1. MVAR IMPULSE RESPONSE FUNCTIONS COMPARED TO A VAR MODEL: A FIRST ASSESSMENT OF THE MACRO-FINANCIAL LINKAGES OF THE BANKING SECTOR IN LUXEMBOURG

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ABSTRACT

In this study we provide a first assessment of the macro-financial linkages of the Luxembourg banking sector. To capture the links between banking sector counterparty credit risk and the macroeconomic environment, both a normal VAR model as well as an MVAR model have been estimated using data that links multiple macroeconomic variables to banking sector counterparty risk. The macroeconomic data include a Luxembourg residential property price index, euro area and Luxembourg real GDP growth and the EURIBOR 3 months interest rate. Based on the model estimation output, impulse response functions (IRFs) have been computed to illustrate the response of counterparty credit risk to Cholesky one standard deviation macroeconomic shocks. The results of the IRF analysis conform to the expected effects of the shocks on banking sector counterparty default risk. The results also highlight the importance of accounting for macro-financial linkages as input into macro-prudential policymaking decisions given their ability to shed light on the interactions between financially relevant variables and the broader economy.

INTRODUCTION

Following the lessons learned during the crisis, stress testing has become common practice among financial supervisory authorities and it represents an important component in the overall process of macro-prudential surveillance and assessment of risks. Stress testing helps to facilitate authorities' understanding and assessment of how the regulatory capital ratio of banks may respond to severe but plausible macroeconomic shocks. If a bank's capital ratio is assessed, based on both the quantitative evidence of a stress test complemented by expert judgment, to be insufficient to withstand such shocks, supervisory authorities may require the bank to hold additional capital as a buffer against adverse events.

To assess the resilience and counterbalancing capacity of the Luxembourg banking sector to adverse macroeconomic shocks, the Banque centrale du Luxembourg (BCL) regularly employs a macro-prudential stress testing framework. This framework is based on a mixture vector autoregressive model (MVAR). The MVAR model is well-suited for stress testing applications as it uses a weighted combination of VAR models in order to better capture the tail risks that are associated with systemic risk and increased fragility. The MVAR model and the stress testing framework used here are both described in more detail in Guarda, Rouabah and Theal (2013)[2].

In addition to facilitating the assessment of the effects of adverse shocks on banks' core equity tier 1 (CET1) ratios, the MVAR model can also help to provide some insight into the channels by which banks'

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2 *P. Guarda, A. Rouabah and J. Theal. "A mixture vector autoregressive framework to capture extreme events in macroprudential stress tests", Journal of Risk Model Validation, 12/2013; 7(4):1-31.*

counterparty credit risk may be affected by the macroeconomic environment. These interconnections are called macro-financial linkages and they represent the channels through which financial stability indicators, like the probability of default and relevant macroeconomic variables, such as GDP growth, interest rates and property prices, interact and affect one another.

For macro-prudential authorities, it is important to understand those macro-financial linkages that could have a detrimental effect on the banking sector and its subsequent ability to extend credit to the economy. In particular, the creditworthiness of a bank's loan counterparties is one of the significant factors that determine a bank's willingness to lend. In addition, counterparty credit risk can be used in the evaluation of the resilience of a bank to adverse economic or financial shocks. If counterparty risk is elevated during a period of stress, banks may need to increase their CET1 levels in order to bolster their counterbalancing capacity in the event that an adverse macroeconomic scenario materializes. In addition, if banks' lending activities are assessed to be systemically relevant for the stability of the financial system, a deepened understanding of the macro-financial linkages may also be used to gauge the potential need for authorities to apply macro-prudential instruments, such as the countercyclical capital buffer.

DATA AND ESTIMATION OF THE VAR AND MVAR MODELS

In this study, both a VAR and MVAR model were estimated in order to establish the relationships between the financial and macro variables. The setup of both models was similar and they consist of a joint system of five linear equations for the probability of default, euro area real GDP growth, the real growth rate of Luxembourg GDP, the real interest rate and the growth rate of a Luxembourg property price index. In the MVAR case, the model is a weighted combination of two individual VAR models rather than a single VAR estimation. This specification allows the component VAR models to capture feedback effects between the macroeconomic variables and the probability of default series. Furthermore, the use of two lags of the endogenous variable in each equation of the respective models allows us to capture the persistence and transmission of exogenous shocks through the system.

Mathematically, the basic VAR model specification used in this study takes the following form for both the VAR and MVAR models:

$$Y_t = c + \Theta_1 Y_{t-1} + \Theta_2 Y_{t-2} + \cdots + \Theta_p Y_{t-p} + e_t$$

The data used to estimate the models consisted of proxies for historical probabilities of default (PD) calculated on a quarterly basis over the period spanning the first quarter of 1995 until the fourth quarter of 2014. In addition to the probability of default, the MVAR and VAR models incorporated data on euro area real GDP growth, the real interest rate and the change in real property prices for a Luxembourg residential property price index. Given that Luxembourg is a small, open economy with a large number of foreign banks, the series for euro area real GDP growth effectively provides an appropriate explanatory variable for the profitability of the banking sector in Luxembourg. Property prices and the real interest rate have been used to capture balance sheet effects as well as changes in counterparty creditworthiness. The choice of variables permits the stress testing framework to capture the feedback effects between the probability of default series and the macroeconomic variables and hence facilitates an assessment of the macro-financial linkages and possible variable interactions.

The results of the estimation of the VAR model are given in the accompanying table 1. The column headings define the dependent variable equations while those in the rows show the lagged independent variables for each equation in the VAR. A total of two lags were used for the estimation. Coefficients displayed in bold text indicate statistical significance while quantities in italic text provide the standard errors of the coefficient estimates.

Table 1:
VAR Model Coefficient Estimates

	YJT	EURO AREA REAL GDP GROWTH	LUX. REAL GDP GROWTH	REAL INTEREST RATE	REAL PROPERTY PRICE GROWTH
Yjt (-1)	**0.924680**	**0.014415**	**0.089937**	-0.003952	-0.002164
	(0.11559)	*(0.00685)*	*(0.02450)*	*(0.00731)*	*(0.01523)*
Yjt(-2)	0.046602	**-0.015577**	**-0.095200**	0.004462	-0.000255
	(0.11290)	*(0.00669)*	*(0.02393)*	*(0.00714)*	*(0.01487)*
euro area real GDP growth (-1)	3.471511	**0.389929**	0.772439	-0.197604	-0.054502
	(2.57836)	*(0.15282)*	*(0.54652)*	*(0.16308)*	*(0.33961)*
euro area real GDP growth (-2)	2.828341	-0.140901	-0.609060	**0.278556**	-0.262784
	(2.14864)	*(0.12735)*	*(0.45543)*	*(0.13590)*	*(0.28301)*
Lux. Real GDP growth (-1)	-0.562061	0.062972	**-0.448872**	0.068693	**0.175949**
	(0.62346)	*(0.03695)*	*(0.13215)*	*(0.03943)*	*(0.08212)*
Lux. Real GDP growth (-2)	-0.363419	0.061000	0.093955	0.009486	0.126832
	(0.64770)	*(0.03839)*	*(0.13729)*	*(0.04097)*	*(0.08531)*
real interest rate(-1)	0.202098	-0.206449	-0.168042	**1.041587**	-0.427649
	(1.99069)	*(0.11799)*	*(0.42195)*	*(0.12591)*	*(0.26220)*
real interest rate(-2)	-0.757861	**0.238551**	0.344225	-0.121623	0.382018
	(1.99606)	*(0.11831)*	*(0.42309)*	*(0.12625)*	*(0.26291)*
property price growth (-1)	0.174925	-0.015176	-0.312664	-0.107577	**0.469779**
	(0.97210)	*(0.05762)*	*(0.20605)*	*(0.06149)*	*(0.12804)*
property price growth (-2)	**1.983934**	0.066227	0.346888	0.075447	0.248547
	(0.99085)	*(0.05873)*	*(0.21002)*	*(0.06267)*	*(0.13051)*
C	0.112427	0.006019	0.033052	-0.002671	0.013064
	(0.08928)	*(0.00529)*	*(0.01892)*	*(0.00565)*	*(0.01176)*

Source: BCL.

The estimation results show that increases in the growth rate of euro area GDP result in an increase in the value of the transformed variable Y_t which is inversely related to the probability of default. Correspondingly, a decrease in euro area economic growth could result in a positive increase in the probability of default, thereby increasing the risk for the Luxembourg banking sector given its sensitivity to the euro area macroeconomic environment owing to the large number of foreign banking groups in the financial sector. A similar effect can be observed for the property price index growth. In addition, an increase in the real interest rate will negatively affect Y_t given that the sum of the coefficients of the real interest rate variable is less than one. Finally, although not statistically significant, the coefficient on the lagged value of Y_t was found to be positive, suggesting that exogenous shocks will persist for a time horizon exceeding the duration of the shock. For the remaining macroeconomic variable equations the model seems to capture the expected dynamics between the macroeconomy and the probability of default. We note, however, that the sign on Luxembourg real GDP growth is the inverse of that which is expected; i.e. it is negative rather than positive. This is due to the presence of a large number of foreign branches and subsidiaries that, although located in Luxembourg, do not undertake activities that are linked to Luxembourg real GDP growth. In this manner, they may also be potentially subject to inward spillovers from the euro area rather than being negatively affected by economic developments in the Luxembourg economy.

Having estimated the models, we can now compute the impulse response functions in order to assess the macro-financial linkages between Luxembourg counterparty credit risk and the economic variables.

IMPULSE RESPONSE FUNCTIONS OF THE MVAR MODEL

To illustrate the impulse response functions (IRFs) and how they can help to understand the linkages between financial stability variables like the probability of default and macroeconomic developments, we first consider the specification of the MVAR model which can be written as a weighted combination of VAR(p) models in the following manner:

$$F\left(y_t \middle| \Im_{t-1}\right) = \sum_{k=1}^{K} \alpha_k \Phi\left(\Omega_k^{-1/2}\left(Y_t - \Theta_{k0} - \Theta_{k1}Y_{t-1} - \Theta_{k2}Y_{t-2} - \ldots - \Theta_{k1p}Y_{t-p_k}\right)\right)$$

Here y_t is the conditional expectation of Y_t, p_k is the autoregressive lag order of the k^{th} component, $\Im_{t-1}$ is the available information set up to time $t-1$, $\phi(\cdot)$ is the cumulative distribution function of the multivariate Gaussian distribution, α_k is the mixing weight of the k^{th} MVAR component distribution, Θ_{k0} is an n-dimensional vector of constant coefficients and $\Theta_{k1},\ldots,\Theta_{kp_k}$ are the autoregressive coefficient matrices of the k^{th} component distribution. Lastly, Ω_k is the $n \times n$ variance-covariance matrix of the k^{th} component distribution.

The IRF from a VAR model represents the deterministic response of the model variables to a standardized shock applied to one of the variables used in the estimation of the model. Because the variables of a VAR form a system of equations, studying the IRF functions of an econometric model helps to facilitate an understanding of the response of a variable (or variables) to an impulse – in this case a exogenous macroeconomic shock – on one of the other variables of the model. In the context of analyzing macro-financial linkages an increase in, for example, the interest rate or a negative shock to GDP may lead to an increase in counterparty credit risk levels. Depending on their regulatory capital level, the resulting increased credit risk may oblige banks to enhance their resilience through various measures, including through the application of macro-prudential measures by national authorities under the CRD IV/CRR framework.

The actual IRF functions are derived based on the estimated coefficient matrices of the MVAR model. In order to obtain a general expression for the impulse response function a VAR(p) model (or equivalently the component VAR models of the MVAR) can first be written in moving average (MA) form as follows;

$$y_t = \sum_{i=0}^{\infty} \Psi_i e_{t-1}$$

Here $\psi_0 = I_n$ and ψ_i is the i^{th} coefficient matrix of the MA representation of the VAR model. By extending the formula to n periods (i.e. the horizon of the impulse function) we obtain a general expression y_{jt} for over the entire impulse function horizon:

$$y_{t+n} = \sum_{i=0}^{\infty} \Psi_i e_{(t+n)-1}$$

It follows that the actual IRF at period n is therefore given as:

$$\{\Psi_n\}_{i,j} = \frac{\partial y_{it+n}}{\partial e_{jt}}$$

This equation gives the response of $y_{i,t+n}$ to a shock in y_{jt} under the condition that all other variables are held constant, thereby isolating the response of individual variables. In practice, the IRF can be computed by using Cholesky decomposition in order to orthogonalize the impulses and subsequently trace the effect of a one standard error shock through the VAR system. Given that the MVAR model consists of a weighted combination of VAR models, the IRFs for each component of the MVAR can be evaluated individually and then be combined according to the MVAR weighting factors, α_k.

Following the estimation of the MVAR stress testing model, the IRFs for each component VAR were computed by applying a one standard deviation shock to the individual macroeconomic variables then evaluating the model equations (i.e. by computing the responses of y_{jt} to the impulse) over a period of 25 quarters. As described above, the individual MVAR IRFs were combined according to the estimated model weights, α_k. In addition to the MVAR model a normal VAR(p) was also estimated and the IRFs for the VAR were computed for purposes of comparison. In computing the IRFs, the variable Y_{jt} was used as the shock target. We recall that this variable is related to the probability of default by the following equation:

$$y_t = \ln\left(\frac{1-p_t}{p_t}\right)$$

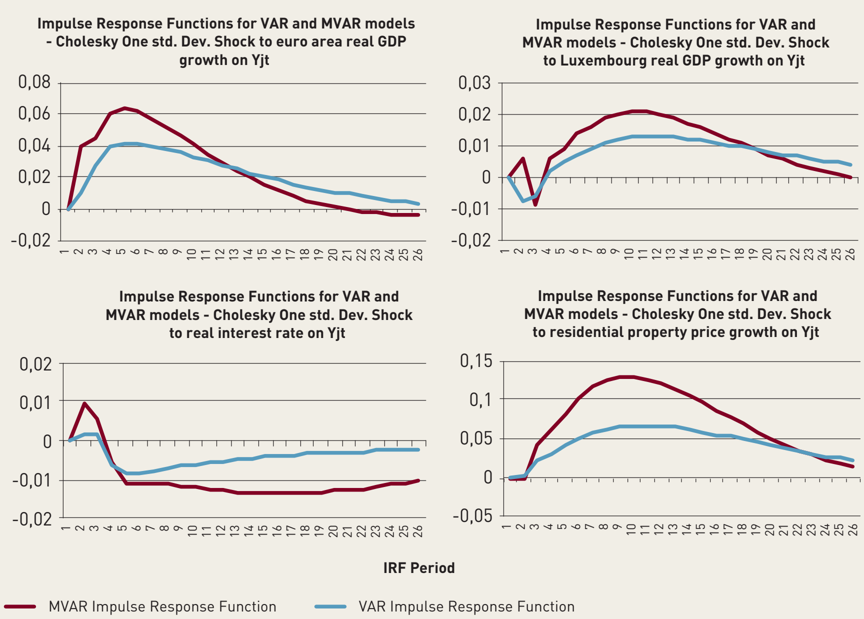
Here the probability of default, p_t, is transformed such that y_t takes on values in the interval $-\infty < y_t < \infty$. The result is that, after the transformation, y_t and p_t will be inversely related; a relationship that will also apply to the first difference of the y_t series.

COMPUTATION OF THE IMPULSE RESPONSE FUNCTIONS

The impulse response functions of the MVAR model were estimated and used to provide an indication of the possible channels of interaction between counterparty creditworthiness and the relevant macroeconomic variables that are of importance for the banking sector in Luxembourg. These latter factors include euro area real GDP growth, Luxembourg real GDP growth, the real (EURIBOR 3 months) interest rate and a residential real estate price index for Luxembourg.

Figure 1 below shows the IRFs for both the MVAR and VAR models resulting from a Cholesky one standard deviation shock to the four individual macroeconomic variables and the resulting response of Y_{jt}; the logit-transformed value of the probability of default proxy described previously.

Figure 1
Comparison of VAR and MVAR IRFs over a horizon of 25 quarters



Source: BCL Calculations.

The response of y_{jt} to orthogonal shocks to euro area real GDP growth, Luxembourg real GDP growth, the real interest rate and the growth of Luxembourg residential property prices suggest that there are interesting and potentially important differences between the IRFs computed using the two different models. Notably, in all four of the graphs, the amplitude of the MVAR IRF exceeds that of the VAR model equivalent.

It is important to underscore that in the graphs a positive shock to euro area real GDP growth implies a decline in the probability of default since the PD is inversely related to y_{jt}. As a result, the increase in euro area real GDP growth is consistent with a decline in counterparty credit risk. In addition, the effect of the shock is temporary as the impact on y_{jt} begins to decline after about 5 quarters, eventually returning to zero. For the shock resulting from Luxembourg real GDP growth, the effect on y_{jt} is similar for the MVAR (an initial increase followed immediately by a decline), but for the VAR there is an initial decline. The conflicting results are due to the volatile nature of the Luxembourg GDP series. Nevertheless, the impact of the shock remains transitory and the effect eventually declines to zero in a manner similar to that observed in the case of euro area real GDP. In any event, the VAR regression coefficient for Luxembourg real GDP growth in the equation for Y_{jt} is not statistically significant. Furthermore, the wrong sign on this particular regression coefficient can be explained by the disconnection between the banking sector and Luxembourg real GDP; the latter resulting from the large number of foreign banks that are not connected to the domestic economy and whose banking activities are internationally oriented.

For the real interest rate shock, the VAR and MVAR responses of y_{jt} are very similar with the exception that the amplitude of the MVAR IRF exceeds that of the VAR (both on the positive and negative sides). The interpretation here is that the impact of the shock is more significant and more sustained in the MVAR case, illustrating the model's ability to capture the tail events associated with the effects of systemic stress and tail risk. Again, the impact eventually dies out towards the end of the IRF horizon of 25 quarters. It is important to take account of the fact that the response by y_{jt} to an unexpected and substantial interest rate shock may be significant given the long and sustained period of low interest rates within the European Union. The effects of an unexpectedly large interest rate increase could potentially have a substantial impact on counterparty credit risk levels for the banking sector.

For the real property price IRF, the MVAR and VAR models also give similar results. However, the effect of the impulse on y_{jt} only materializes approximately 2 quarters after the onset of the unit shock. The interpretation is that there is a delay in the pass through of the shock to real estate prices which could be attributed to the high net worth of Luxembourg households and their subsequent debt servicing capacities. Nevertheless, the amplitude of the shock under the MVAR remains elevated compared to the VAR, suggesting that if some banks are highly concentrated in mortgage lending the materialization of a possible risk related to real estate lending could not be ruled out, especially against the background of persistently low interest rates.

CONCLUSION

The impulse response functions of a VAR model help to provide insights into the dynamics underlying the links between financial stability indicators and the macroeconomic environment. They permit authorities to assess how counterparty risk may be affected by developments related to macroeconomic conditions and vice versa. In the case of the MVAR, the impulse response functions seem to be able to capture additional aspects of risk that a normal VAR model IRF cannot as has been seen in the increased amplitudes of the comparable IRFs as well as the response of the credit risk variable to a shock in real GDP growth, for example.

A deep understanding of the macro-financial linkages between the economy and the banking sector is an important element in authorities' assessment of systemic risk. The reason for the high importance is because developments in the macroeconomic environment can ultimately help to determine the regulatory requirements of banks. In addition, the linkages need to be understood in order for regulatory authorities to make informed policy decisions that can help to mitigate the severe systemic risk that is known to precipitate financial crisis episodes. In addition, a detailed understanding of these economic and financial linkages can help to guide the use and application of macro-prudential tools and to assess their potential effects on the real economy. Such information will be invaluable to bodies such as national systemic risk committees that are responsible for the implementation of macro-prudential measures in the context of CRD IV and the CRR framework in individual EU Member States.