$.

The non-resident unemployed located at a distance d from the home region receives an unemployment benefit $\bar{w}^{uf}$ and chooses to search for a job in the home region (incurring a cost d) or not (no cost but no chance to get a job):

$$rW^{uf}(d) = \max\left[\bar{w}^{uf} - d + p^o\left(W^{of}(d) - W^{uf}(d)\right), \bar{w}^{uf}\right]. \tag{9}$$

[18] We implicitly assume that public sector jobs offer a higher return to workers than private sector jobs. See section 2 for evidence in Luxembourg.

[19] Public sector jobs are well advertised and all recruitments are only organized through official entry exams. A discrete representation of search intensity seems therefore realistic: individual searches (intensity 1) or not (intensity 0). Since public sector jobs always offer a higher return than unemployment or private sector jobs, search intensity is exogenous and equal to 1 for all home unemployed/workers.

The cross-border employed (commuter) living at a distance d from the home region receives a wage $w^{of}(d)$ and incurs a commuting cost d:

$$rW^{of}(d) = w^{of}(d) - d + \psi^o \left(W^{uf}(d) - W^{of}(d)\right). \tag{10}$$

Subtracting (9) from (10), we obtain:

$$\begin{aligned}(r+\psi^o)\left(W^{of}(d) - W^{uf}(d)\right) =& w^{of}(d) - \bar{w}^{uf} \\ &- \min\left[p^o\left(W^{of}(d) - W^{uf}(d)\right), d\right].\end{aligned} \tag{11}$$

By denoting labor productivity $\bar{y}^o$, the discounted expected return of a private sector job with a cross-border employee located (living) at a distance d is:

$$rJ^{of}(d) = \bar{y}^o - w^{of}(d) + \psi^o\left(J^{vo} - J^{of}(d)\right). \tag{12}$$

With the usual free entry condition, vacancies are open until the discounted expected return is equal to 0:

$$rJ^{vo} = 0, \tag{13}$$

which gives:

$$(r+\psi^o)J^{of}(d) = \bar{y}^o - w^{of}(d). \tag{14}$$

The bargained wage is determined by the standard Nash product problem:

$$\max_{w^{of}(d)} \left(W^{of}(d) - W^{uf}(d)\right)^{\eta}\left(J^{of}(d)\right)^{1-\eta}, \tag{15}$$

where η is the worker bargaining power. It can be rewritten as:

$$(1-\eta)\left(W^{of}(d) - W^{uf}(d)\right) = \eta J^{of}(d). \tag{16}$$

We define $\hat{d}$ as the location where the non-resident unemployed is indifferent between searching for a job in the home region or not. The labor catchment area of the home region is therefore delimited by the distance $\hat{d}$. From equation (9), we immediately get:

$$\hat{d} = p^o\left(W^{of}(\hat{d}) - W^{uf}(\hat{d})\right), \tag{17}$$

where, from equation (11):

$$W^{of}(\hat{d}) - W^{uf}(\hat{d}) = \frac{w^{of}(\hat{d}) - \bar{w}^{uf}}{r+\psi^o+p^o}. \tag{18}$$

The next two propositions give properties of the distance $\hat{d}$ and the wage $W^{of}(d)$:

Proposition 1 *(existence and uniqueness of the catchment area)*
$\exists!\quad \hat{d} = \frac{\eta p^o\left(\bar{y}^o - \bar{w}^{uf}\right)}{r+\psi^o+\eta p^o} > 0 \iff \bar{y}^o > \bar{w}^{uf}$

Proof. The proof is given in appendix A. ■

Proposition 2 *(homogeneity of the foreign worker wage)*
$d < \hat{d} \implies w^{of}(d) = w^{of} = \frac{(1-\eta)(r+\psi^o)\bar{w}^{uf} + \eta(r+\psi^o+p^o)\bar{y}^o}{r+\psi^o+\eta p^o}$

Proof. The proof is given in appendix A. ■

Proposition 1 states that the distance that determines the employment pool (labor catchment area) for the home region is uniquely defined. The distance is positively related to the difference $\bar{y}^o - \bar{w}^{uf}$ and to the probability p^o of finding a job[20]. Let $F(d)$ denote the the total mass of non-residents located within a distance d from the home region. Then the amount of non-residents working or searching for a job in the home region is uniquely determined by:

$$N^{of} + U^f = \int_0^{\hat{d}} dF(d). \tag{19}$$

Proposition 2 says that the wage of a foreign worker does not depend on his specific location and is a weighted average of $\bar{w}^{uf}$ and $\bar{y}^o$. Therefore, the expected discounted return of a private firm with a foreign employee does not depend on the location of the foreign employee: $J^{of}(d) = J^{of}$. In section 6, we discuss an alternative modelization which results in wages increasing with the commuting distance.

3.3 Home matches

We introduce the following notation: the expected discounted return of a home unemployed is given by rW^{uh}. The expected discounted return of a home employed working on a private sector job and a home employed working on a public sector job are respectively given by rW^{oh} and rW^c. The expected discounted return of a private sector firm with a vacancy and a private sector firm with a home employee are respectively given by rJ^{vo} and rJ^{oh}.

The home unemployed receives an unemployment benefit $\bar{w}^{uh}$ and searches both for a private sector job and a public sector job. There is a disutility $\mathcal{D}(s)$ associated with the job search intensity s:

$$rW^{uh} = \bar{w}^{uh} + s\,p^o\left(W^{oh} - W^{uh}\right) + p^c\left(W^c - W^{uh}\right) - \mathcal{D}(s). \tag{20}$$

The home employed on a private sector job receives a wage w^{oh}:

$$rW^{oh} = w^{oh} + p^o\left(W^c - W^{oh}\right) + \psi^o\left(W^{uh} - W^{oh}\right). \tag{21}$$

Search intensity s is determined by the first order condition $\partial rW^{uh}/\partial s = 0$, which gives:

$$\mathcal{D}'(s) = p^o\left(W^{oh} - W^{uh}\right), \tag{22}$$

with:

$$(r + \psi^o + p^c + s\,p^o)\left(W^{oh} - W^{uh}\right) = w^{oh} - \bar{w}^{uh} + \mathcal{D}(s). \tag{23}$$

$\bar{y}^o$ is labor productivity and the expected discounted return of a private sector job with home employee is:

$$rJ^{oh} = \bar{y}^o - w^{oh} + (p^c + \psi^o)\left(J^{vo} - J^{oh}\right). \tag{24}$$

[20]These results are intuitive. See for instance Mathä and Wintr (2007) for an empirical illustration.

We define a as the vacancy opening cost and the expected discounted return of a private sector vacancy is:

$$rJ^{vo} = -a + q^o \frac{U^f}{U^f + s\,U^h}\left(J^{of} - J^{vo}\right) + q^o \frac{s\,U^h}{U^f + s\,U^h}\left(J^{oh} - J^{vo}\right). \quad (25)$$

With the usual free entry condition (13) and using definition (14), equations (24) and (25) become:

$$J^{oh} = \frac{\bar{y}^o - w^{oh}}{r + \psi^o + p^c}, \quad (26)$$

$$\frac{a}{q^o} = \frac{U^f}{U^f + s\,U^h}\,\frac{\bar{y}^o - w^{of}}{r + \psi^o} + \frac{s\,U^h}{U^f + s\,U^h}\,\frac{\bar{y}^o - w^{oh}}{r + p^c + \psi^o}. \quad (27)$$

The bargained wage is determined by the standard Nash product problem:

$$\max_{w^{oh}} \left(W^{oh} - W^{uh}\right)^{\eta} \left(J^{oh}\right)^{1-\eta}, \quad (28)$$

where η is worker bargaining power. This can be rewritten as:

$$(1-\eta)\left(W^{oh} - W^{uh}\right) = \eta J^{oh}. \quad (29)$$

Using equations (23) and (26):

$$w^{oh} = \frac{(1-\eta)(r + \psi^o + p^c)\left(\bar{w}^{uh} - \mathcal{D}(s)\right) + \eta(r + \psi^o + p^c + s\,p^o)\bar{y}^o}{r + \psi^o + p^c + \eta s\,p^o}. \quad (30)$$

The wage of a home worker is a weighted average of $\bar{w}^{uh} - \mathcal{D}(s)$ (unemployment benefits net of search costs) and $\bar{y}^o$ (labor productivity).

3.4 Some analytic results

We first need to check that the agents (workers and/or firms) have no incentive to turn down a match, *i.e.* we need to check that $W^{of} - W^{uf} > 0$, $J^{of} > 0$, $W^{oh} - W^{uh} > 0$ and $J^{oh} > 0$. Proposition 3 states that if labor productivity is higher than net unemployment income, then the match surplus is positive and no agent wants to turn down a match:

Proposition 3 *(equilibrium conditions)*
$\bar{y}^o > \bar{w}^{uf} \implies W^{of} - W^{uf} > 0$ *and* $J^{of} > 0$
$\bar{y}^o > (\bar{w}^{uh} - \mathcal{D}(s)) \implies W^{oh} - W^{uh} > 0$ *and* $J^{oh} > 0$

Proof. Using the wage definitions (proposition 2 and equation (30)), we immediately get:

$$W^{of} - W^{uf} = \eta \frac{\bar{y}^o - \bar{w}^{uf}}{r + \psi^o + \eta p^o},$$

$$J^{of} = (1-\eta)\frac{\bar{y}^o - \bar{w}^{uf}}{r + \psi^o + \eta p^o},$$

$$W^{oh} - W^{uh} = \eta \frac{\bar{y}^o - (\bar{w}^{uh} - \mathcal{D}(s))}{r + \psi^o + p^c + \eta s\,p^o},$$

$$J^{oh} \quad = \quad (1-\eta)\frac{\bar{y}^o - (\bar{w}^{uh} - \mathcal{D}(s))}{r + \psi^o + p^c + \eta s\, p^o}.$$

The proof is straightforward. ■

Equation (27) governs the opening of vacancies. It states that the average hiring cost must be equal to expected income. An increase in expected return (right hand side) decreases q^o (increasing the opening of vacancies). An increase in U^f has two effects: (i) a congestion effect and (ii) a vacancy effect. The congestion effect means that more foreign unemployed generate more competition on the labor market and therefore raise home unemployment U^h. The vacancy effect means that more foreign unemployed modify the firm's expected return and therefore the rate at which it opens vacancies. The sign of the vacancy effect dq^o/dU^f is ambiguous *a priori*[21]:

Proposition 4 *(vacancy effect)*
If $s = 1$ *and* $\mathcal{D}(1) = 0$ *then*
$\frac{dq^o}{dU^f} \; < \; 0 \iff (\bar{y}^o - \bar{w}^{uf}) \; > \; \frac{r+\psi^o+\eta p^o}{p^c}\,(\bar{w}^{uf} - \bar{w}^{uh})$

Proof. Equation (27) can be rewritten as $F(U^f, q^o) = 0$. Applying the implicit function theorem, we get:

$$\frac{dq^o}{dU^f} = \frac{U^h\,(q^o)^2}{a\,(U^f + U^h)^2}\left(J^{oh} - J^{of}\right). \tag{31}$$

The sign of dq^o/dU^f therefore depends on the sign of $J^{oh} - J^{of}$. Using proposition 3, we have $J^{oh} < J^{of}$ if $\frac{\bar{y}^o - \bar{w}^{uh}}{r+\psi^o+p^c+\eta p^o} < \frac{\bar{y}^o - \bar{w}^{uf}}{r+\psi^o+\eta p^o}$, and we demonstrate the proposition by simply rearranging terms. ■

More competition will generate a positive externality (*i.e.* will stimulate the opening of vacancies) for the home unemployed if the expected discounted return of a private firm with a foreign worker is more important than the expected discounted return of a private firm with a home worker. This happens when relative unemployment benefits are sufficiently low in the foreign region. In the next proposition, we compare the wages of home and foreign workers:

Proposition 5 *(wage gap)*
If $s = 1$ *and* $\mathcal{D}(1) = 0$ *then*
$w^{oh} \; > \; w^{of} \iff (1-\eta)(\bar{w}^{uh} - \bar{w}^{uf}) \; > \; \frac{\eta(1-\eta)p^c\, p^o(\bar{y}^o - \bar{w}^{uh})}{(r+\psi^o)(r+\psi^o+p^c+\eta p^o)}$

Proof. We obtain this result directly by subtracting w^{of} defined in proposition 2 from w^{oh} defined in equation (30), and by rearranging the different terms. Since the denominator is always positive, we only need to check the sign of the numerator. ■

Since worker bargaining power $\eta \in [0, 1]$, from proposition 2 and equation (30), we immediately see that $w^{of} \in [\bar{w}^{uf}, \bar{y}^o]$ and $w^{oh} \in [\bar{w}^{uh}, \bar{y}^o]$. Proposition 5 implies that if $\bar{w}^{uf} = \bar{w}^{uh}$ (same unemployment benefits in foreign and home

[21] For simplicity, we assume that search intensity is normalized to 1 and generates no disutility.

region), then $w^{of} > w^{oh}$ (higher wages for commuters). The intuition is that home workers search on-the-job, reducing the expected profit of the firm (because employees are more likely to quit) and as a result the wage. The home workers will get higher pay than foreign workers only if the difference in unemployment benefits is sufficiently high to compensate the negative on-the-job search effect.

A summary of the state equations is presented in appendix B.

4 Calibration

The parameters of the model are displayed in table 2. Some parameters are directly taken from data or studies, while others are computed to obtain desired values for some endogenous variables. The home region h refers to Luxembourg and the foreign region f refers to neighboring countries (simple average of Belgium, France and Germany). We take the month as unit of time. We detail the calibration in the next paragraphs.

labor market flows								
$\bar{N}^c$	=	0.34	ψ^c	=	0.005	ψ^o	=	0.015
$\bar{m}^o$	=	0.13	λ	=	0.5	pop	=	1
α	=	0.12	$\bar{d}$	=	0.68			
wage formation								
$\bar{w}^{uf}$	=	0.48	$\bar{w}^{uh}$	=	0.84	η	=	0.5
productivity, interest rate and vacancy cost								
$\bar{y}^o$	=	1	r	=	0.004	a	=	0.26

Table 2: Parameter values

Labor market flows

Three types of workers co-exist in Luxembourg: the commuters (or cross-border workers), the non-national residents and the national residents. In 2006, total domestic employment in Luxembourg was about 320 000, but 115 000 of them were occupied by cross-border workers. The remaining 205 000 jobs were occupied by residents (source: IGSS, figure 2). In our model, we do not distinguish between non-national and national residents and we instead assume they all are national[22]. Luxembourg offers two types of jobs: private sector jobs and public sector jobs. Both private sector and public sector jobs are open to all workers but in practice, the *Letzebuergesch* language requirements in the public sector severely curtail competition from non-residents. In the model, we assume that public vacancies can only be filled by residents. In 2005, 24%

[22] As already mentioned in the introduction, this is a very rough simplification, since in 2001, 37% of the residents were non-national (source: STATEC). See section 6 for a discussion.

of domestic employment was public sector employment (source: STATEC, figure 3). Depending on the definition, the unemployment rate was around 4% or 6% percent in 2006 (source: IGSS, figure 1). Considering an unemployment rate of 6% implies 13 000 unemployed. Normalizing home population *pop* to 1 gives: $1 = U^h + N^{oh} + \bar{N}^c$, with $U^h = 0.06$ $N^{oh} = 0.60$ and $\bar{N}^c = 0.34$. Moreover $N^{of} = 0.53$. Finally, we impose $U^f = 0.06$, *i.e.* 13 000 foreign job seekers. This figure may look rather small since the *Grande Région* hosts approximately 200 000 unemployed job-seekers - source: STATEC (2006). However, only a small fraction of these potential 200 000 job seekers searches actively and efficiently[23].

Job destruction rates

Luxembourg has the strictest employment protection legislation among the OECD countries. The OECD (2006) computes a synthetic summary indicator. With 3.9, Luxembourg obtains the highest rating, while Belgium, France and Germany respectively obtain 2.5, 2.9 and 2.5. Job destruction rate in Luxembourg is therefore low, and notably lower than in the neighboring countries. We therefore choose a low private sector job destruction rate $\psi^o = 0.015$ and an even lower public sector job destruction rate $\psi^c = 0.005$. OECD (2006) emphazises that generous remuneration, strict employment protection, significant periodic wage increases and seniority-based career advancement induce resident workers to leave private sector jobs and apply for public sector jobs, while the government does not experience difficulties in retaining civil servants. This justifies our flow modelization (see figure 6): *all* residents (unemployed and employed) search for a public sector job but civil servants *do not* search for a private sector job.

Matching and disutility functions

For simplicity, we assume no frictions on the public sector labor market: public sector jobs are well advertised and residents know perfectly where and when to apply. The matching function (2) simplifies to $M^c = V^c$. The private sector matching function (1) is standard and given by: $M^o = \bar{m}^o \left(V^o\right)^{\lambda} \left(sU^h + U^f\right)^{1-\lambda}$. Vacancies and unemployment are assumed to have the same weight in the matching function, *i.e.* $\lambda = 0.5$ (actual estimates of λ lie between 0.4 and 0.7). Search disutility is a convex function $\mathcal{D}(s) = \alpha \frac{s^2}{2}$, and the foreign labor distribution is a linear function is $F(d) = d/\bar{d}$ [24].

Wages

Private sector firms productivity $\bar{y}^o$ is normalized to unity[25]. Comparing to neighboring countries, Luxembourg provides relatively high levels of unemployment benefits. Table 1 shows that the net replacement ratio during the initial phase of unemployment is between 85% and 91% in Luxembourg; while it is between 49% and 80% in Belgium, between 71% and 89% in France and be-

[23]Moreover, to keep a stable steady state, a higher U^h would necessitate a higher job destruction rate for private jobs with foreign workers. Introducing different job destruction rates (for foreign and home workers) would complicate the derivation of analytic properties.

[24]An alternative modelization would be $F(d) = \pi d^2/\bar{d}$.

[25]We implicitly suppose that foreign and home workers have exactly the same skills.

tween 69 and 86% in Germany. Moreover, net wages are about 40% higher in Luxembourg than in the neighboring countries (see OECD (2006) or Portugal and Centeno (2001) for a comparison of net average earnings between Luxembourg, Belgium, France and Germany). We set the home replacement ratio to 88%, the foreign replacement ratio to 70% and assume that the average wage in Luxembourg is 40% higher than the average wage in the neighboring countries. This gives $\bar{w}^{uh}/\bar{w}^{uf} = 1.75$. Finally, worker bargaining power $\eta = \lambda$ as in most papers on the subject (Hosios-Pissarides efficiency condition) and the interest rate is set to 0.004, which gives an annual interest rate of 4.9%.

Closing the calibration

There remain four parameters about which we do not have information: $\bar{m}^o$, α, $\bar{d}$, and the vacancy opening cost a. They are chosen to recover the above mentioned values for N^{of}, N^{oh}, U^h and U^f.

5 Simulations

In this section, we consider the full model (we deviate from some of the analytic results by assuming that search intensity is exogenous and normalized to 1), we use the calibration presented in the previous section and conduct several numeric simulations. We look successively at three different types of shocks: (i) foreign shocks (directly affecting the foreign workers), (ii) home (or domestic) shocks (directly affecting the home workers), and (iii) common shocks (directly affecting both the foreign and home workers). All the results are displayed in table 3.

Foreign shocks

A decrease in $\bar{d}$ means a higher foreign population density (or a higher foreign unemployment rate) or, alternatively, a decrease in the commuting cost[26]. In any case, it means more competition from foreign job-seekers. Two main effects are at work. More competition means that the crowding-out of home workers by foreign workers increases. But more foreign job-seekers also mean that the probability for a firm to fill a vacancy increases, which reduces the average vacancy opening cost and stimulates new job creation[27]. These two effects go in opposite directions for the home unemployed but the net effect is slightly positive (lower unemployment rate). As a conclusion, more competition from foreign workers may also be beneficial for home workers.

A decrease in the foreign unemployment benefits $\bar{w}^{uf}$ raises the number of foreign job seekers (higher $\hat{d}$) and reduces their reservation wage. These two effects strongly stimulate the opening of vacancies. Again, although crowding-out increases, the opening of more vacancies also allows for lower home unemployment. A change in foreign legislation is therefore not neutral for home working conditions.

[26] This last interpretation is maybe more intuitive since infrastructure investment and better transportation networks are improving the ability to commute.

[27] We have seen in the analytic section that this positive effect only happens if the wage of the foreign worker is sufficiently low, which is the case in our calibration.

	$\hat{d}$	s	V^o	w^{of}	w^{oh}	N^{of}	N^{oh}	N^o	N	co	U^h
foreign shocks											
$\bar{d}$: -10%	+0.3%	+0.3%	+6.5%	+0.1%	+0.1%	+11.6%	+0.1%	+5.5%	+4.2%	+2.7	-0.1
$\bar{w}^{uf}$: -10%	+10.4%	+0.9%	+10.0%	-0.3%	+0.1%	+11.0%	+0.5%	+5.4%	+4.2%	+2.3	-0.3
home (or domestic) shocks											
$\bar{w}^{uh}$: -10%	+1.1%	+29.4%	+7.5%	+0.2%	-0.6%	+1.6%	+2.5%	+2.1%	+1.6%	-0.2	-1.5
$\bar{N}^c$: $+10\%$	+0.1%	-0.3%	-1.4%	+0.1%	-0.1%	+0.1%	-5.4%	-2.8%	+0.2%	+1.4	-0.2
pop : $+10\%$	-0.4%	-0.4%	+6.3%	-0.1%	-0.1%	-0.6%	+14.9%	+7.61%	+5.9%	-3.6	+1.0
common shocks											
$\bar{y^o}$: $+10\%$	+22.3%	+35.9%	+29.3%	+10.2%	+9.8%	+24.5%	+3.4%	+13.3%	+10.2%	+4.7	-2.0
a : -10%	+1.3%	+1.2%	+7.5%	+0.3%	+0.1%	+1.9%	+0.6%	+1.2%	+1.0%	+0.3	-0.4
η : -10	-0.5%	-0.5%	+24.0%	-1.4%	-0.6%	+1.3%	+1.6%	+1.5%	+1.2%	-0.1	-1.0

Deviations are expressed in % or in basis points. $\bar{d}$: foreign population density, $\bar{w}^{uf}$: foreign unemployment benefits, $\bar{w}^{uh}$: home unemployment benefits, $\bar{N}^c$: public sector jobs, pop : active population, $\bar{y^o}$: productivity, a : vacancy cost, η : worker bargaining power, $\hat{d}$: employment pool, s : search intensity, V^o : private sector vacancies, w^{of} : wage for commuter, w^{oh} : wage for home worker, N^{of} : commuters, N^{oh} : home workers, N^o : private sector employment, N : total employment, co : crowding-out (percentage of private sector jobs occupied by commuters), U^h : home unemployment.

Table 3: Effects of various shocks

Home shocks

A 10% decrease in the home unemployment benefits $\bar{w}^{uh}$ raises the search intensity s and reduces the home worker reservation wage. These two effects (higher probability to fill a vacancy and lower labor cost) strongly stimulate the opening of vacancies and unemployment fall by 1.5 percentage points (*i.e.* fall from 6% to 4.5%). Foreign employment also benefits from the reduction in home unemployment benefits.

A larger public sector $\bar{N}^c$ generates a reduction in private sector employment. New public sector jobs are mainly filled by residents previously employed in the private sector, and these are not replaced by resident unemployed (who are not interested to work in the private sector as revealed by the fall in their search intensity). As a result, the unemployment rate decreases only slightly. Creating public jobs to reduce unemployment therefore seems a counterproductive policy: it is costly and total employment effects are weak.

An increase in domestic supply of labor reduces wages and stimulates the opening of vacancies. However, a higher level of vacancies and lower job competition are not sufficient to absorb the increase in labor supply and unemployment rate also increases[28].

Common shocks

Not surprisingly, a rise in productivity $\bar{y}^o$ is the most efficient way to boost the economy and employment. A 10% increase in productivity triggers a 29% increase in vacancies and this benefits both home and foreign workers. The crowding-out is 4.7% higher but the unemployment rate is lowered by 2.0 percentage points. Moreover, wages increase by 10%[29].

A lower vacancy opening cost a (for instance through a hiring subsidy) directly affects the opening of vacancies, stimulates employment (foreign and home) and lowers home unemployment. Finally, a lower worker bargaining power η indirectly affects the opening of vacancies through wage reduction and has the same positive effects on the labor market.

Welfare analysis

It is worth noting that the positive effect of a labor market policy on unemployment does not necessarily improve the welfare of the economy. For instance, we have seen that a 10% decrease in the home unemployment benefits reduces unemployment rate from 6% to 4.5%. This suggests that by lowering benefits further, unemployment could drop to almost zero. But would total welfare be

[28] An increase in active population is often due to migration. Most of the time, new migrants do not have the nationality or the skills required to work in the public sector. We cannot make this distinction in our model (all domestic labor may work in the public sector). Moreover, it is worth noting that the model could be further improved by making the migration decision endogenous. We here only focus on the commuting decision and leave the migration question for future research.

[29] This will further stimulate consumption and growth. This channel is however not introduced in this simple model.

improved? In the general equilibrium business cycle literature (see for instance King and Rebelo (1999), Merz (1995) or Andolfatto (1996)), welfare is usually measured by the difference between utility of consumption and the different disutilities (searching, working,...). In our model, if we assume no government budget deficit (unemployment benefits are financed by home workers), all income consumed and consumption utility linear, then home welfare is given by:

$$\mathcal{W}^h = w^{oh} N^{oh} + w^c \bar{N}^c - \mathcal{D}(s),$$

where w^c is the public sector wage. Under these assumptions, figure 7 illustrates that lower benefits do indeed reduce unemployment, although the marginal decrease in unemployment fades away with the size of the benefit reduction, because of higher labor market tightness. When benefits are too low, welfare starts decreasing. Two effects are at work: on the one hand employment (and income) increases, but on the other hand individual wage falls and search intensity (and therefore disutility) increases. With small reductions in benefits, the employment effect is stronger than the wage effect and welfare increases, but when the reductions are larger, employment creation is no longer strong enough to compensate the wage effect and welfare decreases.

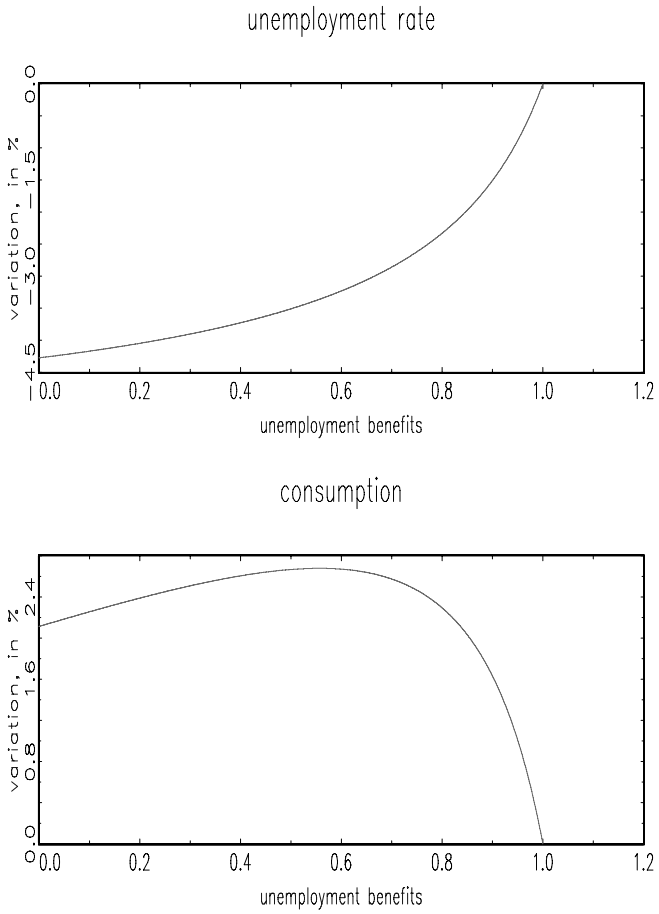


Figure 7: Unemployment, welfare and unemployment benefits

Rise in employment and unemployment

As shown in table 3, with a single shock it is difficult to reproduce what is currently observed in Luxembourg: a joint rise in employment and unemployment (the so-called unemployment paradox) and an increase in the share of commuters (crowding-out of the residents by the cross-border commuters). However, we may generate this phenomenon by combining a foreign shock and a domestic shock. For instance, table 4 shows that lower foreign unemployment benefit or lower commuting costs (higher foreign labor supply) combined with an increase in active population (higher domestic labor supply) may explain both the unemployment puzzle (joint increase in domestic employment and unemployment) and the higher crowding-out. The same results could be generated with similar foreign shocks combined with lower domestic unemployment benefits. However, we miss empirical evidence for such a domestic unemployment benefit shock.

$\bar{w}^{uf} : -10\%$	$\bar{w}^{uh} : +5\%$	$\longrightarrow$	$N : +3.0\%$	$U^h : +0.9$	$co : +2.8$
$\bar{d} : -10\%$	$\bar{w}^{uh} : +5\%$	$\longrightarrow$	$N : +2.7\%$	$U^h : +1.2$	$co : +3.0$
$\bar{w}^{uf} : -10\%$	$pop : +5\%$	$\longrightarrow$	$N : +7.1\%$	$U^h : +0.2$	$co : +0.6$
$\bar{d} : -10\%$	$pop : +5\%$	$\longrightarrow$	$N : +7.2\%$	$U^h : +0.4$	$co : +0.8$

Table 4: Combined shocks

6 Discussion and robustness

In this section, we first sum up our main findings and then try to draw some policy conclusions:

- More foreign job seekers have two effects on home employment: (i) a congestion effect and (ii) a vacancy effect. The congestion effect means that more foreign job-seekers generate more competition on the labor market and raise home unemployment. The vacancy effect means that more foreign job-seekers modify the strategic behavior of firms (vacancy opening). We demonstrate that the vacancy effect is positive (more vacancies) if the reservation wage of foreign workers is sufficiently low. Our numerical simulations show that more foreign job-seekers do not increase home unemployment but instead generate positive externalities that lower home unemployment. Foreign competition alone is therefore unable to explain the joint rises in employment and unemployment (unemployment paradox).
- The combination of a foreign shock (higher foreign labor supply) and a domestic shock is probably needed to explain the recent joint increases in employment and unemployment, the so-called "unemployment paradox", and the stronger job competition (crowding-out). For instance, more foreign job-seekers (generating more employment and stronger job competition)

and an increase in active population (increasing domestic unemployment) would replicate these stylized facts.

- Closing the border to any new cross-border commuters would progressively eliminate all foreign employment in Luxembourg (and therefore reduce the crowding-out to zero). But this would also drastically reduce the opening of vacancies and increase local unemployment. Our model suggests that such a policy would raise Luxembourg's unemployment rate from 6% to 8.1%.

- Reducing the current generosity (relative to international standards) of unemployment benefits would help cut unemployment, by lowering the reservation wage and increasing the willingness to find a job. However, unemployment benefits that are too low would be counter-productive since they would shrink the total welfare of the economy.

- One could be tempted (as in some other countries) to solve the unemployment problem by creating more public sector jobs. We show this would only result in a transfer of workers from private sector jobs to public sector jobs, since the resulting vacancies in the private sector would be filled by non-resident rather than by resident unemployed. Moreover, this policy would be costly for the government.

- Not surprisingly, the best way to boost the economy and lower unemployment is to increase productivity. This would also result in higher wages, that would in turn stimulate further consumption and growth.

We now discuss some of our modeling assumptions:

- All home workers (employed or unemployed) search for a public sector job with the same exogenous intensity. This assumption seems realistic (the expected return of a public sector job is on average higher than the expected return of a private sector job; and the application procedure is standard and well-known). This also allows us to simplify the public sector representation. We could however add an endogenous search intensity. This representation would require the introduction of a search disutility and a public sector wage. Within this setup, a lower public sector wage would reduce search intensity and allow the private sector to offer lower wages for resident workers. Private sector employment would increase and Luxembourg unemployment decrease. In conclusion, the effect of a lower public sector wage on unemployment depends on the assumption we make on search intensity: we have no effect in our model with exogenous search and we would have a positive effect (lower unemployment) in a model with endogenous search.

- Among the foreign population, only the unemployed search for a job in Luxembourg. Allowing on-the-job search for non-resident workers (employed in the foreign country) would not change our results. The only difference would be to add a third type of workers - and therefore a third type of wages - in the private sector: foreign workers previously employed abroad in addition to foreign workers previously unemployed and home workers.

- In our model, the commuting cost d is paid both when unemployed (in this case d can be seen as a searching cost) and when working. As a result, d disappears when looking at the difference between the employed and the unemployed expected returns, so the wage is independent of the location d. This result is not only elegant (no wage distribution) but also intuitive (the commuter from a city located closer to the border probably does not earn higher pay than the commuter located further away). An alternative assumption would be to pay the commuting cost only when working (and not when searching). In this case, we would have a wage distribution (the lower the wage, the closer the commuter resides to Luxembourg) which may not be empirically relevant.

- Distinguishing between nationals (residents filling the requirements to work in the public sector) and non-nationals (residents not filling the requirements to work in the public sector) would not add anything to our main conclusions.

7 Conclusion

In this paper, we construct a small theoretical model of the Luxembourg labor market, with search unemployment and job competition. We show both analytically and numerically that job competition alone cannot explain the unemployment paradox (simultaneous rise in employment and unemployment). The combination of a foreign shock (higher foreign labor supply) and a domestic shock (higher domestic labor supply) is probably needed to explain the unemployment paradox and the stronger job competition (crowding-out).

Our model could be developed further in several directions. Firstly, we could allow for an endogenous decision to migrate, and try to explain both the rise in active population and commuters. Secondly, we could try to find empirical data to see if the skill mismatch (skill demand *vs.* skill supply) has increased over the last years. In case of positive answer, the model should be adapted to make a skill distinction. Thirdly, the model could be closed by introducing a government budget constraint and private consumption. The final objective would be to introduce this framework into a full dynamic general equilibrium model (see for instance Pierrard and Sneessens (2003) or Pierrard (2005) for such a model with skill competition), which would allow us to conduct complete welfare analysis and provide more precise policy recommendations. This would also allow us to introduce other types of shocks (as demand shocks that are probably as important as supply shocks to explain the creation of new vacancies in the short run). Finally, because of the growing active population and the continuous expansion of the financial sector in Luxembourg, it could be fruitful to examine at the forces behind migration and agglomeration and close the gap between the search unemployment literature and the New Economic Geography literature (see for instance see Neary (2001) for a review of the New Economic Geography literature). These developments are however left for further research.

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A Proof of propositions 1 and 2

Assume that there exists a $\hat{d} > 0$. If a foreign worker chooses to search for a job in the home region, *i.e.* if his location $d < \hat{d}$, his expected discounted return is given by:

$$rW^{uf}(d) = \bar{w}^{uf} - d + p^o \left(W^{of}(d) - W^{uf}(d)\right), \tag{32}$$

while $rW^{of}(d)$ is still given by equation (10). The surplus becomes:

$$(r + \psi^o + p^o)\left(W^{of}(d) - W^{uf}(d)\right) = w^{of}(d) - \bar{w}^{uf}. \tag{33}$$

The job equation (12) is unchanged and, by solving equation (16), we finally obtain a Nash bargain wage independent of the location d:

$$w^{of}(d) = w^{of} = \frac{(1-\eta)(r+\psi^o)\bar{w}^{uf} + \eta(r+\psi^o+p^o)\bar{y}^o}{r+\psi^o+\eta p^o}. \tag{34}$$

This proves proposition 2.

We now determine the conditions under which there exists a unique $\hat{d} > 0$. Figure 8 plots the expected discounted return for a foreign unemployed located at a distance d from the home region. If the unemployed does not search for a job in the home region, his expected return is $rW^{uf}(d) = \bar{w}^{uf}$ (horizontal line 'non participation' in figure 8). If the unemployed searches for a job in the home region, his expected return is:

$$rW^{uf}(d) = \bar{w}^{uf} - d + p^o \left(W^{of}(d) - W^{uf}(d)\right). \tag{35}$$

Using equations (33) and (34), we obtain:

$$rW^{uf}(d) = \bar{w}^{uf} - d + \frac{\eta p^o \left(\bar{y}^o - \bar{w}^{uf}\right)}{r+\psi^o+\eta p^o}. \tag{36}$$

We get $\partial rW^{uf}(d)/\partial d = -1$ (line 'participation' in figure 8). If $rW^{uf}(0) > \bar{w}^{uf}$, there exists a single $\hat{d} > 0$, and this condition is met if $\bar{y}^o > \bar{w}^{uf}$. It is straightforward to derive $\hat{d} = \eta p^o \left(\bar{y}^o - \bar{w}^{uf}\right)/(r+\psi^o+\eta p^o)$. This proves proposition 1.

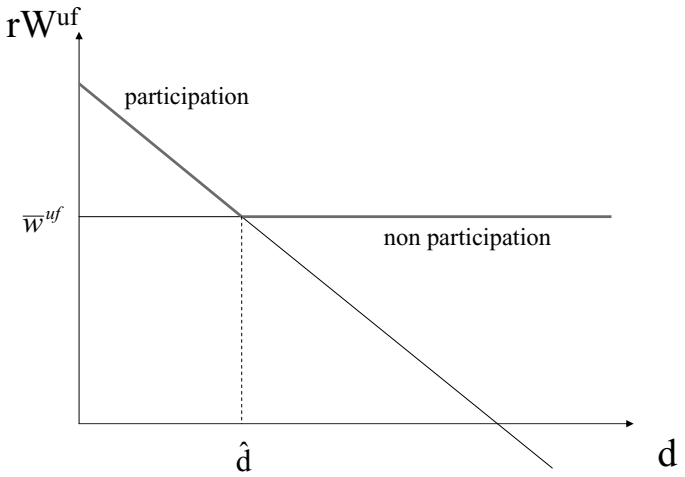

Figure 8: Expected discounted return of the foreign unemployed

B Summary of model equations

We have a system of 16 equations for the 16 endogenous variables $\{M^o, M^c, q^o, q^c, p^o, p^c, V^o, V^c, U^f, U^h, N^{of}, N^{oh}, w^{of}, w^{oh}, s, \hat{d}\}$:

Definitions

$$\begin{aligned} M^o = \mathcal{M}^o\left(V^o, sU^h + U^f\right) \quad &\text{and} \quad M^c = \mathcal{M}^c\left(V^c, U^h + N^{oh}\right), \\ q^o = \frac{M^o}{V^o} \quad &\text{and} \quad q^c = \frac{M^c}{V^c}, \\ p^o = \frac{M^o}{sU^h + U^f} \quad &\text{and} \quad p^c = \frac{M^c}{U^h + N^{oh}}. \end{aligned}$$

Flows

$$\begin{aligned} pop &= U^h + N^{oh} + \bar{N}^c, \\ \psi^o N^{of} &= p^o U^f, \\ \left(\psi^o + p^c\right) N^{oh} &= p^o s U^h, \\ \psi^c \bar{N}^c &= p^c\left(U^h + N^{oh}\right), \\ N^{of} + U^f &= \int_0^{\hat{d}} dF(d). \end{aligned}$$

Wages

$$\begin{aligned} w^{of} &= \frac{(1-\eta)(r+\psi^o)\bar{w}^{uf} + \eta(r+\psi^o+p^o)\bar{y}^o}{r+\psi^o+\eta p^o}, \\ w^{oh} &= \frac{(1-\eta)(r+\psi^o+p^c)\left(\bar{w}^{uh} - \mathcal{D}(s)\right) + \eta(r+\psi^o+p^c+s\,p^o)\bar{y}^o}{r+\psi^o+p^c+\eta s\,p^o}. \end{aligned}$$

First order conditions

$$\begin{aligned} \hat{d} &= \frac{\eta p^o\left(\bar{y}^o - \bar{w}^{uf}\right)}{r+\psi^o+\eta p^o}, \\ \mathcal{D}'(s) &= p^o \frac{w^{oh} - \bar{w}^{uh} + \mathcal{D}(s)}{r+\psi^o+p^c+s\,p^o}, \\ \frac{a}{q^o} &= \frac{U^f}{U^f + s\,U^h}\,\frac{\bar{y}^o - w^{of}}{r+\psi^o} + \frac{s\,U^h}{U^f + s\,U^h}\,\frac{\bar{y}^o - w^{oh}}{r+p^c+\psi^o}. \end{aligned}$$