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LOLA 2.0: LUXEMBOURG OVERLAPPING GENERATION MODEL FOR POLICY ANALYSIS

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LOLA 2.0: Luxembourg OverLapping generation model for policy Analysis *

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Abstract

Beyond cyclical movements, it may be helpful to understand how structural forces or policies shape an economy in the longer term. With such remote horizons, it is crucial to base analysis on an appropriate tool. In this paper, we build an overlapping generation structure with New Open Economy Macroeconomics and labour market frictions à la Diamond-Mortensen-Pissarides. The main novelty over LOLA 1.0 is the integration of current account and exchange rate dynamics according to the New Open Economy Macroeconomics approach. We calibrate the model on Luxembourg data. By way of illustration, we study the interactions between expected demographic changes, labour market dynamics and public finance, and we look at the recently proposed policy responses.

Keywords: Overlapping generations, Long-run projections, Imbalances, Luxembourg.

JEL-Code: D91, E24, E62, F41, J11.

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Résumé non-technique

L’approche méthodologique actuellement privilégiée pour l’analyse de problèmes de politique macroéconomique est celle des modèles d’équilibre général dynamiques. L’équilibre général signifie que tous les marchés sont liés entre eux et ceci est indispensable si l’on veut appréhender les effets de chocs/politiques structurel(le)s dans leur ensemble. L’aspect intertemporel est tout aussi important puisque les agents économiques n’ont pas une approche statique, mais prennent au contraire des décisions en sachant qu’elles auront un effet sur leur futur. Ces modèles sont à interpréter comme des représentations stylisées (maquettes) du fonctionnement de l’économie. Ils sont construits à partir de représentations cohérentes et rigoureuses des mécanismes de marché et du comportement des agents économiques, fondées sur la théorie microéconomique. Hormis quelques cas particuliers hyper simplifiés, les propriétés et implications de ces modélisations de la réalité économique peuvent rarement être étudiées en termes analytiques généraux. Typiquement, les maquettes sont “calibrées” et leurs propriétés étudiées par simulations numériques, en veillant à spécifier et calibrer le modèle initial (scénario de base) de façon à reproduire des caractéristiques bien établies de l’économie considérée. Les effets de politiques économiques ou autres modifications de l’environnement économique sont simulés en élaborant des variantes du scénario de base. Le modèle LOLA 2.0 représente l’économie luxembourgeoise et se conforme à cette approche d’équilibre général dynamique. Il vise principalement à étudier tant les effets de chocs structurels comme les chocs de démographie, que les effets de politiques structurelles telles qu’une réforme des pensions.

Outre l’aspect d’équilibre général dynamique, le modèle LOLA 2.0 présente cinq caractéristiques principales. Premièrement, c’est un modèle à générations imbriquées. Cela signifie que plusieurs générations (travailleurs jeunes, travailleurs moins jeunes, retraités, ...) avec des situations différentes coexistent à chaque instant. Cela introduit un cycle de vie et permet, entre autres, d’étudier des problématiques comme le financement des pensions. Plus précisément, dans LOLA, la vie d’un individu (de 20 ans à 99 ans) est divisée en 16 périodes. Une période représente donc 5 ans. Deuxièmement, pour modéliser le marché du travail, nous suivons l’approche Diamond-Mortensen-Pissarides, qui représente explicitement les comportements de demande et d’offre de travail, les processus de formation des salaires et leurs impacts sur les probabilités d’embauche. Pratiquement, cela nous permet d’introduire des notions comme le chômage involontaire ou bien encore les postes vacants et donc d’avoir une représentation du marché du travail assez réaliste. Troisièmement, l’emploi n’est pas uniquement résident mais il est aussi frontalier. Ces travailleurs frontaliers peuvent avoir des productivités différentes de celles des travailleurs résidents, de même que des pouvoirs de négociation (des salaires) différents. Quatrièmement, l’économie est ouverte, dans la lignée des travaux de Obstfeld-Rogoff. Cela génère une dynamique de la balance courante, qui elle-même dépend des chocs étrangers, de notre compétitivité et de nos préférences. Cinquièmement, le modèle comporte

un système de pension par répartition (“pay as you go”) mais une pension complémentaire peut être financée par l’épargne individuelle. Outre les pensions, le gouvernement doit également financer les prépensions, les allocations de chômage et les autres dépenses publiques. Toutes ces dépenses sont, en partie ou non, financées par différentes taxes sur le travail, le capital et la consommation. Tout déséquilibre du solde primaire se répercute sur le niveau de la dette publique.

Ce modèle est ensuite calibré, c’est-à-dire que des valeurs sont données à tous les paramètres de manière à reproduire le mieux possible la situation de l’économie luxembourgeoise en 2010, comme par exemple le taux de chômage, les finances publiques ou encore les avoirs extérieurs. De plus, dans un souci de réalisme, nous introduisons dans notre modèle les évolutions démographiques attendues (probabilités de passage d’une génération à une autre, taux de natalité, immigration et frontaliers) entre 2010 et 2100. En d’autres termes, la taille de la population par âge évolue au cours du temps selon les dernières projections démographiques disponibles. Enfin, nous supposons également que la productivité du travail augmente de manière continue. Nous fixons cette amélioration de productivité afin d’obtenir une croissance potentielle annuelle moyenne de l’ordre de 2% entre 2015 et 2060, ce qui est comparable au chiffre du rapport de l’Ageing Working Group (Commission Européenne, 2012).

A titre d’illustration, nous utilisons le modèle LOLA afin d’analyser et de comprendre les effets à moyen et long terme que les évolutions démographiques (vieillissement de la population et importance des frontaliers) pourraient avoir sur l’économie du Luxembourg. D’ici 2100, on remarque une baisse continue du taux de croissance potentielle du PIB et de l’emploi, un taux de chômage qui demeure relativement stable entre 5% et 6% et des finances publiques qui se détériorent fortement, à cause du vieillissement de la population, tant résidente que frontalière, et d’un alourdissement progressif et non négligeable des dépenses de pensions. Ensuite, nous analysons la récente réforme des pensions proposée par le gouvernement. Nous montrons qu’elle va dans le bon sens mais qu’elle est insuffisante au vu de l’ampleur du problème qui nous attend.

Par définition, un modèle, aussi bon soit-il, ne parviendra jamais à reproduire parfaitement une économie. Le développement de LOLA doit donc être vu comme un processus continu dont chaque nouvelle version vise à améliorer la précédente. Par rapport à la version LOLA 1.0, la version 2.0 a ajouté une génération supplémentaire, détaillé la partie finance publique, affiné et rafraichi la calibration et, surtout, introduit une dynamique de la balance courante dans la lignée des travaux de Obstfeld-Rogoff. Ce modèle sera encore développé dans le futur. Dans un premier temps, nous voudrions réduire la durée d’une période de 5 à 1 an. Cela n’aura aucun effet sur nos conclusions mais devrait permettre de rendre nos résultats plus “lisibles”. Dans un deuxième temps, nous aimerions ajouter un secteur productif dans notre modèle afin de pouvoir distinguer le secteur des services (dont le secteur financier) du secteur manufacturier.

Enfin dans un troisième temps, la taille de l'offre de travail frontalière potentielle pourrait être représentée par une relation économétrique estimée plutôt que d'être complètement exogène.

1 Introduction

In this paper, we build a dynamic general equilibrium model of the Luxembourg economy to (i) understand the long-term effects of structural forces, (ii) identify – or not – the development of potential imbalances and (iii) look at appropriate policy responses if necessary. First, to properly incorporate the aging process as well as the potential development of imbalances, we construct an overlapping generation (OLG) model *à la* Auerbach and Kotlikoff (1987). Since Ricardian equivalence does not hold, unlike most models with infinitively lived agents, this OLG framework is also suitable for the evaluation of alternative fiscal policies. Second, to better match the complexity and the importance of the labour market, as well as the role of cross-border workers, we do not consider a streamlined perfectly competitive labour market but instead introduce a more realistic labour market with imperfections, along the lines of Diamond-Mortensen-Pissarides (DMP, see Pissarides (2000) for an extensive exposition). Third, we introduce the OLG structure in a New Open Economy Macroeconomics (NOEM) model, whose starting point is usually considered to be the Redux model of Obstfeld and Rogoff (1995), to consider trade openness and capital mobility.

A few papers have introduced frictional labour markets into life-cycle models. For instance, Hairault et al. (2010) and Chéron et al. (2011) study early-retirement decision choices, whereas de la Croix et al. (2012) show that neglecting labour market frictions and employment rate changes may seriously bias the evaluation of pension reforms when they have an impact on the interest rate. However, these models work in closed-economy settings and therefore do not investigate capital flows and current account dynamics. Bringing an overlapping generation model into a NOEM model can therefore deliver interesting insights. Ganelli (2005) and Botman et al. (2006) build such a model to evaluate a wide range of fiscal policies. However, these two papers introduce neither an aging process nor a labour market with search and matching. The methodological contribution of this paper is therefore to combine both the DMP and NOEM paradigms within a life-cycle setup.

The model is calibrated to match the main macroeconomic features of the Luxembourg economy in 2010. We also feed the model with expected technological progress, fertility rates, migration flows and survival probabilities until 2100. Then we illustrate the functioning of the model through two experiments. First, we show that the – non sustainable – employment dynamics along with the aging process are a nest for fiscal imbalances that will materialize from 2030 onwards. Given the current projections, the primary deficit will move from presently 0.4% of GDP to around 12% in 2050 and 20% in 2100. Second, we show that only deep – and unpopular – fiscal reforms could solve the whole deficit problem. This underlines the need of closely monitoring the expected economic evolutions, and of reacting at the earliest possible stage to any imbalance development. This also underlines the need of building adequate models.

We briefly review the existing models of the Luxembourg economy in section 2 and we detail our new model in section 3. We explain the calibration in section 4 and simulate the effects of aging and policy shocks in section 5. Section 6 concludes.

2 Existing models for the Luxembourg economy

The STATEC has developed a macroeconometric model (see Adam (2004) for an overview), estimated from annual data (from 1970 onwards) and used for forecasting and scenario analysis. Similarly the BCL, see Guarda (2005), has developed the Luxembourg block of the euro-area multi-country model. The model is estimated with annual data from 1985 onwards and may also be used for projections and policy simulations. These are large-scale and detailed models that may prove useful for short-term forecasts. However, these two models do not belong to the micro-founded literature and are therefore subject to the Lucas critique.

There exist two micro-founded models representing the Luxembourg economy. Deák et al. (2012) develop a model with overlapping generations along the lines of Blanchard (1985) and Yaari (1965). The labour market is represented by a "right-to-manage" setup and wages are bargained between firms and unions. Jobs may be occupied by residents or cross-border commuters and the interest rate is determined according to the NOEM approach. The second version of their model extends the previous setting with a banking sector. Pierrard and Sneessens (2009) propose a model with a "pure" OLG representation, rather than the Blanchard (1985) and Yaari (1965) one, which allow them to study demography related questions as the activity rate of seniors or the cost of pensions. The labour market is *à la* DMP, there are cross-border commuters, the interest rate is fixed and net exports are simply the residual between home production and home demand. The model presented in this paper extends Pierrard and Sneessens (2009) by representing the current account and exchange rate dynamics according to the New Open Economy Macroeconomics. The model is therefore richer since it allows to study shocks as foreign demand, price markups or risk premium.

3 The Model

We develop a dynamic general equilibrium model featuring overlapping generation (OLG) dynamics. The labour market includes frictions à la Diamond-Mortensen-Pissarides (see Pissarides (2000) for an extensive exposition), as well as resident and cross-border employment. To model the current account and exchange rate dynamics, we follow the New Open Economy Macroeconomic (NOEM) literature initially developed by Obstfeld and Rogoff (1995).

3.1 Demographics

We assume a country with an open labour market, meaning that both the population in this country (we call it resident or home population and denote it by the superscript h hereafter) and the population in the bordering countries (we call it foreign population and denote it by the superscript f hereafter) may supply their labour force – in the home country. We therefore need to describe the demography of the home and foreign populations/countries. In each country, each member of a generation can live for up to sixteen periods of 5 years each (from age 20 till 99), indexed by a from 0 to 15. Let $Z^x_{a,t}$ denote the size of the generation reaching age a at period t in country $x \in \{h, f\}$. The size of new generations changes over time at an exogenous fertility rate x^x_t:

$$Z^x_{0,t} = (1 + x^x_t)\, Z^x_{0,t-1}\,,\ \forall t > 0\,. \tag{1}$$

The evolution of the size of a given generation born in time t is determined by a cumulative survival probability vector $\beta^x_{a,t+a}$ as well as a migration flow vector $X^x_{a,t+a}$ so that:

$$Z^x_{a,t+a} = \beta^x_{a,t+a}\, Z^x_{0,t} + X^x_{a,t+a}\,,\ \forall a \in [0, 15]\,, \tag{2}$$

with $\beta^x_{0,t} = 1$. Total (adult) population at time t is equal to $Z^x_t = \sum_{a=0}^{15} Z^x_{a,t}$. The fertility rate, the survival probability vector and the migration vector can vary exogenously over time.

We use the variable $z^x_{a,t+a}$ to define the population of working age:

$$P^x_{a,t+a} = z^x_{a,t+a}\, Z^x_{a,t+a}\,, \tag{3}$$

where $z^x_{a,t+a} < 1$ for $a = 0$ (post-secondary education), $z^x_{a,t+a} = 1$ for $a \in [1, 8]$ and $z^x_{a,t+a} = 0$ for $a \in [9, 15]$ (compulsory retirement). Moreover, we assume that between ages 55 and 64, workers may choose to retire early. People of working age are thus either employed (N^x_a for $0 \le a \le 8$), unemployed (U^x_a for $0 \le a \le 8$) or on a early retirement scheme (E^x_a for $7 \le a \le 8$):[1]

$$\begin{aligned} P^x_{a,t} &= N^x_{a,t} + U^x_{a,t} + E^x_{a,t}\,, \\ &= \left[n^x_{a,t} + u^x_{a,t} + e^x_{a,t}\right] P^x_{a,t}\,. \end{aligned} \tag{4}$$

Lower-case letters denote the proportion of individuals in each group. Let $\lambda^x_{7,t}$ denote the fraction of people who choose to retire and leave the labor market between 55 and 64, so that the number of early retired workers of that age group is $E^x_{7,t} = \lambda^x_{7,t}\, P^x_{7,t}$. Similarly, let $\lambda^x_{8,t}$ denote the fraction of active workers of age 60-64 who decide to leave the labor market. The total number of workers on an early retirement scheme at time t is then equal to:

$$\begin{aligned} E^x_{7,t} + E^x_{8,t} &= e^x_{7,t}\, P^x_{7,t} + e^x_{8,t}\, P^x_{8,t}\,, \\ \text{with:} \quad & e^x_{7,t} = \lambda^x_{7,t}\,, \\ & e^x_{8,t} = \lambda^x_{7,t-1} + \lambda^x_{8,t}\,(1 - \lambda^x_{7,t-1})\,. \end{aligned} \tag{5}$$

[1]We do not introduce the other participation rate decisions as for instance the female participation rate. See de la Croix and Docquier (2007) for further motivation of this choice.

3.2 Labour Market Flows

We assume a constant returns to scale matching function:

$$M_t = M(V_t, \Omega_t)\,, \tag{6}$$

where V_t and Ω_t stand respectively for the total number of vacancies and job seekers at the beginning of period t. Job seekers may be located in the home country h or in the foreign bordering countries f, such that $\Omega_t = \Omega_t^h + \Omega_t^f$. The pool of job seekers in each country $x \in \{h, f\}$ is equal to the new entrants $P_{0,t}^x$, plus the total number of unemployed workers in all older active generations:

$$\begin{aligned}
\Omega_t^x &= \sum_{a=0}^{8} \Omega_{a,t}^x\,, \\
&= P_{0,t}^x + \sum_{a=1}^{8} \left[1 - (1-\chi)\, n_{a-1,t-1}^x\right] P_{a,t}^x \\
&\quad + (1-\lambda_{7,t}^x)\left[1-(1-\chi)\, n_{6,t-1}^x\right] P_{7,t}^x \\
&\quad + (1-\lambda_{8,t}^x)\,\left[(1-\lambda_{7,t-1}^x) - (1-\chi)\, n_{7,t-1}^x\right]\, P_{8,t}^x\,.
\end{aligned} \tag{7}$$

where $\Omega_{a,t}^x$ is the number of job seekers of age a and χ is the exogenous job destruction rate. The probabilities of finding a job and of filling a vacancy will be given respectively by:

$$p_t = \frac{M_t}{\Omega_t} \qquad \text{and} \qquad q_t = \frac{M_t}{V_t}\,.$$

In each country, the number of employed workers in age group a is determined by the sum of non-destroyed jobs (when $a > 0$) and of new hires:

$$\begin{aligned}
n_{a,t}^x &= p_t\,\frac{\Omega_{a,t}^x}{P_{a,t}^x}\,, && \text{for } a = 0\,, \\
&= (1-\chi)\, n_{a-1,t-1}^x + p_t\,\frac{\Omega_{a,t}^x}{P_{a,t}^x}\,, && \text{for } 1 \le a \le 6\,, \\
&= (1-\lambda_{a,t}^x)\,(1-\chi)\, n_{a-1,t-1}^x + p_t\,\frac{\Omega_{a,t}^x}{P_{a,t}^x}\,, && \text{for } 7 \le a \le 8\,.
\end{aligned}$$

After substituting for $\Omega_{a,t}^x$, these equations become:

$$\begin{aligned}
n_{a,t}^x &= p_t\,, && \text{for } a = 0\,, \\
&= (1-p_t)(1-\chi)\, n_{a-1,t-1}^x + p_t\,, && \text{for } 1 \le a \le 6\,, \\
&= (1-p_t)(1-\lambda_{a,t}^x)\,(1-\chi)\, n_{a-1,t-1}^x + p_t(1-\lambda_{a,t}^x)\,, && \text{for } a = 7\,, \\
&= (1-p_t)(1-\lambda_{a,t}^x)\,(1-\chi)\, n_{a-1,t-1}^x + p_t(1-\lambda_{a,t}^x)(1-\lambda_{a-1,t-1}^x)\,, && \text{for } a = 8\,.
\end{aligned} \tag{8}$$

The same equations can be written in terms of the probability of filling a vacancy q_t by using $p_t = q_t\, V_t/\Omega_t$. Total employment is equal to:

$$N_t = N_t^h + N_t^f = \sum_{a=0}^{8} \left(n_{a,t}^h\, P_{a,t}^h + n_{a,t}^f\, P_{a,t}^f\right)\,.$$

3.3 Households in the home country

For simplicity, in this section we drop the superscript h from all variables. Each individual is assumed to belong to a representative household, one for each age category. There is no aggregate uncertainty and all households have perfect foresight. However, we introduce a role for idiosyncratic uncertainty through an imperfect annuity market. More precisely, if an individual dies at the end of a period, his financial wealth is not fully redistributed among surviving agents from the same generation but partially goes to the government.[2] Given a sequence of contingent wages and prices, an individual/household born at time t will determine his optimal contingent consumption and early retirement plans by maximizing his expected utility, subject to his intertemporal budget constraint.

Let $c_{a,t+a}$ represent the consumption level of an individual consumer of generation t and age a, while $n_{a,t+a}\, z_{a,t+a}$ and $e_{a,t+a}\, z_{a,t+a}$ represent respectively the proportion of employed and early retired workers in the total population of age a born at time t. The objective function of the household (effectively of one cohort) is written as follows:

$$W_t^H = \max_{c_{a,t+a},\, \lambda_{7,t+7},\, \lambda_{8,t+8}} \sum_{a=0}^{15} \beta^a\, \beta_{a,t+a} \left\{ \mathcal{U}(c_{a,t+a}) - d^n\, n_{a,t+a}\, z_{a,t+a} + d_a^e\, \frac{(e_{a,t+a})^{1-\phi}}{1-\phi}\, z_{a,t+a} \right\} Z_{0,t}\,, \quad (9)$$

where $0 < \beta < 1$ is the subjective discount factor. Instantaneous utility is assumed to be separable in c, n and e. The utility of per capita consumption is represented by a standard concave function. Marginal labour disutility is assumed to be constant, equal to $d^n > 0$. The extra utility derived from early retirement is represented by a concave function of the early retirement rate with $d_a^e > 0$ and $0 < \phi < 1$.[3] The decision variables are c_a, λ_7 and λ_8. The last two variables refer to the fraction of agents in the corresponding age groups who decide to go on early retirement and leave the labour market, respectively at age 55 and 60. Inactivity and employment rates are given by (5) and (8).

The household's flow budget constraint at time $t + a$ takes the form:

$$I_{a,t+a} + \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}} \right)^{\varpi} [1 + r_{t+a}(1 - \tau_{t+a}^k)] \cdot s_{a-1,t+a-1} = (1 + \tau_{t+a}^c) c_{a,t+a} + s_{a,t+a}, \quad (10)$$

where $\varpi \in [0,1]$. $\varpi = 1$ implies a perfect insurance against lifetime uncertainty whereas a lower ϖ reduces the distribution within a generation. $I_{a,t+a}$ comprises labor income and various transfers:

$$I_{a,t+a} = z_{a,t+a} \left[(1 - \tau_{a,t+a}^w) w_{a,t+a} \cdot n_{a,t+a} + b_{a,t+a}^u \cdot u_{a,t+a} + b_{a,t+a}^e \cdot e_{a,t+a} \right] + (1 - z_{a,t+a})\, b_{a,t+a}^i.$$

[2] An imperfect annuity market is a convenient way to generate a concave consumption shape across generations and to introduce precautionary savings. However, as shown in de la Croix et al. (2012) and Marchiori et al. (2011), this does not fundamentally change the real effects of aggregate shocks.

[3] This formulation implies – without loss of generality – that the disutility associated with the search activities of the unemployed is normalized to zero.

Wage, consumption and capital tax rates are given by τ_a^w, τ^c and τ^k respectively. τ_a^w may vary across ages to allow for targeted tax cuts. $b_{a,t+a}^u, b_{a,t+a}^e, b_{a,t+a}^i$ are the replacement benefits received respectively by the unemployed, early retired or statutory retirement age worker on a legal pension scheme; $s_{a,t+a}$ is the financial wealth accumulated at time $t+a$, in per capita terms. This financial wealth is held either in the form of shares, physical capital rented out to firms, net foreign assets or domestic debt. The non arbitrage condition ensures that all forms of savings pay a similar interest rate r_{t+a} before taxes.

The optimal consumption plan must satisfy the usual Euler equation:

$$\frac{\mathcal{U}'_{c_{a,t+a}}}{1+\tau^c_{t+a}} = \beta \left(\frac{\beta_{a+1,t+a+1}}{\beta_{a,t+a}}\right)^{1-\varpi} \left[1+r_{t+a+1}(1-\tau^k_{t+a+1})\right] \frac{\mathcal{U}'_{c_{a+1,t+a+1}}}{1+\tau^c_{t+a+1}} . \tag{11}$$

After substitution and rearrangements, and assuming a logarithmic utility of consumption, the condition determining the optimal proportion of early retired workers aged 60-65 can be shown to be:

$$\begin{aligned} \frac{b^e_{8,t+8}}{(1+\tau^c_{t+8})\, c_{8,t+8}} + d^e_8\, (e_{8,t+8})^{-\phi} \quad &= \quad \pi_{8,t+8} \left[\frac{(1-\tau^w_{8,t+8})\, w_{8,t+8}}{(1+\tau^c_{t+8})\, c_{8,t+8}} - d^n\right] \\ &\quad + (1-\pi_{8,t+8}) \left[\frac{b^u_{8,t+8}}{(1+\tau^c_{t+8})\, c_{8,t+8}}\right] , \end{aligned}$$

where π is the unconditional probability that an active worker will be employed. A similar condition holds for early retirement at age 55-60. Details are given in the appendix A.

For later use, we also note that the value of an additional job for a household of age a is given by:

$$\begin{aligned} \frac{1}{\mathcal{U}'_{c_{a,t}}} \frac{\partial W^H_t}{\partial N_{a,t}} \quad &= \quad \frac{1}{\mathcal{U}'_{c_{a,t}}} \frac{1}{z_{a,t}\, Z_{a,t}} \frac{\partial W^H_t}{\partial n_{a,t}} \\ &= \quad \sum_{j=0}^{7-a} \frac{\beta_{a+j,t+j}}{\beta_{a,t}} \beta^j \frac{\mathcal{U}'_{c_{a+j,t+j}}}{\mathcal{U}'_{c_{a,t}}} \left\{ \frac{(1-\tau^w_{a+j,t+j})\, w_{a+j,t+j} - b^u_{a+j,t+j}}{(1+\tau^c_{t+j})} - \frac{d^n}{\mathcal{U}'_{c_{a+j,t+j}}} \right\} \frac{\partial n_{a+j,t+j}}{\partial n_{a,t}} , \end{aligned} \tag{12}$$

where $\partial n_{a+j,t+j}/\partial n_{a,t}$ can be obtained from (8).

3.4 Households in the foreign country

Cross-border workers are employed and pay taxes (on wages) in the home country but consume in the foreign country. Unemployment benefits are paid by the foreign country but early-retirement and retirement benefits are paid by the home country. Because we are only interested in the home country, we consider foreign country household decisions exogenous. More precisely, we take as given inactivity choices $\lambda^f_{a,t}$ as well as wages $w^f_{a,t}$.[4]

[4]An extension of this model would be to endogenize the cross-border commuters' behaviour, along the lines of Pierrard (2008).

3.5 Firms

Intermediate monopolistic firms located in the home country are denoted by h. There are uniformly distributed between $[0,1]$ and indexed by i, each of whom produces a single differentiated good, also indexed by i. Intermediate monopolistic firms located in the rest of the world – foreign country – are denoted by f. There are uniformly distributed between $[0,1]$ and indexed by j, each of whom produces a single differentiated good, also indexed by j. Time subscripts are ignored when there is no risk of confusion.

Demand

The aggregate demand from the home country is:

$$D = \left[\omega_1 \left(\int_0^1 \left(D_h(i)\right)^\theta di\right)^{\frac{\rho}{\theta}} + \omega_2 \left(\int_0^1 \left(D_f(j)\right)^\theta dj\right)^{\frac{\rho}{\theta}}\right]^{\frac{1}{\rho}}, \tag{13}$$

with $0 < \theta < 1$ and $0 < \rho < 1$. The elasticity of substitution between the differentiated goods is $1/(1-\theta)$ and the one between home and foreign goods is $1/(1-\rho)$.[5]

Final Firm

In the home country, the final firm maximizes:

$$\max_{D_h(i), D_f(j)} \; P\, D - \int_0^1 P(i)\, D_h(i)\, di - \int_0^1 e\, P^*(j)\, D_f(j)\, dj\,, \tag{14}$$

under the constraint (13), which gives:

$$\frac{D_h(i)}{D} = \left(\frac{1}{\omega_1}\right)^{\frac{1}{\theta-1}} \left(\frac{P(i)}{P}\right)^{\frac{1}{\theta-1}} \left(\frac{D_h}{D}\right)^{\frac{\theta-\rho}{\theta-1}}, \tag{15}$$

$$\frac{D_f(j)}{D} = \left(\frac{1}{\omega_2}\right)^{\frac{1}{\theta-1}} \left(\frac{eP^*(j)}{P}\right)^{\frac{1}{\theta-1}} \left(\frac{D_f}{D}\right)^{\frac{\theta-\rho}{\theta-1}}, \tag{16}$$

where P is the home consumption price index (CPI), $P(i)$ is the home-currency price of good i, $P^*(j)$ is the foreign-currency price of good j, and e is the nominal exchange rate between h and f, i.e. the price of foreign currency in home currency.

We denote D^* the aggregate demand from the rest of the world and we define it similarly to (13). The final firm in the rest of the world maximizes a problem similar to (14). This yields

[5]When $\theta \to 1$ and $\sigma \to +\infty$, we get the linear (perfect substitutes) function; when $\theta \to 0$ and $\sigma \to 1$, we get the Cobb-Douglas function; when $\theta \to -\infty$ and $\sigma \to 0$, we get the Leontief (perfect complements) function.

the following demands, assuming that $\theta^* = \theta$ and $\rho^* = \rho$:

$$\frac{D_f^*(j)}{D^*} = \left(\frac{1}{\omega_1^*}\right)^{\frac{1}{\theta-1}} \left(\frac{P^*(j)}{P^*}\right)^{\frac{1}{\theta-1}} \left(\frac{D_f^*}{D^*}\right)^{\frac{\theta-\rho}{\theta-1}}, \tag{17}$$

$$\frac{D_h^*(i)}{D^*} = \left(\frac{1}{\omega_2^*}\right)^{\frac{1}{\theta-1}} \left(\frac{P(i)}{e\,P^*}\right)^{\frac{1}{\theta-1}} \left(\frac{D_h^*}{D^*}\right)^{\frac{\theta-\rho}{\theta-1}}. \tag{18}$$

Prices

The law of one price says that identical goods should sell for the same price in two separate markets (or in short, identical goods must have identical prices). This means:

$$P(i) = e\,P^*(i)\,, \tag{19}$$

where $P(i)$ is the home-currency price of good i and $P^*(i)$ is the foreign-currency price of the same good. Moreover, let P (resp. P^*) be the home (resp. foreign) consumption price index (CPI). The real exchange rate is:

$$\gamma = \frac{e\,P^*}{P}\,. \tag{20}$$

Whereas the law of one price applies to individual commodities, purchasing power parity (PPP) applies to the general price level/index. PPP holds when $\gamma = 1$. Finally, let us define $\phi(i) = P(i)/P$ and $\phi^*(j) = P^*(j)/P^*$.

Intermediate Firms

Intermediate firms located in the home country use two productive factors, labor and capital. Labour is measured in efficiency units. Efficiency varies across age (because of experience and abilities), but may also vary across time (easier access to education) and country of residence. We define total labour input in the home firm i as follows:

$$H_t(i) = \sum_{a=0}^{8} \left(h_{a,t}^h\, N_{a,t}^h(i) + h_{a,t}^f\, N_{a,t}^f(i) \right) .$$

We assume a constant-return-to-scale production function in labor and capital:

$$Y_t(i) = A_t\, F(K_t(i), \bar{h}_t\, H_t(i))\,,$$

where A_t stands for total factor productivity and $\bar{h}_t = \psi \bar{h}_{t-1}$ where $\psi > 1$ is an exogenous labour augmenting technical progress.[6] Firms rent capital from households at cost $r_t + \delta$ and

[6]Note that with a Cobb-Douglas production function, total factor augmenting, capital augmenting and labour augmenting technical progresses are interchangeable and consistent with balanced growth. For other production functions, only the labour augmenting progress is consistent with balanced growth.

pay a gross wage $w^x_{a,t}(i)$ to workers of age a from country $x \in \{h, f\}$. The wage results from firm-specific Nash bargain and is therefore indexed by i. We allow the employer wage tax ζ to vary across age groups (to allow for social security tax cuts targeted on specific age groups). The representative firm maximizes the discounted value of all the dividends (profits) that will be distributed to shareholders. Profits at time t are given by:

$$
\begin{aligned}
\Pi_t(i) &= \phi_t(i) Y_t(i) - (r_t + \delta) K_{t-1}(i) - \sum_{a=0}^{8} (1 + \zeta_{a,t}) \left(w^h_{a,t}(i)\, N^h_{a,t}(i) + w^f_{a,t}(i)\, N^f_{a,t}(i) \right) \\
&\quad - a_t\, V_t(i) - FC_t\,, \qquad (21)
\end{aligned}
$$

where δ is the capital depreciation rate, a_t stands for the exogenous cost of posting a vacancy and FC_t for an exogenous fixed cost. Moreover, $V_t = \int_0^1 V_t(i)di$ and $N^x_{a,t} = \int_0^1 N^x_{a,t}(i)di$, which implies $\partial V_t / \partial V_t(i) = \partial N^x_{a,t} / \partial N^x_{a,t}(i) = 1$. It is worth noting that because of our 5-year period, capital is not predetermined but firms pay interests as usual in the following period. The value of the firm can thus be written as follows:[7]

$$
\begin{aligned}
W^F_t(i) = \max_{\phi_t(i), K_t(i), V_t(i)} \quad & \phi_t(i) \left[D_{ht}(i) + D^*_{ht}(i) \right] - (r_t + \delta) K_{t-1}(i) \\
& - \sum_{a=0}^{8} (1 + \zeta_{a,t}) \left(w^h_{a,t}(i)\, N^h_{a,t}(i) + w^f_{a,t}(i)\, N^f_{a,t}(i) \right) - a_t\, V_t(i) - FC_t \\
& + \mathrm{mc}_t(i) \{ Y_t(i) - [D_{ht}(i) + D^*_{ht}(i)] \} \\
& + R^{-1}_{t+1}\, W^F_{t+1}(i). \qquad (22)
\end{aligned}
$$

subject to (8), with $p_t = q_t\, V_t / \Omega_t$, and subject to (15) and (18). The first-order optimality conditions are:

$$
\phi_t(i) = \frac{mc_t(i)}{\theta}\,, \qquad (23)
$$

$$
\frac{r_{t+1} + \delta}{R_{t+1}} = mc_t(i) A_t F_{K_t(i)}\,, \qquad (24)
$$

$$
a_t = q_t \sum_{a=0}^{8} \left(\frac{\Omega^h_{a,t}}{\Omega_t} \frac{\partial W^F_t(i)}{\partial N^h_{a,t}} + \frac{\Omega^f_{a,t}}{\Omega_t} \frac{\partial W^F_t(i)}{\partial N^f_{a,t}} \right), \qquad (25)
$$

where $\partial W^F_t(i) / N^x_{a,t}$ is the value at time t of an additional worker of age a from country $x \in \{h, f\}$. With a job destruction rate χ, this is equal to:

$$
\begin{aligned}
\frac{\partial W^F_t(i)}{\partial N^x_{a,t}} &= \sum_{j=0}^{8-a} \frac{\beta^x_{a+j,t+j}}{\beta^x_{a,t}} R^{-1}_{t,t+j} \left(1 - \lambda^x_{a+j-1,t+j-1}\right) \left(1 - \lambda^x_{a+j,t+j}\right) (1 - \chi)^j \\
&\quad \times \left\{ \mathrm{mc}_{t+j}(i)\, \bar{h}_{t+j}\, h^x_{a+j,t+j}\, A_{t+j} F_{H_{t+j}(i)} - (1 + \zeta_{a+j,t+j})\, w^x_{a+j,t+j}(i) \right\}, \qquad (26)
\end{aligned}
$$

where $\lambda^x_{a+j,t+j} = 0$ for $a + j < 7$, $R_{t,t} = 1$ and $R_{t,t+j} = \Pi^j_{k=1} R_{t+k}$ for $j \geq 1$.

[7]Shareholders may belong to different age groups and have different consumption levels. However, they all have the same discount factor given by $R^{-1}_{t+1} = \beta \left(\frac{\beta_{a+1,t+1}}{\beta_{a,t}} \right)^{1-\varpi} \frac{\mathcal{U}'_{c_{a+1,t+1}}}{\mathcal{U}'_{c_{a,t}}} = \left(1 + r_{t+1}(1 - \tau^k_{t+1}) \right)^{-1}$, $\forall\, a \in \{0, 15\}$.

3.6 Wages

Wages are renegotiated in every period. They are determined by a standard Nash bargaining rule:

$$\max_{w^h_{a,t}(i)} \left(\frac{\partial W^F_t(i)}{\partial N^h_{a,t}} \right)^{1-\eta_a} \left(\frac{1}{\mathcal{U}'_{c_{a,t}}} \frac{\partial W^H_t}{\partial N^h_{a,t}} \right)^{\eta_a} .$$

The first-order optimality condition can then be written as:

$$(1-\eta_a)\, \frac{1}{\mathcal{U}'_{c_{a,t}}} \frac{\partial W^H_t}{\partial N^h_{a,t}} = \eta_a \, \frac{1-\tau^w_{a,t}}{(1+\zeta_{a,t})(1+\tau^c_t)} \frac{\partial W^F_t(i)}{\partial N^h_{a,t}} \, . \tag{27}$$

3.7 Aggregation

In equilibrium, all intermediate firms in a country are identical and we may drop the index i in the home country and the index j in the foreign country.

Demands

From (15) and (16), we derive respectively the home demand for home goods and the home demand for foreign goods:

$$D_{ht} = \left(\frac{1}{\omega_1} \, \phi_t \right)^{\frac{1}{\rho-1}} D_t, \tag{28}$$

$$D_{ft} = \left(\frac{1}{\omega_2} \, \gamma_t \, \phi^*_t \right)^{\frac{1}{\rho-1}} D_t. \tag{29}$$

Similarly, from (18), we derive the foreign demand for home goods:

$$X_t = D^*_{ht} = \left(\frac{1}{\omega^*_2} \, \frac{\phi_t}{\gamma_t} \right)^{\frac{1}{\rho-1}} D^*_t. \tag{30}$$

We assume a small open economy setup and we therefore take the price mark-up in the foreign country ϕ^*_t as well as the total demand in the foreign country D^*_t as exogenously given.

Real exchange rate

It may be interesting to derive an alternative expression for the real exchange rate γ_t. By plugging (15) and (16) into (13), and using equations (28) and (29), we finally obtain:

$$\gamma_t = \frac{1}{\phi^*_t} \; {\omega_2}^{\frac{1}{\rho}} \left[1 - \left(\frac{1}{\omega_1} \right)^{\frac{1}{\rho-1}} \phi_t^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} .$$

We immediately see that the real exchange rate γ_t decreases (equivalent to an appreciation of the domestic currency) with the price mark-ups in the domestic and foreign countries. Indeed, both domestic and foreign goods enter as inputs for the production of the final goods. An increase in the prices of inputs decreases the competitiveness of the home economy, which is equivalent to a appreciation of the real exchange rate, that is an decrease in γ_t.

National accounts

Let us define the price of intermediate goods in the home country by P_{ht} and the price of intermediate goods in the foreign country by P^*_{ft}. Intermediate firms in the home country sell their production at a price P_{ht} to domestic agents and the rest of the world:

$$P_{ht}\, Y_t = P_{ht}\, D_{ht} + P_{ht}\, X_t \Longrightarrow Y_t = D_{ht} + X_t\,. \tag{31}$$

The income from selling production at a price P_{ht} is equal to the demand for consumption goods, investment goods, net exports goods and to pay sunk costs, all at a price P_t:

$$\begin{aligned} & P_{ht}\, Y_t = P_t C_t + P_t G_t + P_t I_t + P_t\, NX_t + P_t a_t V_t + P_t\, FC_t \\ \Longrightarrow\quad & \phi_t Y_t - a_t V_t - FC_t = C_t + G_t + I_t + NX_t && (32) \\ \Longrightarrow\quad & GDP_t = C_t + G_t + I_t + NX_t\,. && (33) \end{aligned}$$

Equation (32) is the national account identity and equation (33) defines GDP. Aggregate consumption is $C_t = \sum_a c_{a,t}\, Z^h_{a,t}$ and aggregate investment is:

$$I_t = K_t - (1-\delta)K_{t-1}\,, \tag{34}$$

where K_t represents firms' physical capital. Finally, net exports are:

$$P_t\, NX_t = P_{ht} X_t - e_t P^*_{ft} D_{ft} \quad\Longrightarrow\quad NX_t = \phi_t\; X_t - \gamma_t\, \phi^*_t\; D_{ft}\,. \tag{35}$$

Note that equation (13) combined with the national accounts idendities implies:

$$D_t = C_t + G_t + I_t + aV_t + FC_t\,.$$

Government

We assume that unemployment and (early or legal) retirement benefits are determined by an exogenous fraction of the relevant gross wage, so that:

$$\begin{aligned} b^u_{a,t} &= \rho^u_t\, w^h_{a,t} && \text{for } 0 \le a \le 8\,, \\ b^{e,x}_{a,t} &= \rho^e_t\, w^x_{a,t} && \text{for } 7 \le a \le 8\,, \\ b^{i,x}_{a,t} &= \rho^i_t\, \bar{w}^x_t && \text{for } 9 \le a \le 15\,. \end{aligned}$$

The legal retirement benefit is calculated on the basis of a lifetime average wage $\bar{w}$. Total transfer expenditures are then equal to:

$$T_t = \left[\sum_{a=0}^{8} b^u_{a,t}\, u^h_{a,t}\, z^h_{a,t}\, Z^h_{a,t} \right] + \left[\sum_{x\in\{h,f\}} \sum_{a=7}^{8} b^{e,x}_{a,t}\, e^x_{a,t}\, z^x_{a,t}\, Z^x_{a,t} \right] + \left[\sum_{x\in\{h,f\}} \sum_{a=9}^{15} b^{i,x}_{a,t}\, (1 - z^x_{a,t})\, Z^x_{a,t} \right]. \tag{36}$$

Public consumption is assumed to be a fraction of output, i.e. $G_t = \bar{g}_t\, GDP_t$. Government revenues Γ_t, are defined as:

$$\begin{aligned} \Gamma_t \;=\;& \tau^c_t\, C_t + \sum_x \sum_a \left(\tau^w_{a,t} + \zeta_{a,t}\right) w^x_{a,t}\, n^x_{a,t}\, z^x_{a,t}\, Z^x_{a,t} + \tau^k_t\, r_t \left(\sum_a \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}} \right)^{\varpi} s_{a-1,t+a-1}\, Z^h_{a,t+a} \right) \\ & + (1 + r_t) \left(\sum_a \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}} \right) \left(1 - \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}} \right)^{\varpi - 1} \right) s_{a-1,t+a-1}\, Z^h_{a,t+a} \right), \end{aligned} \tag{37}$$

where the last term represents involuntary income resulting from the imperfect annuity market. When $\varpi = 1$ (perfect annuity market), this last term disappears. If current expenditures are higher than current income, the government has a primary deficit (net borrowing requirements) NBR_t:

$$NBR_t + \Gamma_t = T_t + G_t\,. \tag{38}$$

The primary deficit adds to the existing stock of public debt (liabilities) L_t, along with the interest rate repayments:

$$L_t = (1 + r_t) L_{t-1} + NBR_t\,. \tag{39}$$

International capital market

Total savings in the home economy may be directed to the home economy as physical capital K_t and equities Q_t, or to the rest of the world as net foreign assets NFA_t:[8]

$$K_t + Q_t + NFA_t = \sum_{a=0}^{15} s_{a,t}\, Z_{a,t}\,. \tag{40}$$

Moreover, in our deterministic setup, the return on equities must be equal to the market interest rate. In other words, the value of equities must be such that:

$$\frac{Q_{t+1} + \Pi_{t+1}}{Q_t} = 1 + r_{t+1}\,. \tag{41}$$

Finally, as in Schmitt-Grohe and Uribe (2004), we assume the interest rate that is decreasing in the country's net foreign asset position:

$$r_t = \bar{r} + \xi \left[\exp\left(\overline{nfa} - \frac{NFA_t}{GDP_t} \right) - 1 \right], \tag{42}$$

[8]We assume that domestic debt is owned by the rest of the world and therefore enters – negatively – in the net foreign asset position.

where $\xi > 0$ and $\bar{r}$ is the long run interest rate when the country runs its steady-state net foreign asset position to GDP ($\overline{nfa}$).

Current account and GNP

The current account surplus (or the net capital outflow from the domestic country to foreign) is given by the change in the net foreign asset position,

$$CA_t = NFA_t - NFA_{t-1}. \tag{43}$$

GNP_t is calculated as follows:

$$GNP_t = GDP_t + r_t NFA_t - \sum_a (1 - \tau^w_{a,t}) w^f_{a,t} - T^f_t \,, \tag{44}$$

where transfers to non-resident individuals, T^f_t, are equal to:

$$T^f_t = \left[\sum_{a=7}^{8} b^{e,f}_{a,t} \, e^f_{a,t} \, z^f_{a,t} \, Z^f_{a,t} \right] + \left[\sum_{a=9}^{15} b^{i,f}_{a,t} \, (1 - z^f_{a,t}) \, Z^f_{a,t} \right].$$

Net trade must therefore be equal to:

$$NX_t = CA_t - r_t NFA_t + \sum_a (1 - \tau^w_{a,t}) w^f_{a,t} + T^f_t \,. \tag{45}$$

3.8 Balanced growth path

We remove exogenous growth by scaling all trending variables by the labour augmenting technological progress. Assuming that $\bar{h}_0 = 1$, we transform all endogenous trending variables $v_t = \{c_{a,t},\ s_{a,t},\ I_{a,t},\ w^h_{a,t},\ b^u_{a,t},\ b^{e,x}_{a,t},\ b^{i,x}_{a,t},\ Y_t,\ K_t,\ \Pi_t,\ W^F_t,\ X_t,\ D_{ht},\ D_{ft},\ D_t,\ GDP_t,\ C_t,\ G_t,\ I_t,\ NX_t,\ Q_t,\ T_t,\ \Gamma_t,\ L_t,\ NBR_t,\ NFA_t,\ CA_t,\ GNP_t,\ T^f_t\}$ according to:

$$\tilde{v}_t = \frac{v_t}{\bar{h}_t} = \frac{v_t}{\psi^t}. \tag{46}$$

Moreover, we assume that the vacancy cost a_t, the fixed cost FC_t, the exogenous foreign wage $w^f_{a,t}$ and the exogenous total foreign demand D^*_t are also growing at the same exogenous rate than the labour augmenting technological progress, which allows us to define:

$$\begin{aligned}
\tilde{a} &= a_t / \psi^t \,, && (47)\\
\tilde{FC} &= FC_t / \psi^t \,, && (48)\\
\tilde{w}^f_{a,t} &= w^f_{a,t} / \psi^t \,, && (49)\\
\tilde{D}^*_t &= D^*_t / \psi^t \,. && (50)
\end{aligned}$$

This modifies equations (10), (11), (22), (26), (34), (37), (39), (38), (40), (41) and (43). All transformed equations are displayed in appendix B.

In the simulation section, we produce welfare evaluations of alternative shocks or policies. To eliminate the mechanical effects of the technical progress and longer life-time duration, and assuming a log-utility, we re-scale the – per capita – welfare for a new-born generation in t as follows:

$$\frac{\tilde{W}_t^H}{Z_{0,t}} = \frac{\sum_{a=0}^{15} \beta^a \beta_{a,t+a} \left\{ \ln(\tilde{c}_{a,t+a}) - d^n n_{a,t+a} z_{a,t+a} + d_a^e \frac{(e_{a,t+a})^{1-\phi}}{1-\phi} z_{a,t+a} \right\}}{\sum_{a=0}^{15} \beta^a \beta_{a,t+a}}.$$

4 Calibration

In this section, we explain how we give values to the exogenous variables (vectors) and to the exogenous parameters (scalars). These values are either borrowed from empirical studies or existing projections, or fixed to reflect the economic conditions of Luxembourg in 2010. The model starts from an initial steady state in 1970 and reaches the final steady state in 2300. The size of each vector is equal to the number of periods, that is 67 since one period corresponds to 5 years. Our analysis focuses on the subperiod from 2010 to 2100 within the transitional path.[9] Among the exogenous variables, we have the demographic variables $\{\beta_{a,t}^h, x_t^h, X_{a,t}^h, \beta_{a,t}^f, x_t^f, X_{a,t}^f\}$, the policy variables $\{\tau_t^k, \tau_t^c, \tau_{a,t}^w, \zeta_{a,t}, \rho_t^u, \rho_t^e, \rho_t^i, \bar{g}_t\}$, the productivity variables $\{A_t, h_{a,t}^h, h_{a,t}^f\}$ and the remaining foreign variables $\{\lambda_{a,t}^f, \tilde{w}_{a,t}^f, \tilde{D}_t^*, \phi_t^*\}$. It is worth noting that apart from the demographic variables, most of the other variables are simply vectors with identical elements, at least in the baseline simulations (for instance the capital rate rate τ_t^k is constant from 1970 till 2300). The exogenous parameters are related to the production $\{\delta, \alpha, \theta, \rho, \tilde{FC}, \psi\}$, to the preferences $\{\beta, \phi, d^n, d_7^e, d_8^e, \varpi, \omega_1, \omega_2, \omega_2^*\}$, to the labor market $\{\tilde{a}, \nu, \eta_a, \chi\}$ or to the interest rate $\{\bar{r}, \xi, \overline{nfa}\}$. All these values are detailed hereafter.

Demographic variables. Survival probabilities $\beta_{a,t}^h$ from 1970 to 2100 are constructed from French mortality rates (Vallin and Meslé, 2001) assuming that the Luxembourg mortality rates are not too different from French ones. After 2100, survival probabilities are held constant. Panel a of Figure 1 shows selected mortality rates for the older generations. Population by age classes over the 1970-2050 period is taken from the United Nations (2010), whose projections are close to the ones computed by the STATEC (2010). From 2050 onwards, we assume constant the size of the first generation and population evolves according to mortality rates. Panel c of Figure 1 shows the population level across ages at different periods of time. We observe that Luxembourg's population (aged 20-99) is expected to rise over the whole 21^{st} century. Panel

[9] Starting the simulations in 1970 and ending them in 2300 allows us to isolate the period in which we are interested from the initial and final conditions.

b of Figure 1 shows the evolution of the natality rate (population in the first – yearly – generation divided by the whole population), which is directly computed from the population data. The natality rate relates directly to the x_t^h vector we have in the model. The migration shocks $X_{a,t}^h$ are calibrated as differences between the population by age group projected by the United Nations and the population (by age group) generated by the $\beta_{a,t}^h$. Obviously, from 2050 onwards, migration shocks are zero. Survival probabilities $\beta_{a,t}^f$ associated to the population in the bordering countries are assumed to be identical to the resident population, whereas migration shocks $X_{a,t}^f$ are supposed to be 0. However, the growth rate of the first cohort of bordering population (fertility x_t^f) is calibrated so that our baseline scenario reflects the evolution of the foreign-to-total-employment ratio as given by the medium projection scenario of STATEC (2010) and shown in panel d of Figure 1. We see that the share of cross-border workers in total employment started rising in the 80s attaining 41% in 2010 and stabilizing around 55% from 2060 onwards.

Technology. We assume a constant returns-to-scale Cobb-Douglas production function. The elasticity of output with respect to capital is set to 0.30 and TFP is constant. The depreciation rate of capital is set at 3.5% per quarter. There are fixed costs in production amounting to 1.4% of GDP in 2010. Table 1 summarizes the value of these parameters.

Human capital. Labour augmenting technical progress grows at a yearly 1% implying $\psi = 1.051$ on a 5-year basis. With this value, we match an annual GDP growth of 3.7% in 2010 (Ministry of Finance, 2012). Annual GDP growth is 2.3% over the period 2015-2060, close to the yearly potential output growth of 1.9% over the period 2010-2060 estimated by the Ageing Working Group (European Commission, 2012, Table 1.8). Moreover, wage growth in 2010 amounts to 0.9% close to the 0.8% of annual average real wage growth over the period 2000-2009 (Ministry of Finance, 2012). Moreover, we assume that a worker's efficiency $h_{a,t}^x$ increases with age until 60 and then slightly decreases. We consider similar efficiencies for both resident and cross-border workers. Wages follow broadly the same pattern but they start decreasing one period earlier, that is at age 55, as backed by empirical findings (see, for instance, Kotlikoff and Gokhale, 1992; Johnson and Neumark, 1996; Aubert and Crépon, 2003). The main reason is that older workers have a lower expected job tenure and they generate lower expected profits for the firm.

Preferences. Utility is logarithmic in consumption, so that the wealth and substitution effects of a change in the interest rate cancel each other. There is no bequest motive and the labor disutility parameter d_n is set equal to 0.25, which represents, for different generations, a marginal disutility of employment (divided by the marginal disutility of consumption) of 11% to 24% of the wage income in 2010 (similar values in the final steady state).[10] Parameter ϕ is

[10]This relative value of domestic activity is hard to measure empirically. Using German data, Frick et al. (2011) show that the average wage income advantages from home production is between 30% and 60%, depending on

set to 0.20, implying a Frisch elasticity of about 0.6, in line with estimated values (den Haan and Kaltenbrunner, 2009). The leisure (early retirement) parameters are set at $d_7^e = 0.079$ and $d_8^e = 0.112$, and contribute to reproducing senior activity (see below and Table 2). We fix the annual discount factor to 0.988 to obtain an annual capital-output ratio of 2.50 in 2010, and we assume an imperfect insurance against lifetime uncertainty by setting $\varpi = 0.77$. With these values, individual consumption rises until the age of 85 and then slowly decreases. Finally, θ is set at 0.94 yielding an elasticity of substitution between different varieties of a given category of goods of 16.67 and comparable to $\theta = 1/1.1 = 0.909$ in Gavilán et al. (2011).

Taxes. Data on capital taxation are taken from Bosca et al. (2005).[11] The capital tax rate τ^k equals 5.93%. Data on employer's and employee's wage taxes (ζ and τ^w, respectively) originate from the OECD Tax Database (OECD, 2010b). More precisely, we use averages over the 2000-2009 period of the "Employer SSC" item to compute ζ and of the "Employee SSC" item for τ^w. The employer's wage tax is 11.5%, whereas the employee's wage tax is 12.3%. These two taxes are similar for all generations of workers. Consumption tax rate is fixed at 27.75%.[12] These values allow us to match the average primary government budget deficit 2010-2012 (Ministry of Finance, 2012).

Transfers. Government consumption is a constant fraction $\bar{g} = 16.4\%$ of GDP. *Gross* replacement rates over a five-year unemployment spell are calculated from OECD (2009a, Table 1.6, population-weighted averages). They are set to a value corresponding to 30% of the gross replacement rate in the first year of an unemployment spell and are displayed in Table 1. This value is set above the five year average (the replacement rate is 87% in the first year of unemployment and 8% in years 2 to 5) since the number of unemployed decreases with duration (Brosius, 2011a). The reference wage used to compute pension benefits is an average over the years of activity. At a given replacement rate, our formulation implies that pensions are indexed on current wages. The value for the gross replacement rate ρ_t^i is set to 81.5% in 2010 (OECD, 2009b), while the (gross) replacement rate at age 55-64 (early retirement), ρ^e, is fixed at 36.7%, based on OECD computations (see Duval, 2003, Figure 1).[13]

These values allow us to reproduce the different senior activity (together with the leisure parameters d_7^e and d_8^e). Table 2 shows that activity rate for the group aged 55-64 years in 2010, resulting from these parameter values, is in line with the one calculated from the OECD (2010a) data. Moreover, pension expenditures to GDP are 9.2% of GDP, matching the 9.2% estimated by the European Commission (2012).

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the methodology. However, using a GDP approach, Giannelli et al. (2012) find that Germany has by far the highest value of home production among the 24 EU countries. Our values therefore seem acceptable in light of these results.

[11] See *Cuadro 1* (p.128) of Bosca et al. (2005). Their study belongs to the research line initiated by Mendoza et al. (1994), but improves on the latter by providing data for a larger set of OECD countries.

[12] τ^c must be regarded as more general than a pure consumption tax. For instance, when all firm profits are distributed to households/shareholders, this is also equivalent to a tax on firm profit.

[13] See also Zahlen (2011, Tables 6 and 7) for more detailed numbers.

Table 1: Exogenous variable and parameter values

Production function		**Preferences**	
A_t	13.527	β (quarterly)	0.997
δ (quarterly)	0.035	ϕ	0.2
α	0.30	d^n	0.25
θ	0.94	d^e_7	0.07877
$\tilde{FC}$	48.447	d^e_8	0.11175
		ϖ	0.77
Taxes (in %)			
$\tau^w_{a,t}$	12.30	**Labor market variables**	
$\zeta_{a,t}$	11.50	$\tilde{a}$	44.821
τ^k_t	5.9253	ν	7.803
τ^c_t	27.75	η_a	0.6
		χ (quarterly)	0.02
Transfers (in %)			
$\bar{g}_t$	16.40	**Human capital**	
ρ^u_t	26.89	$h^x_{0,t}$	17.4
ρ^i_t	81.52	$h^x_{1,t}$	18.8
ρ^e_t	36.68	$h^x_{2,t}$	23.2
		$h^x_{3,t}$	25.5
Open Economy		$h^x_{4,t}$	27.3
ρ	0.8	$h^x_{5,t}$	29.1
ω_1	0.93	$h^x_{6,t}$	30.4
ω_2	0.07	$h^x_{7,t}$	30.5
ω_2^*	0.46	$h^x_{8,t}$	27.5
$\tilde{D}^*_t / \tilde{D}_t$	20	ψ	1.0510
ϕ^*_t	0.5045		
$\lambda^f_{7,t}$	$\lambda^h_{7,t}$	**Interest rate**	
$\lambda^f_{8,t}$	$\lambda^h_{8,t}$	$\bar{r}$	0.276
$\tilde{w}^f_{a,t}$	$\tilde{w}^h_{a,t}$	ξ	0.5
		$\overline{nfa}$	0.253

Subscript t represents an exogenous variable (vector) with identical elements (time invariant). Subscript a means that the exogenous variable or parameter has the same value for each generation $a \in \{0, 1, 2, ..., 8\}$. Superscript x means that the exogenous variable has the same value for $x \in \{h, f\}$, that is for the resident and the cross-border workers. $\lambda^f_{7,t}$ $\lambda^f_{8,t}$ and $\tilde{w}^f_{a,t}$ are simply fully indexed on the corresponding home endogenous variables. The exogenous demographic variables $\{\beta^h_{a,t}, x^h_t, X^h_{a,t}, \beta^f_{a,t}, x^f_t, X^f_{a,t}\}$ are shown or explained in Figure 1.

Labor market parameters. Following den Haan et al. (2000), we adopt the following constant

Table 2: Data match given parameter settings

Variable	Data	Model	Variable	Data	Model
Activity Rate (55-64)	53	52	Unemployment Rate	5.5	5.5
Pension Exp. (% GDP)	9.2	9.2	Cons. (% GDP)	37	37
Prim. Budg. Def. (% GDP)	0.4	0.4	Public Cons. (% GDP)	16	16
Public Debt (% GDP)	19	19	Investment (% GDP)	23	29
NFA (% GDP)	92	94	Net Trade (% GDP)	24	18

Data refer to 2010 and numbers are in percentages. Sources: BCL and Statec.

returns-to-scale matching function:

$$M(V_t, \Omega_t) = \frac{V_t \, \Omega_t}{(V_t^\nu + \Omega_t^\nu)^{\frac{1}{\nu}}}. \tag{51}$$

The major advantage of this approach, compared with the standard Cobb-Douglas specification used in the literature is that it guarantees matching probabilities between zero and one for all Ω_t and V_t ($0 < p_t, q_t < 1$).[14] In contrast, RBC models, which study the effects of (smaller) shocks in the short term, tend to use the Cobb-Douglas specification. However, function (51) is more appropriate in our case, where labor markets are subject to large demographic changes over a longer period.

Job destruction rates are difficult to find for Luxembourg. Brosius (2011b) uses Social Security data for Luxembourg and finds quarterly job *separation* rates of 2 to 2.5% over the period 2009-2010. Bassanini and Marianna (2009, Figure 4) report an average job destruction rate of about 8% *per annum* in some European countries (Germany, Finland and Sweden). In their model applied to the euro area, Christoffel et al. (2009) use a *quarterly* rate of 6 per cent. We fix the quarterly job destruction rate χ at 2%.

The bargaining power of workers η_a is set to the value of 0.6 for all generations, within the range of usually estimated values. Vacancy costs $\tilde{a}$ and the parameter of the matching function ν are used to reproduce unemployment rates of workers ages 20-64 in 2010 (own calculations based on data from the OECD, 2010a). These parameter values yield a steady-state probability of filling a vacancy (over a five-year period) of 94% and a probability of finding a job of 88%.

Finally, we assume that inactivity decisions and wages of the cross-border workers are similar to those of the resident workers, that is $\lambda^f_{7,t} = \lambda^h_{7,t}$, $\lambda^f_{8,t} = \lambda^h_{8,t}$ and $\tilde{w}^f_{a,t} = \tilde{w}^h_{a,t}$.

Open Economy. Like Gavilán et al. (2011), we follow Adolfson et al. (2007) and fix $\rho = 0.8$ so

[14] Function (51) reflects the following matching procedure. Its denominator ($\equiv J_t$) represents the number of channels through which matches occur at each period. A firm and a worker assigned (randomly) to the same channel are successfully matched, otherwise agents remain unmatched. A worker locates a firm with probability V_t/J_t, a firm locates a worker with probability Ω_t/J_t, and the total mass of matches is $V_t\Omega_t/J_t$ (den Haan et al., 2000, p.485).

to obtain an elasticity of substitution between home and foreign goods of 5. As a comparison, Deák et al. (2012) choose $\rho = 1/1.2 = 0.833$.[15] We fix ϕ^* at 0.5045 to obtain $\gamma = 1$ at the initial steady state, and we assume that $\tilde{D}^*$ is such that foreign to domestic demand is D^*/D is 20 in 2010, which corresponds to the population ratio of Luxembourg to the rest of the Grande Région (OIE, 2009).[16] We assume $\omega_2 = 1 - \omega_1$ and the preference parameters ω_1 and ω_2^* are set to match net-trade-to-GDP and export-to-GDP ratios.[17]

Interest rate. We suppose an annual real interest rate of 5% at the initial steady state, implying a 5-year interest rate $\bar{r} = 0.27628$. We set $\xi = 0.5$ which means that risk premia depends negatively on the foreign assets to GDP ratio. $\overline{nfa}$ is normalized such that there is no premium at the initial steady state.

Implied values. Table 2 and Figure 2 show the implied values in 2010 for selected variables. When possible, we compare these values to real Luxembourg data.

5 Simulations

First, we show the main macroeconomic implications of the aging process and check if imbalances are expected to develop. The only forces driving the model are the 6 exogenous demographic shocks explained in the previous section. Second, as we showed that fiscal imbalances will happen, we assess the efficiency of different fiscal reforms.

5.1 Aging and imbalances

Figure 3 shows the expected evolution of the Luxembourg economy. Between 2010 and 2100, we observe a continuous decrease in the growth rates of GDP and employment, a stable unemployment rate between 5% and 6% and a strong deterioration in public finances.

As regards to the labour market, the expected decrease in the inflow of immigrant and cross-border workers as well as the lower fertility rate will strongly depress the labor supply. The reduction in the labor supply will affect total employment by decreasing its growth rate from 2% in 2010-2015 to less than 1% from 2040 onwards. The unemployment rate will remain relatively stable as the slowdown in total employment growth is compensated by more people leaving for retirement due to population aging. Concerning GDP, the reduction in employment growth induces a slowdown in GDP growth, which reaches an annual rate of 1.3% in 2100. The slowdown affects all GDP components although consumption resists. The resilience

[15] For a discussion on the values of ρ, see (Lane, 2011, p. 245).

[16] The *Grande Région* or *Greater Region* is an area composed by the following regions/countries: Saarland, Lorraine, Luxembourg, Rhineland-Palatinate and Wallonia.

[17] Since we model a small open economy rather than a 2-country model, equation (17), defining the foreign demand of foreign goods, is useless and there is therefore no need to give a value to the parameter ω_1^*.

Table 3: The different measures of the pension reform proposal

Measure 1	Gradual reduction in the replacement rate up to 8.6% by 2050 (and beyond)
Measure 2	Partial disadjustment (by 50%) of pension incomes on wages from 2030 onwards
Measure 3a	Increase by 2 percentage points in employees' wage taxes from 2030 onwards
Measure 3b	Increase by 2 percentage points in employers' wage taxes from 2030 onwards

of consumption comes from the stable retirement income, financed by the government, of the ever growing older population. Put it differently, public debt finances private consumption. It can be noticed that reduced GDP growth should depress wage growth. However, wage growth is also sustained by the reduced labor supply. These two effects cancel each other out and wage growth remains basically constant. Finally, positive net trade strengthens the net foreign asset position which implies a lower interest rate. In turn, this lower interest rate (by decreasing marginal cost and the price mark-up) improves competitiveness and increases the real exchange rate (equivalent to a depreciation of the domestic currency).

So far so good. However, as already said, population aging combined with the slowdown in employment growth will make the financing of public pensions very difficult and severely deteriorate public finances. Without policy reforms, the primary public deficit will pass from 0% to 20% in 2100.

5.2 Fiscal reforms

To try to remove the development of this fiscal imbalance, we now explore the effects of policy changes such as a pension reform. In Luxembourg, no major structural pension reform has been implemented yet. However, the government has recently submitted a reform proposal comprising a variety of measures: a gradual reduction in the replacement rate ("Measure 1"), a partial disadjustment of pensions ("Measure 2") and increases in employees' ("Measure 3a") and employers' wage taxes ("Measure 3b"), see Table 3. It is assumed that the pension reform is implemented in 2015. Figure 4 shows the effects of different combinations of these various measures.

The major measure of the pension reform is the gradual reduction in the replacement rate ("Measure 1"). The measure is supposed to start in 2013 and reach a 8.6% cut over a 40 year period. The cut in the replacement rate reduces pension expenditures but encourages also senior workers to remain longer in activity. This measure stimulates also savings and asset accumulation (not shown) leading to a reduction in unemployment. Employment and GDP rise, while public finances improve. All in all, pension expenditures are decreased by 1.74 percentage points (pp) in 2050 and 2.52pp in 2100 compared to the baseline without reforms (not shown), implying an almost identical improvement in the primary budget (-2.28pp in 2100 compared to the baseline).

Scenario "Measure 1+2" combines the first measure with a partial adjustment of only 50% of pension incomes on real wages. The reform proposal states that the pension disadjustment is implemented when the finances of pension systems deteriorate, i.e. when pension expenditures become larger than pension contributions.[18] This situation arises in 2030 in our "Measure 1" scenario. Given a real wage annual growth of about 1%, "Measure 2" implies that pensions (of newly retirees) augment by 0.5%.[19] Scenario "Measure 1+2" induces a reduction in the (average) replacement rate of 4.9% in 2030 and 13.3% in 2050 and beyond. This scenario has similar but larger effects than scenario "Measure 1" since this latter scenario induces a smaller reduction in the (average) replacement rate (4.3% in 2030 and 8.6% in 2050 and beyond). The primary budget deficit decreases by up to 3.52pp in 2100.

Scenario "Measure 1+2+3a" adds an increase in employees' wage taxes to the two previous measures. As "Measure 2", "Measure 3a" should be implemented only when the financing of pension systems deteriorates, i.e. in 2030. The primary budget deficit decreases slightly more than in the previous scenario (by 4.12pp in 2100). The increment in the deficit reduction is modest because higher employees' wage taxes have several negative side-effects. First, they induce workers to bargain a higher gross wage, which raises labor costs and thus unemployment. Second, they depress net wages and reduce workers' incentives to stay longer on their job. Third, a higher gross wage raises pension expenditures since pension benefits are indexed on gross wages.

Scenario "Measure 1+2+3b" combines a rise in employers' wage taxes with "Measure 1" and "Measure 2". This measure is also implemented when pension expenditures become larger than contributions to pension systems. Employer tax hikes raise labor costs and depress bargained wages. They have a small consequence on unemployment and inactivity rates com-

[18]Pension contributions are set to a percentage of all tax revenues. The percentage is calibrated to match the prmary deficit of pension systems in 2012, equal to 1.5% of GDP (i.e. a surplus).

[19]It can be noticed that the disadjustment concerns only pensions of newly retirees. Moreover, the partial indexation needs to be maintained until 2140 in order for pension expenditures to become again smaller than pension contributions.

pared to the scenario "Measure 1+2". However, they lead to a larger primary deficit reduction, of up to -4.64pp by 2100.

In conclusion, we see that a lower replacement ratio combined with higher taxes can solve 23% of the deficit problem ("Measure 1+2+3b").[20] To solve the remaining 77%, we would need – unpopular – deeper changes. This underlines the need of closely monitoring the expected economic evolutions, and of reacting at the earliest possible stage to any imbalance development. This also underlines the need of building adequate models to analyze the evolution of the economy and to evaluate the impact of reforms and alternative policies.

6 Conclusions

In this paper we extend the first version of LOLA by improving the calibration, developing the public finance block, adding one more – young – generation and introducing current account and exchange rate dynamics through the NOEM. We then show the interest of this kind of model for policy purposes.

Again, this is not a definitive model. In the future, we would like to reduce the periodicity of the model from 5 years to one year. Moreover, to better make the distinction between services (including financial services) and manufacturing, we will move from a one-sector to a two-sector production function. Finally, as already explained, it is difficult to derive foreign households' behaviour from first order conditions. But we could instead introduce reduced-form equations rather than the exogenous behaviour we have currently.

[20]Combining all the measures, i.e. scenario "Measure 1+2+3a+3b" (not shown), could solve up to 26% of the deficit problem and would lead to the largest primary budget deficit reduction, by up to 5.23pp in 2100. However, as explained above, the inclusion of "Measure 3a" implies that scenario "Measure 1+2+3a+3b" has less favorable effects on unemployment and senior activity rates than scenario "Measure 1+2+3b" or even scenario "Measure 1+2".

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Appendix A: Household Optimization Problem

With initial and final financial wealth equal to zero (no bequests), the household's intertemporal budget constraint can be written as follows:

$$\sum_{a=0}^{15} R_{t,t+a}^{-1}\,\beta_{a,t+a}^{\varpi}\Big\{\Big[(1-\tau_{a,t+a}^{w})\,w_{a,t+a}\,n_{a,t+a}+b_{a,t+a}^{u}\,u_{a,t+a}+b_{a,t+a}^{e}\,e_{a,t+a}\Big]\,z_{a,t+a} \\ +b_{a,t+a}^{i}\,(1-z_{a,t+a})-(1+\tau_{t+a}^{c})\,c_{a,t+a}\Big\}=0\,. \tag{52}$$

The discount factor $R_{t,t+a}$ is defined by $R_{t,t}=1$ and $R_{t,t+a}=\Pi_{j=1}^{a}R_{t+j}$ for $a\geq 1$, with $R_{t+j}=1+r_{t+j}(1-\tau_{t+j}^{k})$. The values of $c_{a,t+a}$, $\lambda_{7,t+7}$ and $\lambda_{8,t+8}$ maximizing the household objective function (9) subject to (5) and (8) and the intertemporal budget constraint (52) can thus be obtained from the maximization of the following Lagrangean function:

$$\frac{W_t^H}{Z_{0,t}}=\max_{c_{a,t+a},\lambda_{7,t+7},\lambda_{8,t+8}}\sum_{a=0}^{14}\beta_{a,t+a}\Big\{\beta^{a}\Big(\mathcal{U}(c_{a,t+a})-d^{n}\,n_{a,t+a}\,.\,z_{a,t+a}+d_a^{e}\,\frac{(e_{a,t+a})^{1-\phi}}{1-\phi}\,z_{a,t+a}\Big) \\ +\mu_t\beta_{a,t+a}^{\varpi-1}\,R_{t,t+a}^{-1}\Big(\Big[b_{a,t+a}^{u}+\Big((1-\tau_{a,t+a}^{w})\,w_{a,t+a}-b_{a,t+a}^{u}\Big)\,n_{a,t+a}+(b_{a,t+a}^{e}-b_{a,t+a}^{u})\,e_{a,t+a}\Big]\,.\,z_{a,t+a} \\ +\,b_{a,t+a}^{i}\,(1-z_{a,t+a})-(1+\tau_{t+a}^{c})\,c_{a,t+a}\Big)\Big\}\,,$$

where μ_t is the Lagrange multiplier associated to the intertemporal budget constraint. The optimal values of $c_{a,t+a}$, $\lambda_{7,t+7}$ and $\lambda_{8,t+8}$ must satisfy the following first-order optimality conditions:

$$\beta^{a}\,\frac{\mathcal{U}'_{c_{a,t+a}}}{1+\tau_{t+a}^{c}}=\mu_t\,\beta_{a,t+a}^{\varpi-1}\,R_{t,t+a}^{-1}\,, \tag{53}$$

$$\Big[d^{n}\,\frac{\partial n_{7,t+7}}{\partial\lambda_{7,t+7}}-d_7^{e}\,(e_{7,t+7})^{-\phi}\,\frac{\partial e_{7,t+7}}{\partial\lambda_{7,t+7}}\Big]+\beta\,\frac{\beta_{8,t+8}}{\beta_{7,t+7}}\Big[d^{n}\,\frac{\partial n_{8,t+8}}{\partial\lambda_{7,t+7}}-d_8^{e}\,(e_{8,t+8})^{-\phi}\,\frac{\partial e_{8,t+8}}{\partial\lambda_{7,t+7}}\Big] \\ =\frac{\mathcal{U}'_{c_{7,t+7}}}{1+\tau_{t+7}^{c}}\Big[\Big((1-\tau_{7,t+7}^{w})\,w_{7,t+7}-b_{7,t+7}^{u}\Big)\,\frac{\partial n_{7,t+7}}{\partial\lambda_{7,t+7}}+\Big(b_{7,t+7}^{e}-b_{7,t+7}^{u}\Big)\,\frac{\partial e_{7,t+7}}{\partial\lambda_{7,t+7}}\Big] \\ +\beta\,\frac{\beta_{8,t+8}}{\beta_{7,t+7}}\,\frac{\mathcal{U}'_{c_{8,t+8}}}{1+\tau_{t+8}^{c}}\Big[\Big((1-\tau_{8,t+8}^{w})\,w_{8,t+8}-b_{8,t+8}^{u}\Big)\,\frac{\partial n_{8,t+8}}{\partial\lambda_{7,t+7}}+\Big(b_{8,t+8}^{e}-b_{8,t+8}^{u}\Big)\,\frac{\partial e_{8,t+8}}{\partial\lambda_{7,t+7}}\Big]\,, \tag{54}$$

$$\Big[d^{n}\,\frac{\partial n_{8,t+8}}{\partial\lambda_{8,t+8}}-d_8^{e}\,(e_{8,t+8})^{-\phi}\,\frac{\partial e_{8,t+8}}{\partial\lambda_{8,t+8}}\Big] \\ =\frac{\mathcal{U}'_{c_{8,t+8}}}{1+\tau_{t+8}^{c}}\Big[\Big((1-\tau_{8,t+8}^{w})\,w_{8,t+8}-b_{8,t+8}^{u}\Big)\,\frac{\partial n_{8,t+8}}{\partial\lambda_{8,t+8}}+\Big(b_{8,t+8}^{e}-b_{8,t+8}^{u}\Big)\,\frac{\partial e_{8,t+8}}{\partial\lambda_{8,t+8}}\Big]\,. \tag{55}$$

In these expressions,

$$\frac{\partial e_{7,t+7}}{\partial \lambda_{7,t+7}} = 1\,, \qquad \frac{\partial e_{8,t+8}}{\partial \lambda_{7,t+7}} = (1-\lambda_{8,t+8})\,, \qquad \frac{\partial e_{8,t+8}}{\partial \lambda_{8,t+8}} = (1-\lambda_{7,t+7})\,,$$
$$\frac{\partial n_{7,t+7}}{\partial \lambda_{7,t+7}} = -\frac{n_{t+7}}{1-\lambda_{t+7}}\,, \qquad \frac{\partial n_{8,t+8}}{\partial \lambda_{7,t+7}} = -\frac{n_{t+8}}{1-\lambda_{t+7}}\,, \qquad \frac{\partial n_{8,t+8}}{\partial \lambda_{8,t+8}} = -\frac{n_{t+8}}{1-\lambda_{t+8}}\,.$$

The first optimality condition (11) is the usual Euler condition. It implies:

$$\frac{u'_{c_{a,t+a}}}{1+\tau^c_{t+a}} = \beta \left(\frac{\beta_{a+1,t+a+1}}{\beta_{a,t+a}}\right)^{1-\varpi} R_{t+a+1}\, \frac{u'_{c_{a+1,t+a+1}}}{1+\tau^c_{t+a+1}}\,.$$

The other two optimality conditions are specific to this model and determine the activity rate of senior workers. After substitution and rearrangements (where we also use (5)) and with the assumption that $\mathcal{U}(c_{a,t+a})$ is logarithmic, these optimality conditions can be recast as follows:

$$\begin{aligned}
&\left[\frac{b^e_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} + d^e_7\,\left(e_{7,t+7}\right)^{-\phi}\right](1-e_{7,t+7}) \\
&\qquad + \beta\,\frac{\beta_{8,t+8}}{\beta_{7,t+7}}\left[\frac{b^e_{8,t+8}-b^u_{8,t+8}}{(1+\tau^c_{t+8})\,c_{8,t+8}} + d^e_8\,\left(e_{8,t+8}\right)^{-\phi}\right](1-e_{8,t+8}) \\
&\qquad = \left[\frac{(1-\tau^w_{7,t+7})\,w_{7,t+7}-b^u_{7,t+7}}{(1+\tau^c_{t+7})\,c_{7,t+7}} - d^n\right] n_{7,t+7} \\
&\qquad + \beta\,\frac{\beta_{8,t+8}}{\beta_{7,t+7}}\left[\frac{(1-\tau^w_{8,t+8})\,w_{8,t+8}-b^u_{8,t+8}}{(1+\tau^c_{t+8})\,c_{8,t+8}} - d^n\right] n_{8,t+8}\,,
\end{aligned} \tag{56}$$

and

$$\left[\frac{b^e_{8,t+8}-b^u_{8,t+8}}{(1+\tau^c_{t+8})c_{8,t+8}} + d^e_8\,\left(e_{8,t+8}\right)^{-\phi}\right](1-e_{8,t+8}) = \left[\frac{(1-\tau^w_{8,t+8})w_{8,t+8}-b^u_{8,t+8}}{(1+\tau^c_{t+8})c_{8,t+8}} - d^n\right] n_{8,t+8}\,. \tag{57}$$

The economic interpretation of these optimality conditions becomes easier if we notice that the unconditional probability of having a job is given by:

$$\pi_{a,t+a} = \frac{n_{a,t+a}}{n_{a,t+a}+u_{a,t+a}} = \frac{n_{a,t+a}}{1-e_{a,t+a}}\,,$$

so that the last optimality condition for instance can be written as follows:

$$\frac{b^e_{8,t+8}}{(1+\tau^c_{t+8})\,c_{8,t+8}} + d^e_8\,\left(e_{8,t+8}\right)^{-\phi} = \pi_{8,t+8}\left[\frac{(1-\tau^w_{8,t+8})\,w_{8,t+8}}{(1+\tau^c_{t+8})\,c_{8,t+8}} - d^n\right] + (1-\pi_{8,t+8})\left[\frac{b^u_{8,t+8}}{(1+\tau^c_{t+8})\,c_{8,t+8}}\right],$$

and similarly for the other optimality condition.

Appendix B: Balanced growth path

Equations (10), (11), (21), (22), (26), (34), (37), (39), (41) and (43) become respectively:[21]

$$I_{a,t+a} + \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}}\right)^{\varpi} [1 + r_{t+a}(1 - \tau^k_{t+a})] \cdot \frac{s_{a-1,t+a-1}}{\psi} = (1 + \tau^c_{t+a})c_{a,t+a} + s_{a,t+a}\,,$$

$$\frac{\psi\, \mathcal{U}'_{c_{a,t+a}}}{1 + \tau^c_{t+a}} = \beta \left(\frac{\beta_{a+1,t+a+1}}{\beta_{a,t+a}}\right)^{1-\varpi} [1 + r_{t+a+1}(1 - \tau^k_{t+a+1})]\, \frac{\mathcal{U}'_{c_{a+1,t+a+1}}}{1 + \tau^c_{t+a+1}}\,,$$

$$\begin{aligned}\Pi_t(i) \quad = \quad & \phi_t(i) Y_t(i) - \frac{(r_t + \delta)\, K_{t-1}(i)}{\psi} - \sum_{a=0}^{8} (1 + \zeta_{a,t}) \left(w^h_{a,t}(i)\, N^h_{a,t}(i) + w^f_{a,t}(i)\, N^f_{a,t}(i) \right) \\ & - a_t\, V_t(i) - FC_t\,,\end{aligned}$$

$$\begin{aligned}W^F_t(i) = \max_{\phi_t(i), K_t(i), V_t(i)} \quad & \phi_t(i)\, [D_{ht}(i) + D^*_{ht}(i)] - \frac{(r_t + \delta)\, K_{t-1}(i)}{\psi} \\ & - \sum_{a=0}^{8} (1 + \zeta_{a,t}) \left(w^h_{a,t}(i)\, N^h_{a,t}(i) + w^f_{a,t}(i)\, N^f_{a,t}(i) \right) - a_t\, V_t(i) - FC_t \\ & + \mathrm{mc}_t(i) \{ Y_t(i) - [D_{ht}(i) + D^*_{ht}(i)] \} \\ & + R^{-1}_{t+1}\, \psi\, W^F_{t+1}(i)\,,\end{aligned}$$

$$\begin{aligned}\frac{\partial W^F_t(i)}{\partial N^x_{a,t}} \quad = \quad & \sum_{j=0}^{8-a} \frac{\beta^x_{a+j,t+j}}{\beta^x_{a,t}}\, R^{-1}_{t,t+j}\, (1 - \lambda^x_{a+j-1,t+j-1})\, (1 - \lambda^x_{a+j,t+j})\, (1 - \chi)^j \\ & \times \psi^j \left\{ \mathrm{mc}_{t+j}(i)\, \bar{h}_{t+j}\, h^x_{a+j,t+j}\, A_{t+j} F_{H_{t+j}(i)} - (1 + \zeta_{a+j,t+j})\, w^x_{a+j,t+j}(i) \right\},\end{aligned}$$

$$I_t = K_t - \frac{(1 - \delta) K_{t-1}}{\psi}\,,$$

$$\begin{aligned}\Gamma_t \quad = \quad & \tau^c_t\, C_t + \sum_x \sum_a (\tau^w_{a,t} + \zeta_{a,t})\, w^x_{a,t}\, n^x_{a,t}\, z^x_{a,t}\, Z^x_{a,t} + \tau^k_t\, r_t \left(\sum_a \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}}\right)^{\varpi} \frac{s_{a-1,t+a-1}}{\psi}\, Z^h_{a,t+a} \right) \\ & + (1 + r_t) \left(\sum_a \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}}\right) \left(1 - \left(\frac{\beta_{a-1,t+a-1}}{\beta_{a,t+a}}\right)^{\varpi-1} \right) \frac{s_{a-1,t+a-1}}{\psi}\, Z^h_{a,t+a} \right),\end{aligned}$$

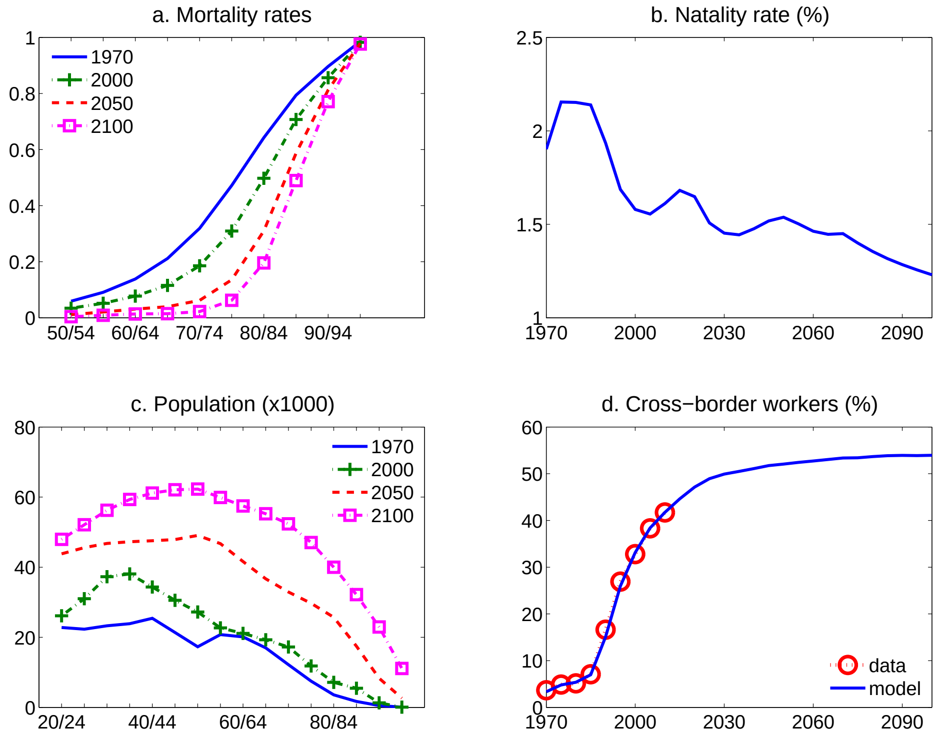
$$L_t = (1 + r_t)\, \frac{L_{t-1}}{\psi} + NBR_t\,,$$

[21] To avoid a too complex notation, we voluntary omit the tilde above all detrended variables.

$$\frac{Q_{t+1} + \Pi_{t+1}}{Q_t} = \frac{1 + r_{t+1}}{\psi},$$

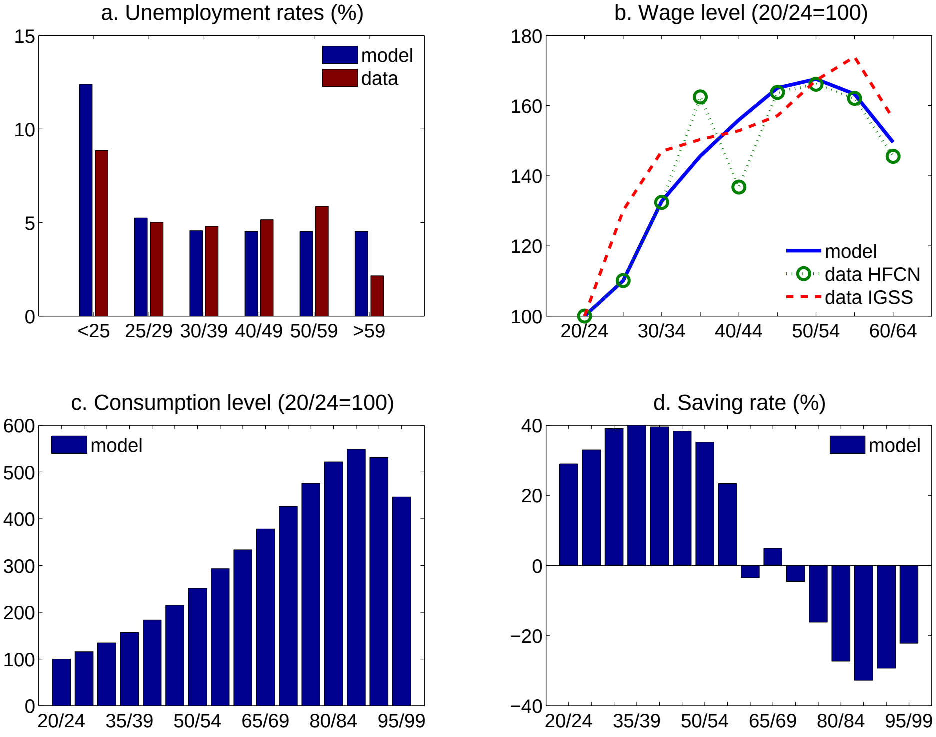
$$CA_t = NFA_t - \frac{NFA_{t-1}}{\psi}.$$

Figure 1: Exogenous demographic variables



Panel a shows the mortality rates (probability to survive to the next period) for the older generations and for selected periods. They are use to compute $\beta^h_{a,t}$ and we simply assume that $\beta^h_{a,t} = \beta^f_{a,t}$. Panel b shows the natality rate (ratio between births and total population) which is used to compute x^h_t. We compute migration flows $X^h_{a,t}$ to reproduce the population per age displayed in panel c. We assume no migration in the bordering countries, that is $X^f_{a,t} = 0$. We compute the foreign fertility rate x^f_t to reproduce the share of cross border commuters shown in panel d.

Figure 2: Selected variables by age given parameter settings, in 2010



Data refer to 2010 when available, and to the most recent year when 2010 not yet available. *Panel a*: Data from Statec. *Panel b*: the series 'data HFCN' is computed by Mathä et al. (2012) based on data from the Luxembourg household finance and consumption survey (HFCN) and the series 'data IGSS' is calculated by Lünnemann and Wintr (2009) who use data from the Inspection Générale de la Sécurité Sociale (IGSS).

Figure 3: Effects of aging

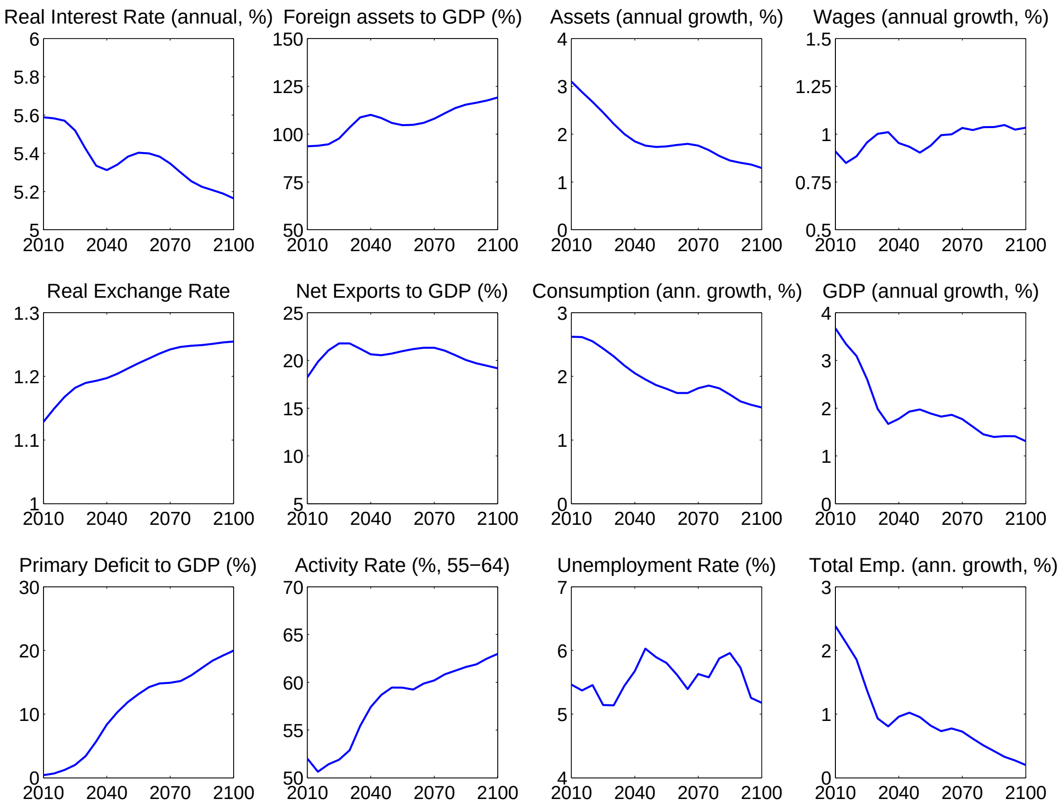
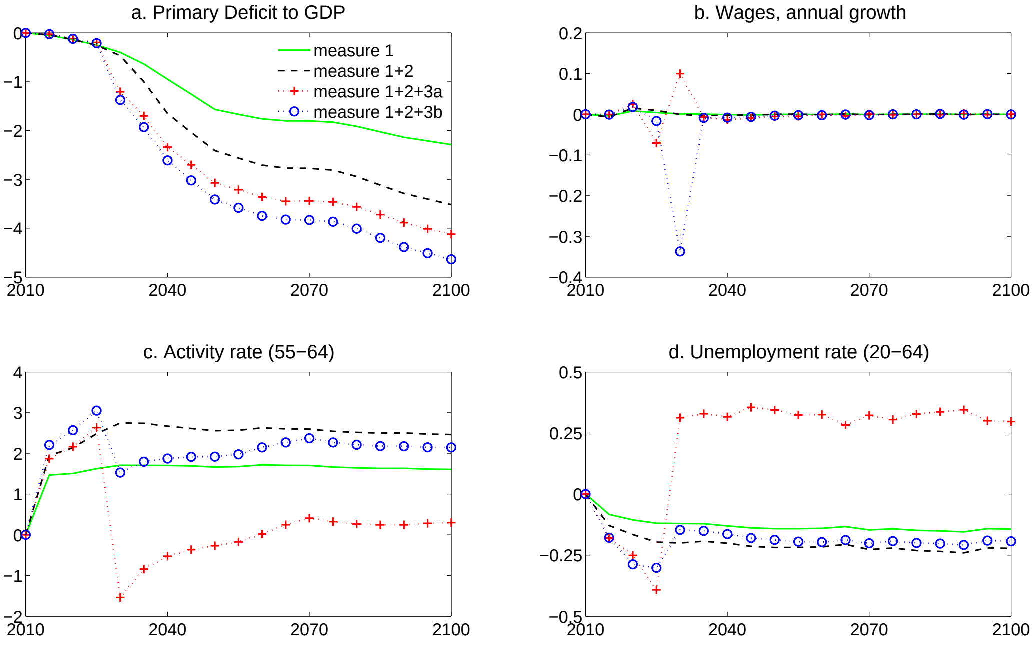


Figure 4: Effects of a pension reform



Percentage point changes (i.e. absolute changes) with respect to the simulations without reforms shown in figure 3.

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