2. AN MVAR FRAMEWORK TO CAPTURE EXTREME EVENTS IN MACRO-PRUDENTIAL STRESS TESTS

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1. INTRODUCTION

In the period following the financial crisis, the use of stress testing to assess the effect of adverse economic shocks on bank capitalization levels has become widespread. These tests have become a permanent fixture in the toolbox of regulatory authorities. However, reduced form implementations of these tests tend to be based on the underlying assumption that the residuals behave according to a univariate Gaussian distribution. Indeed, many of these models are formulated within the context of a classical vector autoregressive (VAR) framework. Although this assumption renders the model tractable, it fails to capture the observed frequency of distant tail events that represent the hallmark of systemic financial stress. Consequently, it seems apparent that these kinds of macro models tend to underestimate the actual level of credit risk. The omission of tail events also leads to an inaccurate assessment of the degree of systemic risk inherent in the financial sector. Clearly this may have significant implications for macro-prudential policy makers. One possible way to overcome such a limitation is to introduce a mixture of distributions model in order to better capture the potential for extreme events.

Based on the methodology developed by Fong, Li, Yau and Wong (2007), we have incorporated a macroeconomic model based on a mixture vector autoregression (MVAR) into the stress testing framework of Rouabah and Theal (2010) that is used at the Banque centrale du Luxembourg. This allows the counterparty credit risk model to better capture extreme tail events in comparison to models based on assuming normality of the distributions underlying the macro models. We believe this approach facilitates a more accurate assessment of credit risk.

The financial crisis that began in 2008 highlighted not only the poor risk-management practices implemented by the financial sector, it also illustrated weaknesses in financial regulatory and oversight frameworks. In particular, three major post-crisis lessons emerged. First, analysing financial stability requires a system-wide perspective rather than a strict micro-prudential approach. Second, there is an important link between macroeconomic conditions and financial stability that, prior to the crisis, was poorly understood and inadequately monitored. Third, statistical models of the linkages between the financial system and the real economy may break down in the face of extreme events. To address these three challenges, this paper applies a mixture vector autoregression (MVAR) in the context of macroeconomic stress tests in an attempt to illustrate the inadequacy of commonly employed VAR models. In forward-looking simulations, the MVAR model can provide multi-modal distributions for counterparty risk in the banking sector, reflecting the possible asymmetries and non-linearities that may manifest in the linkages between macroeconomic developments and financial stability.

In this study, we use the MVAR framework to extend previous work by Rouabah and Theal (2010) evaluating aggregate credit risk for Luxembourg's banking sector. We compare stress-test results based on a mixture of normals (MVAR) model to those obtained with a standard linear VAR. We also calculate Basel II tier 1 capital ratios under the MVAR framework and compare these to the values obtained from the standard linear VAR model.

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2. THE MVAR MODEL: A TOOL TO CAPTURE EXTREME EVENTS

Fong et al. (2007) develop the MVAR model as a multivariate extension of the mixture autoregression model in Wong and Li (2000). An $MVAR(n,K;p_k)$ model with K components for an observed n-dimensional vector Y_t takes the following form:

$$F\left(y_t|\Im_{t-1}\right)=\sum_{k=1}^{K}\alpha_k\Phi\left(\Omega_k^{-1/2}\left(Y_t-\Theta_{k0}-\Theta_{k1}Y_{t-1}-\Theta_{k2}Y_{t-2}-\ldots-\Theta_{k1p}Y_{t-p_k}\right)\right) \quad (1)$$

Where y_t is the conditional expectation of Y_t, p_k is the autoregressive lag order of the k^{th} component, $\Im_{t-1}$ is the available information set up to time $t-1$, $\Phi(\cdot)$ is the cumulative distribution function of the multivariate Gaussian distribution, α_k is the mixing weight of the k^{th} component distribution, Θ_{k0} is an n-dimensional vector of constant coefficients and $\Theta_{k1},\ldots,\Theta_{kp_k}$ are the $n\times n$ autoregressive coefficient matrices of the k^{th} component distribution. Lastly, Ω_k is the $n\times n$ variance-covariance matrix of the k^{th} component distribution. One convenient characteristic of the MVAR is that individual components of the MVAR can be non-stationary while the entire MVAR model remains stationary.

It is possible to estimate the parameters of the MVAR using the expectation-maximization (EM) algorithm of Dempster et al. (1977). This assumes a vector of (generally) unobserved variables $Z_t=\left(Z_{t,1},\ldots,Z_{t,K}\right)^{\mathrm{T}}$ defined as:

$$Z_{t,i}=\begin{cases}1 & \text{if } Y_t \text{ comes from the } i^{th} \text{ component}; \quad 1\le i\le K,\\ 0 & \text{otherwise}\end{cases} \quad (2)$$

Where the conditional expectation of the binary indicator $Z_{t,i}$ gives the probability that an observation originates (or does not originate) from the i^{th} component of the mixture. As shown by Fong et al. (2007), the conditional log-likelihood function of the MVAR model can subsequently be written as follows:

$$l=\sum_{t=p+1}^{T}\left\{\sum_{k=1}^{K}Z_{t,k}\log\left(\alpha_k\right)-\frac{1}{2}\sum_{k=1}^{K}Z_{t,k}\log\left|\Omega_k\right|-\frac{1}{2}\sum_{k=1}^{K}Z_{t,k}\left(e_{kt}^{\mathrm{T}}\Omega_k^{-1}e_{kt}\right)\right\} \quad (3)$$

Where the following variable definitions apply:

$$\begin{aligned}e_{kt}&=Y_t-\Theta_{k0}-\Theta_{k1}Y_{t-1}-\Theta_{k2}Y_{t-2}-\ldots-\Theta_{kp_k}Y_{t-p_k}\\&=Y_t-\tilde{\Theta}_kX_{kt}\\\tilde{\Theta}_k&=\left[\Theta_{k0},\Theta_{k1},\ldots,\Theta_{kp_k}\right]\\X_{kt}&=\left(1,Y_{t-1}^{\mathrm{T}},Y_{t-2}^{\mathrm{T}},\ldots,Y_{t-p_k}^{\mathrm{T}}\right)^{\mathrm{T}}\end{aligned} \quad (4)$$

A number of model parameters need to be estimated. The parameter vector of the MVAR model is, in this case, $\Psi\left(\hat{\alpha}_k,\hat{\tilde{\Theta}}_k^{\mathrm{T}},\hat{\Omega}_k\right)$. Here $\hat{\alpha}_k$ are the estimated mixing weights of the K component distributions, $\hat{\tilde{\Theta}}_k^{\mathrm{T}}$ are the estimated $n\times n$ autoregressive coefficient matrices and $\hat{\Omega}_k$ are estimates of the K $n\times n$ variance covariance matrices. As discussed in Fong et al. (2007), for the purpose of identification, it is assumed that $\alpha_1\ge\alpha_2\ge\cdots\ge\alpha_K\ge0$ and $\sum_k\alpha_k=1$. In the vector X_{kt}, the first element (i.e. the 1) is a scalar quantity.

As shown in Fong et al. (2007), the equations for the expectation and maximization steps can be written as follows. In the expectation step, the missing data Z are replaced by their expectation conditional on the parameters $\tilde{\Theta}$ and on the observed data $Y_1 \ldots Y_T$. If the conditional expectation of the k^{th} component of Z_t is denoted $\tau_{t,k}$ then the expectation step is calculated according to equation (5):

Expectation Step:

$$\tau_{t,k} = \frac{\alpha_k \left|\Omega_k\right|^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2} e_{kt}^{\mathrm{T}} \Omega_k^{-1} e_{kt}\right)}{\sum_{k=1}^{K} \alpha_k \left|\Omega_k\right|^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2} e_{kt}^{\mathrm{T}} \Omega_k^{-1} e_{kt}\right)}, \; k = 1,\ldots,K \tag{5}$$

Following the expectation step, the maximization step can then be used to estimate the parameter vector $\tilde{\Theta}$. The M-step equations are defined in Fong et al. (2007) as:

Maximization Step:

$$\begin{aligned} \hat{\alpha}_k &= \frac{1}{T-p} \sum_{t=p+1}^{T} \tau_{t,k}, \\ \hat{\tilde{\Theta}}_k^{\mathrm{T}} &= \left(\sum_{t=p+1}^{T} \tau_{t,k} X_{tk} X_{tk}^{\mathrm{T}} \right)^{-1} \left(\sum_{t=p+1}^{T} \tau_{t,k} X_{tk} Y_t^{\mathrm{T}} \right), \\ \hat{\Omega}_k &= \frac{\sum_{t=p+1}^{T} \tau_{t,k} \hat{e}_{kt} \hat{e}_{kt}^{\mathrm{T}}}{\sum_{t=p+1}^{T} \tau_{t,k}} \end{aligned} \tag{6}$$

where $1 \ldots . K$. The model parameters are obtained by maximizing the log-likelihood function given in equation (3).

In addition to the MVAR, a VAR(2) model is also estimated. After estimating the models, it is possible to subject them to exogenous, pre-specified adverse macroeconomic shocks. This provides an empirical measure of how the probability of default of counterparties responds to exogenous shocks in the macroeconomic environment. To predict the response of the system, we can use a Monte Carlo simulation to generate both a baseline and a conditional adverse scenario for the probability of default. The baseline scenario is constructed by first drawing a random sample from a standard normal distribution. Through recursion of the respective VAR or MVAR model equations, it is therefore possible to generate simulated forward values of both the probability of default and the macroeconomic variables over some finite horizon period. The end result of this process is that a distribution of the probabilities of default can be constructed. The distribution thus generated can subsequently be considered as the baseline scenario.

The adverse scenario is constructed in a similar manner, except that at various periods throughout the simulation horizon exogenous shocks are applied to the individual macroeconomic variable equations. Consequently, conditional on the shocks, the distribution of the adverse scenario probability of default is governed by the dynamics of the macroeconomic variables in combination with the persistence of the shocks induced by the lagged specification of the model. This ability to generate two separate distributions for the probability of default allows for comparison of the estimated baseline and adverse scenarios when an artificial and exogenous shock is applied to a particular macroeconomic variable. The application of the exogenous shocks to the variables of the model allows us to analyze the sensitivity of the probability of default distribution to specific adverse macroeconomic developments. Under this type of deterministic approach, the response of the distribution can be evaluated for more complex macroeconomic scenarios. In any case, comparing the distributions provides information on the probable impact of macroeconomic shocks on the probability of default and can thus the procedure can be considered as a form of stress test. In order to perform the actual stress test, we must decide on some exceptional but plausible stressed scenarios. It is critical that the scenarios selected are neither too extreme nor too mild in their impact on the system because if the exogenous shocks are chosen inappropriately then the exercise will provide no relevant insight.

Three different stressed scenarios were employed with shocks being applied individually to the selected macroeconomic variables. The scenarios were chosen in order to focus on the various aspects of the transmission mechanism between the macroeconomic environment and the counterparty credit risk of the Luxembourg banking sector. The three specific scenarios include both domestic and EU level effects and are taken over a horizon of 10 quarters starting in 2011 Q3 and with the simulation ending in 2013 Q4. The scenarios are comprised of the following macroeconomic conditions:

1. A decrease in Euro area real GDP growth of magnitude -0.025 in the first quarter of 2012, followed by successive shocks of -0.028, 0.0 and 0.01 in the subsequent quarters
2. An increase in real interest rates of 100 basis points beginning in the first quarter of 2012 and a further increase of 100 basis points in 2012 Q3
3. A reduction in real property prices of magnitude 4% in 2012 Q1 and subsequent losses of 4% over the remaining quarters of 2012

Shocks of this magnitude represent particularly severe disturbances. It is important to note that if the shocks are too small, the test will provide no insight into the possible impact on the probability of default. Conversely, if the shocks are too large in magnitude, then the probability of such an event occurring would be too small and the testing exercise risks being uninformative. All shocks are applied on a quarter-to-quarter basis over the separate scenarios. For both the baseline and adverse scenarios we performed 5000 Monte Carlo simulations of the model and used the 5000 simulated probabilities of default in the last quarter of 2013 to construct the histograms. The actual simulation results for the four scenarios are displayed in figures 1 through 3.

Figure 1
Distributions of Adverse and Baseline scenarios under adverse shocks to euro area real GDP growth

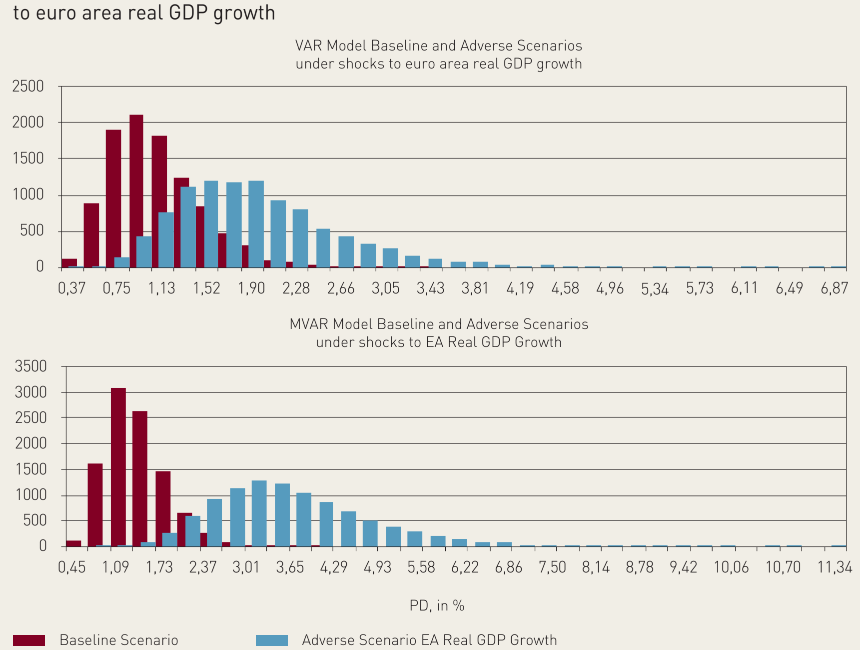


Figure 2
Distributions of Adverse and Baseline scenarios under adverse shocks to the real interest rate

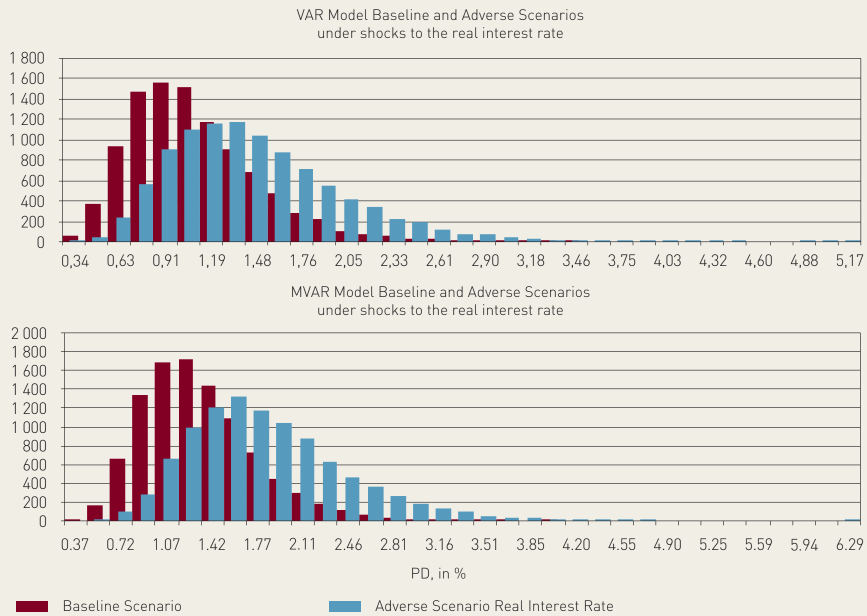
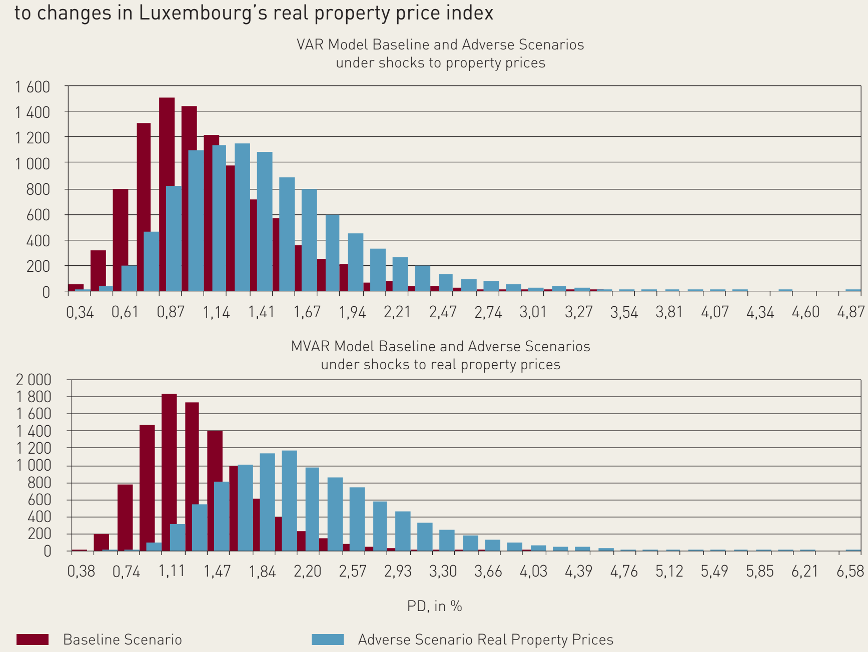


Figure 3
Distributions of Adverse and Baseline scenarios under adverse shocks to changes in Luxembourg's real property price index



For all scenarios, the histograms exhibit a characteristic shift to the right of the stressed distribution, indicating that the average probability of default under the adverse scenario increases relative to the baseline scenario. An associated increase in the standard deviation is also observed along with increased weight in the tails of the distributions. For the shock to euro area real GDP growth, in the VAR case, the mean probability of default increases from approximately 1.09% to 1.70% under the adverse scenario. The corresponding change for the MVAR estimation is from 1.09% to 3.2%. For the remaining scenarios the increase is from 1.05% to 1.42% for the VAR and 1.24% to 1.59% for the MVAR under the real interest rate scenario. For the property price shocks, the VAR distribution increases from 0.9% to 1.27% while the MVAR increases from 1.17% to 2.02%. Tail probabilities under the stressed VAR scenario do not exceed their MVAR counterparts and no scenario displays probabilities of default in excess of approximately 8.14%. Despite the severity of the scenarios, the results for the selected adverse scenarios suggest that exogenous shocks to fundamental macroeconomic variables have a limited and somewhat mild effect on the average probability of default, except in the MVAR euro area real GDP growth and property price scenarios. For instance, the largest change in average counterparty PDs occurs for the MVAR under shocks to euro area GDP growth with a change of 2.11%. Under the VAR scenarios, the largest change between the adverse and baseline scenario also occurs under the GDP scenario, but the magnitude of the change is only 0.61%. The MVAR increase is more than 3.4 times larger than that observed for the VAR model.

3. SIMULATION AND CALCULATION OF CAPITAL REQUIREMENTS

The results of the Monte Carlo simulation can also be used to gain insight into the capitalization level of the entire Luxembourg banking sector. Using equations (7) and (8) for capital requirements for corporate exposures and Basel II tier 1 capital ratios, respectively, it is possible to calculate capital requirements under the adverse scenario.

$$k_c^* = \left(LGD \times N\left[\frac{G(PD)}{\sqrt{(1-R_c)}} + \left(\frac{R_c}{(1-R_c)} \right)^{\frac{1}{2}} \times G(0.999) \right] - PD \times LGD \right)^* \times \left(\frac{1}{1-1.5b} \right) \quad (7)$$

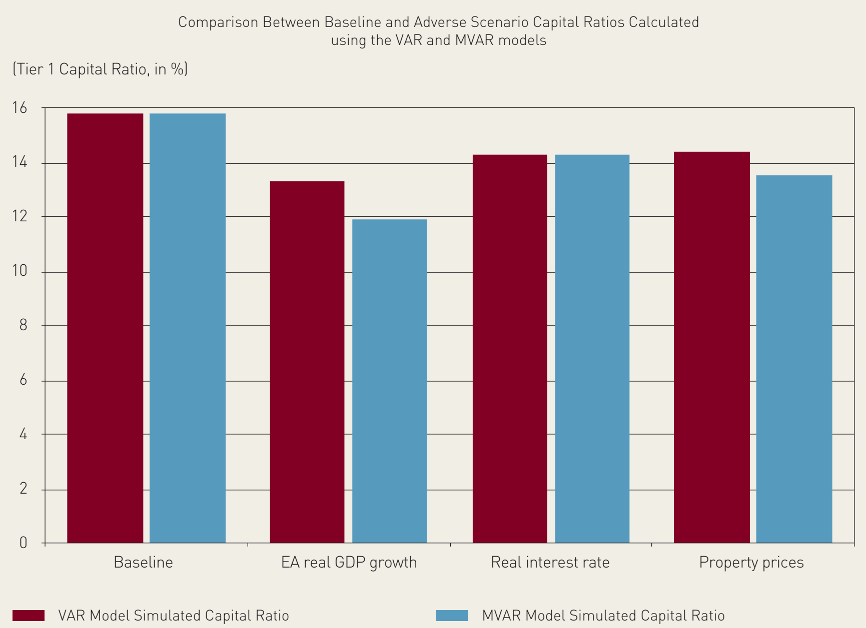
$$capital\ ratio = \frac{K + \Pi}{RWA - 12.5E^c\left(k_c - k_c^*\right)} \quad (8)$$

In equation (7), $G(PD)$ represents the inverse normal distribution with the probability of default, PD, as its argument. Here $N(\cdot)$ is the cumulative normal distribution, R_c denotes asset correlation and b is the maturity adjustment. The asterisk superscript on k denotes capital requirements under the stressed scenario. In equation (8), K denotes tier 1 capital, Π and RWA denote profit and risk weighted assets, respectively, and E^c represents corporate exposures.

To calculate the capital ratio, we use data on bank profitability, risk weighted assets, loans and the amount of tier 1 capital held by banks. As the entire sector is studied, it is important to stress these values represent average quantities. Throughout the analysis, the loss given default (LGD) is assumed to be 0.5, or 50%, and a maturity adjustment is used based on the Basel II regulations for risk-weighted assets for corporate, sovereign and bank exposures. The mean value of the probability of default values obtained from the Monte Carlo simulation is used during the calculation of the Basel II correlation and capital requirements.

Figure 4 presents a bar chart showing the banking sector capital ratios under the four stressed scenarios in comparison to the baseline scenario. There are some noticeable differences between the capital requirements calculation for the VAR and MVAR models. Empirically the difference is 1.37%, suggesting that the VAR(2) model underestimates the required amount of capital in face of exogenous shocks to euro area real GDP

Figure 4
Differences in the Capital Requirements as Evaluated Under the VAR(2) and MVAR(4,2;2,2) Models



growth. Similar, although less dramatic, results can be observed for the other variables. For the real interest rate the magnitude of the difference is 0.10% while for property prices the difference is approximately equal to 0.88%.

4. CONCLUSION

According to the empirical results in this paper, the VAR model consistently underestimates counterparty credit risk. In a simulation that applies adverse macroeconomic shocks to the econometric model, it is found that the level of Tier 1 capital required to withstand these shocks is underestimated by the VAR model. For shocks to euro area real GDP growth the magnitude of this underestimation is approximately 1.4% of Tier 1 capital. Financially, for some banks, this may represent a significant amount of capital. The underestimation of capital requirements in the case of the univariate model may demonstrate that there is an information gain provided by the MVAR model which is not present in the VAR framework. Indeed, the difference between the calculated values has its origins in the distributional assumptions underlying the VAR and MVAR models. In the context of the MVAR, the model is capturing a significant amount of the tail effects that, being based on the assumption of univariate normality, the VAR model does not capture. However, at this time there is no statistical test that we can apply to these results in order to empirically evaluate their significance.

In this study we have shown that, compared to a framework with a unimodal distribution, using the MVAR model to assess counterparty risk provides a more accurate representation of the true risk by better capturing the more extreme movements observed in empirical measures of credit risk. The estimations of Tier 1 capital performed using univariate VAR models consistently underestimate the required amount of Tier 1 capital needed to withstand adverse macroeconomic shocks. These differences need to be taken into account since they have significant consequences from a regulatory perspective.

REFERENCES

Dempster, A.P., N.M. Laird and D.B. Rubin. (1977). "Maximum Likelihood from Incomplete Data via the EM Algorithm," *Journal of the Royal Statistical Society. Series B (Methodological)*, Vol. 39, No. 1. pp. 1-38.

Fong, P.W., W.K. Li, C.W. Yau and C.S. Wong. (2007). "On a mixture vector autoregressive model", *The Canadian Journal of Statistics*, Vol. 35, No. 1, pp. 135-150.

Rouabah, A. and J. Theal. (2010). "Stress Testing: The Impact of Shocks on the Capital Needs of the Luxembourg Banking Sector", *Banque centrale du Luxembourg*, Working Paper No. 47.

Wong, C.S. and W.K. Li. (2000). "On a mixture autoregressive model," *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, Vol. 62, No. 1, pp. 95-115.